Formation Control of Nonholonomic Mobile Robots with Omnidirectional Visual Servoing and Motion Segmentation

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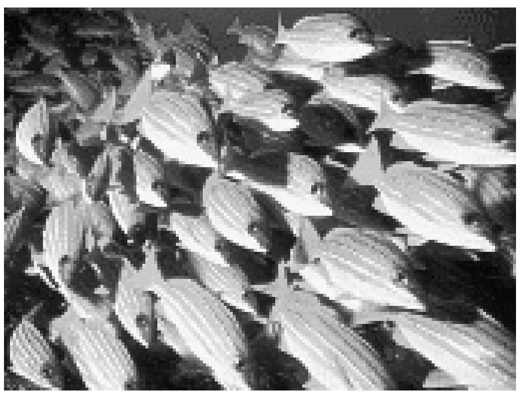
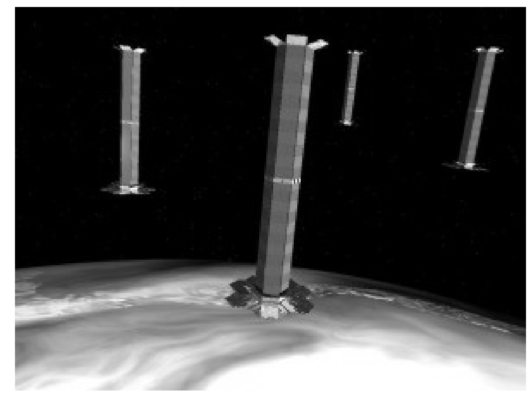
Motivation





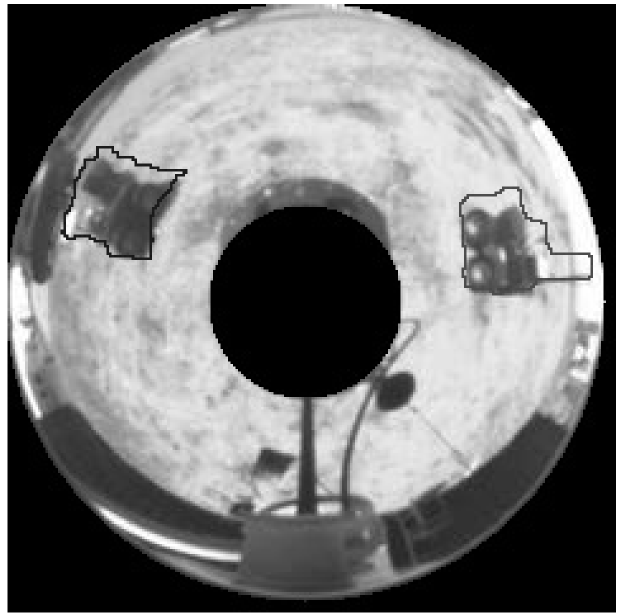
- Formation control is ubiquitous
  - Safety in numbers
  - Decreased aerodynamic drag
  - Higher traffic throughput
  - Applications in defense, space exploration, etc

Previous Work

- Formation control has a very rich literature (short list):

- String stability for line formations [Swaroop et.al. TAC96]
  - Formation can become unstable due to error propagation
- Mesh stability of UAVs [Pant et.al. ACC01]
  - Generalization of string stability to a planar mesh
- Input-to-state stability [Tanner et.al ICRA02]
  - Structure of interconnections and amount of information communicated affects ISS
- Feasible formations [Tabuada et.al. ACC01]
  - Differential geometric conditions on feasibility of formations under kinematic constraints of mobile robots
- Vision-based formation control [Das et.al. TAC02]
  - Leader position estimated by vision; Formation control in task space

Our Approach

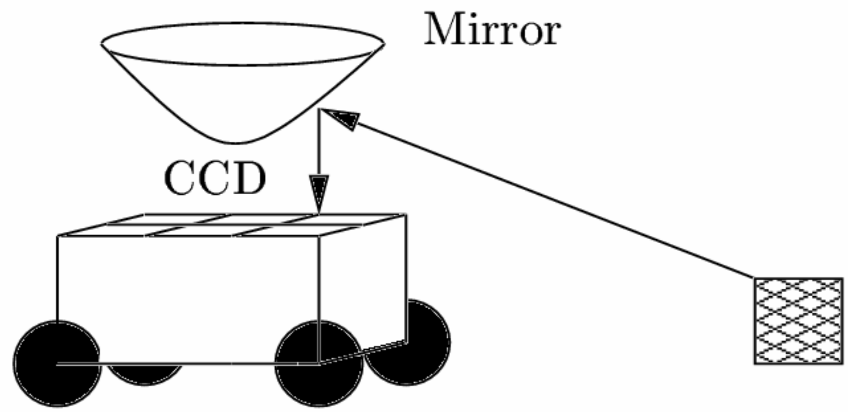


- Distributed formation control (no explicit communication)
- Formation specified in image plane of each follower
- Multi-body motion segmentation to estimate leader position
- Followers employ tracking controller in the image plane
- Naturally incorporate collision avoidance by exploiting geometry of omni-directional images

Outline

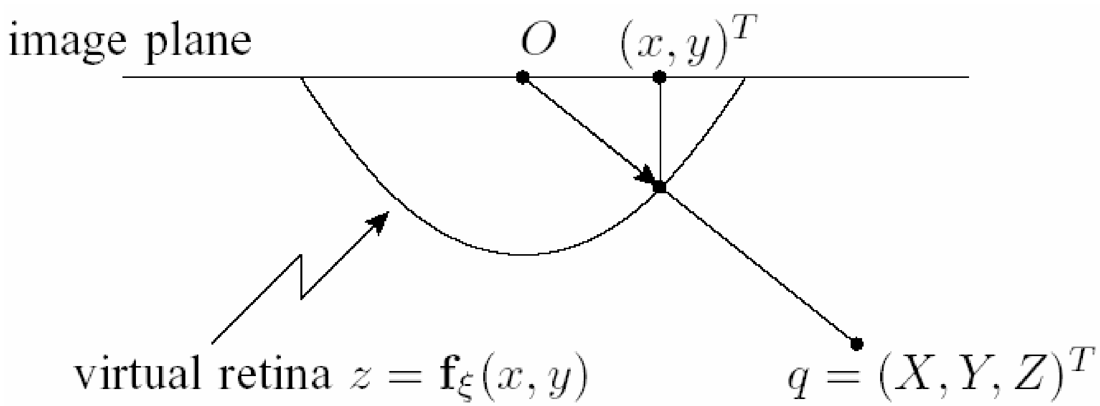
- Omnidirectional vision
  - Central panoramic cameras, back-projection ray
  - Central panoramic optical flow equations
  - Multi-body motion segmentation
- Distributed formation control
  - Leader-follower dynamics in image plane
  - Feedback linearization control design
  - Collision avoidance using navigation functions
- Experimental results
  - Motion segmentation of robots in real sequence
  - Vision-based formation control simulations

Central Panoramic Camera



- Catadioptric camera is lens-mirror combination
- Central panoramic: single effective focal point
  - Parabolic mirror, orthographic lens
  - Hyperbolic camera, perspective lens
- Efficiently compute back-projection ray associated with each pixel in image

Central Panoramic Optical Flow



Central Panoramic Projection Model

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-Z+\xi\sqrt{X^2+Y^2+Z^2}} \begin{bmatrix} X \\ Y \end{bmatrix}$$

- Optical flow induced by a planar camera motion with velocities $\Omega = (0, 0, \Omega_z)^T$ and $V = (V_x, V_y, 0)^T$

Central Panoramic Optical Flow

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -y \\ x \end{bmatrix} \Omega_z + \frac{1}{\lambda} \begin{bmatrix} 1 - \rho x^2 & -\rho xy \\ -\rho xy & 1 - \rho y^2 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix}$$

$$\rho = \frac{\xi}{1+z}$$

Central Panoramic Motion Segmentation

- Optical flows of pixels $i = 1, \ldots, n$ in frames $j = 1, \ldots, m$ live in a 5 dimensional subspace.
- Optical flows can be factorized into structure and motion $\begin{bmatrix} \dot{x}_{ij} & \dot{y}_{ij} \end{bmatrix} = S_i M_j^T$

$$S_i = \begin{bmatrix} x_i & -y_i & \frac{1-\rho_i x_i^2}{\lambda_i} & \frac{-\rho_i x_i y_i}{\lambda_i} & \frac{1-\rho_i y_i^2}{\lambda_i} \end{bmatrix} \in \mathbb{R}^{1\times 5}$$

$$M_j = \begin{bmatrix} 0 & \Omega_{zj} & V_{xj} & V_{yj} & 0 \\ \Omega_{zj} & 0 & 0 & V_{xj} & V_{yj} \end{bmatrix} \in \mathbb{R}^{2\times 5}.$$

- Given k independent motions

$$W \triangleq \left[\begin{array}{cc|c|cc} \dot{x}_{11} & \dot{y}_{11} & \cdots & \dot{x}_{1m} & \dot{y}_{1m} \\ \vdots & \vdots & & \vdots & \vdots \\ \dot{x}_{n1} & \dot{y}_{n1} & \cdots & \dot{x}_{nm} & \dot{y}_{nm} \end{array}\right] = \begin{bmatrix} S_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & S_k \end{bmatrix} \begin{bmatrix} M_1^T \\ \vdots \\ M_k^T \end{bmatrix} = SM^T$$

- Number of independent motions is obtained as:

$$k = \frac{1}{5}\text{rank}(W)$$

Central Panoramic Motion Segmentation

- Independent motions live in 5 dimensional subspaces of a higher-dimensional subspace
- Motion segmentation can be solved using Generalized Principal Component Analysis
  - Project onto a subspace of dimension 6
  - Apply GPCA: fit and differentiate a polynomial

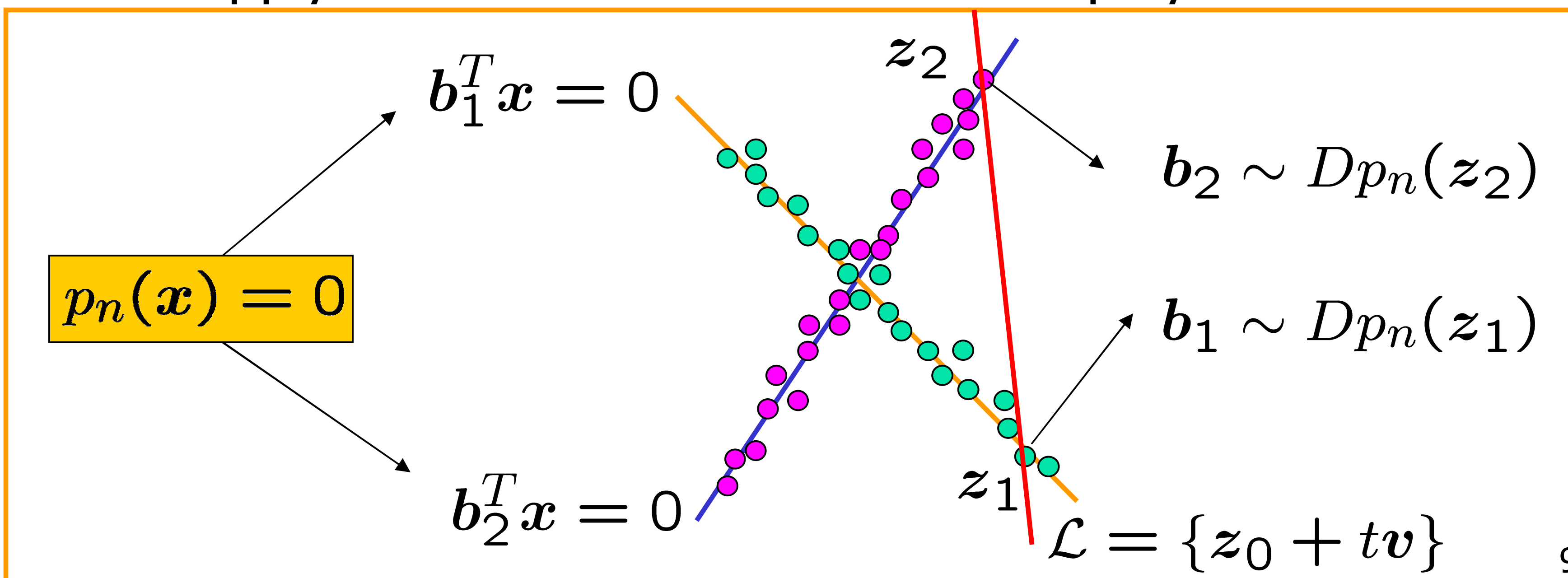
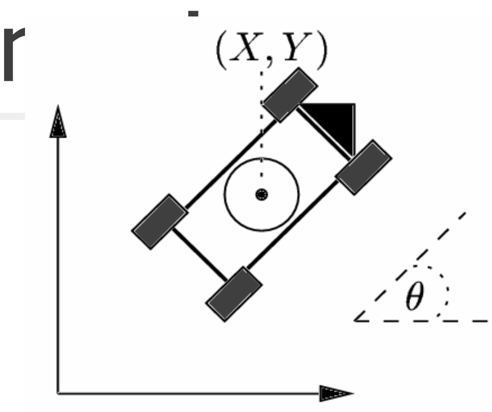


Image Leader-Follower Dyr



- Kinematic model $\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix} = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} v, \quad \dot{\theta} = \omega$
- Inputs $v \in \mathbb{R},\ \omega \in \mathbb{R}$
- Leader position $(x, y)^T$ in follower's camera

Central panoramic leader-follower dynamics

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = -\begin{bmatrix}\frac{1-\rho x^2}{\lambda} & -y\\ \frac{-\rho xy}{\lambda} & x\end{bmatrix}\begin{bmatrix}v_f\\ \omega_f\end{bmatrix} + \begin{bmatrix}\frac{1-\rho x^2}{\lambda} & \frac{-\rho xy}{\lambda} & -y\\ \frac{-\rho xy}{\lambda} & \frac{1-\rho y^2}{\lambda} & x\end{bmatrix}\begin{bmatrix}F_{\ell f}\\ \omega_\ell\end{bmatrix}$$

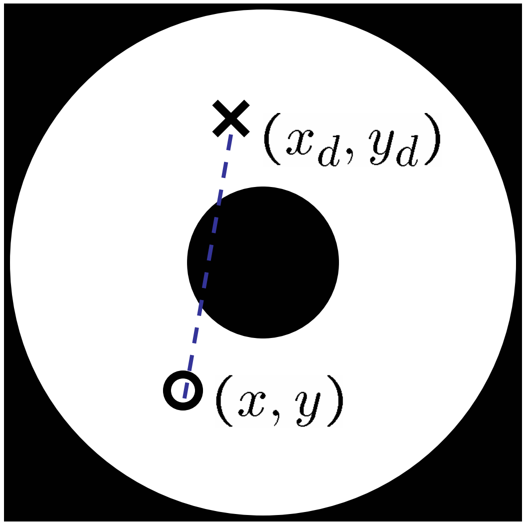
- Write as drift-free control system:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = H(x, y)\mathbf{u}_f + \mathbf{d}_{\ell f}$$

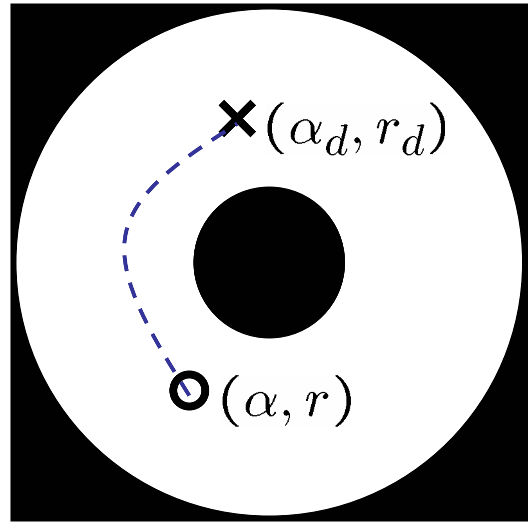
- Recover leader velocity using optical flow of background

$$\mathbf{d}_{\ell f} = \begin{bmatrix}\dot{x}_\ell\\ \dot{y}_\ell\end{bmatrix} - H(x_\ell, y_\ell)H(x_w, y_w)^{-1}\begin{bmatrix}\dot{x}_w\\ \dot{y}_w\end{bmatrix}$$

Omnidirectional Visual Servoing

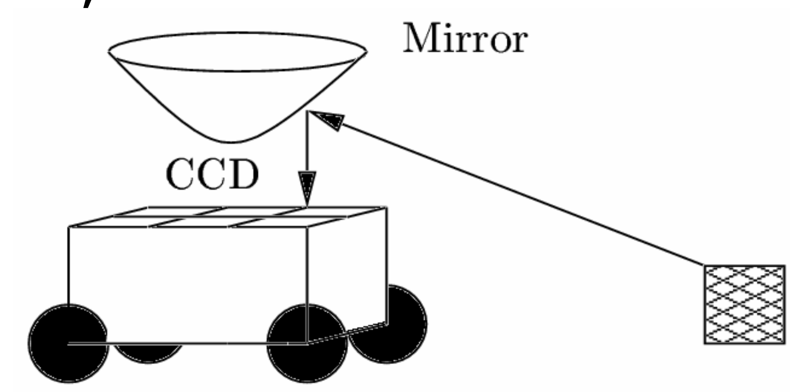


Cartesian coordinates



Polar coordinates

- Controlling in Cartesian coordinates, leader trajectory intersects circle
- Controlling in polar coordinates, follower mostly rotates
- Trajectory passing through inner circle is a collision



Omnidirectional Visual Servoing

- Leader position (α, r), desired leader position (α_d, r_d)

Feedback Linearization Control Law in Polar Coordinates

$$\mathbf{u}_f = \begin{bmatrix} \frac{\lambda}{(1-\rho r^2)\cos(\alpha)} & 0 \\ \frac{\sin(\alpha)}{r(1-\rho r^2)\cos(\alpha)} & 1 \end{bmatrix} \left(\begin{bmatrix} k_1(r - r_d) \\ k_2(\alpha - \alpha_d) \end{bmatrix} + \tilde{\mathbf{d}}_{\ell f} \right)$$

Degenerate configurations

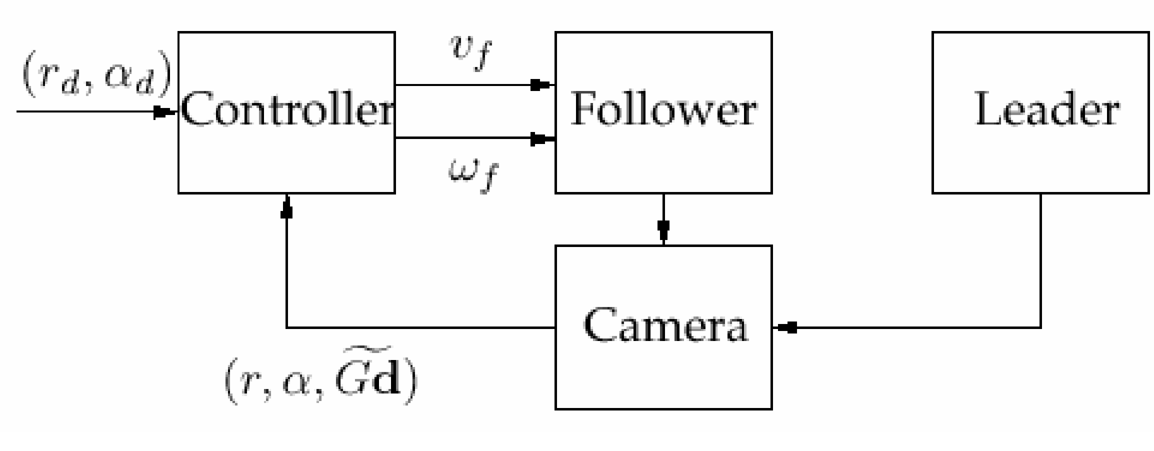
- $\cos(\alpha) = 0$ due to nonholonomy of mobile robot
- Robot can not move sideways instantaneously
- $r = 1$ due to geometry of central panoramic cameras
- Corresponds to horizon points at infinity

Omnidirectional Visual Servoing

- Can avoid degenerate configurations with pseudo-feedback linearizing control law

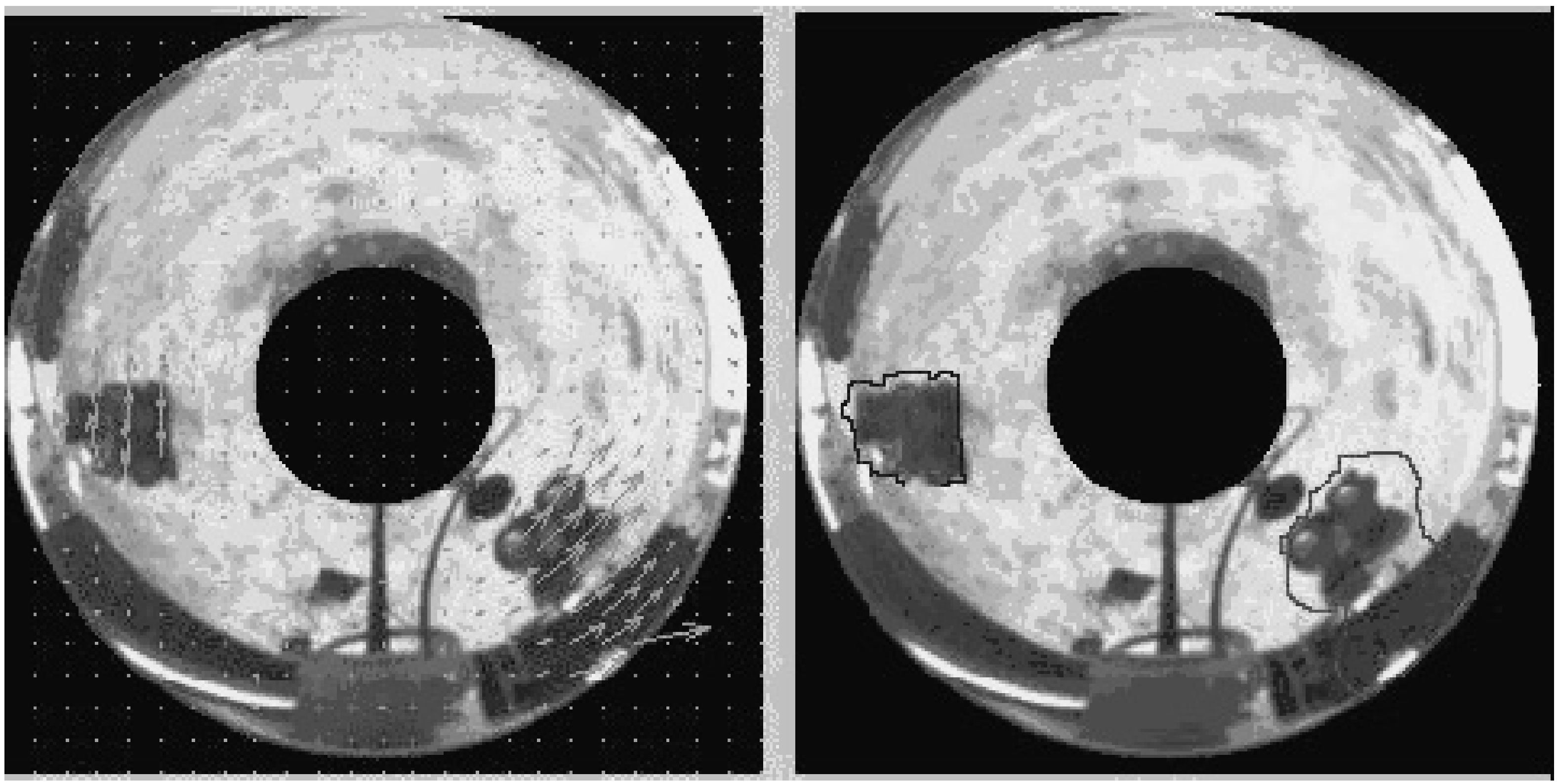
$$\mathbf{u}_f = \begin{bmatrix} \frac{\lambda\cos(\alpha)}{(1-\rho r^2)} & 0 \\ \frac{\sin(\alpha)\cos(\alpha)}{r(1-\rho r^2)} & 1 \end{bmatrix} \left(\begin{bmatrix} k_1(r-r_d) \\ k_2(\alpha-\alpha_d) \end{bmatrix} + \tilde{\mathbf{d}}_{lf} \right)$$

- However, the formation is only Input-to-State Stable (ISS)
- Can easily modify control law to achieve collision avoidance by using a Navigation Function



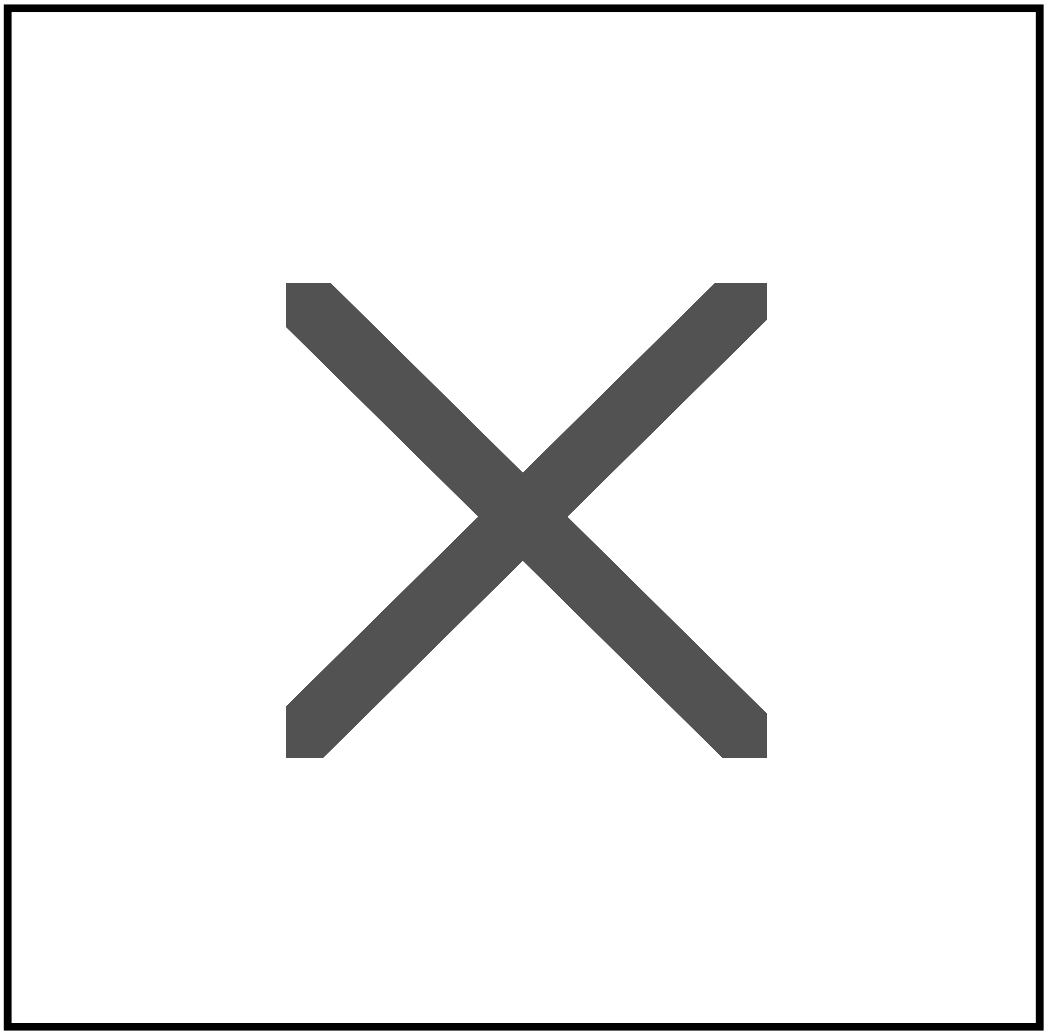
Experimental Results

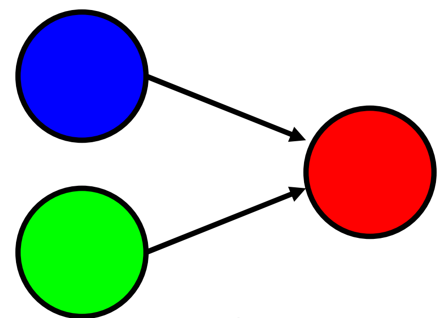
- Multi-body motion segmentation

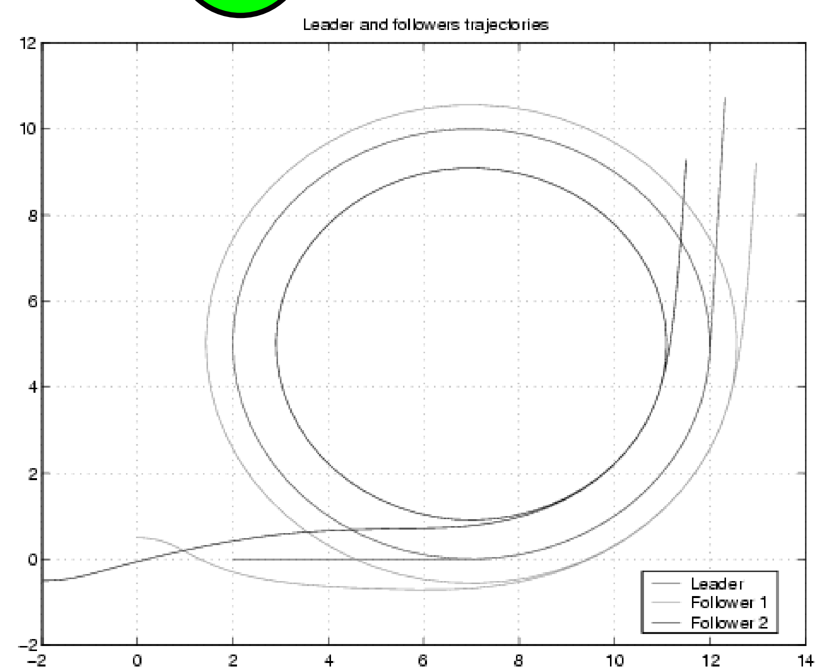


Wedge Formation

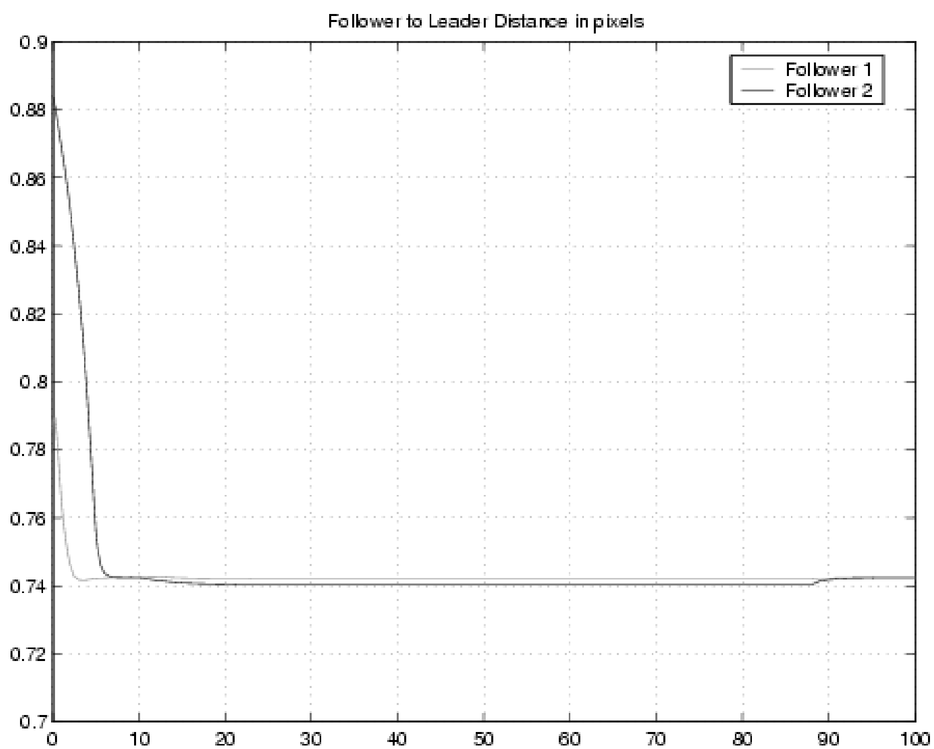
- Green follows red $r_d = 1/\sqrt{2}, \qquad \theta_d = \pi/6$
- Blue follows red $r_d = 1/\sqrt{2}, \qquad \theta_d = -\pi/6$



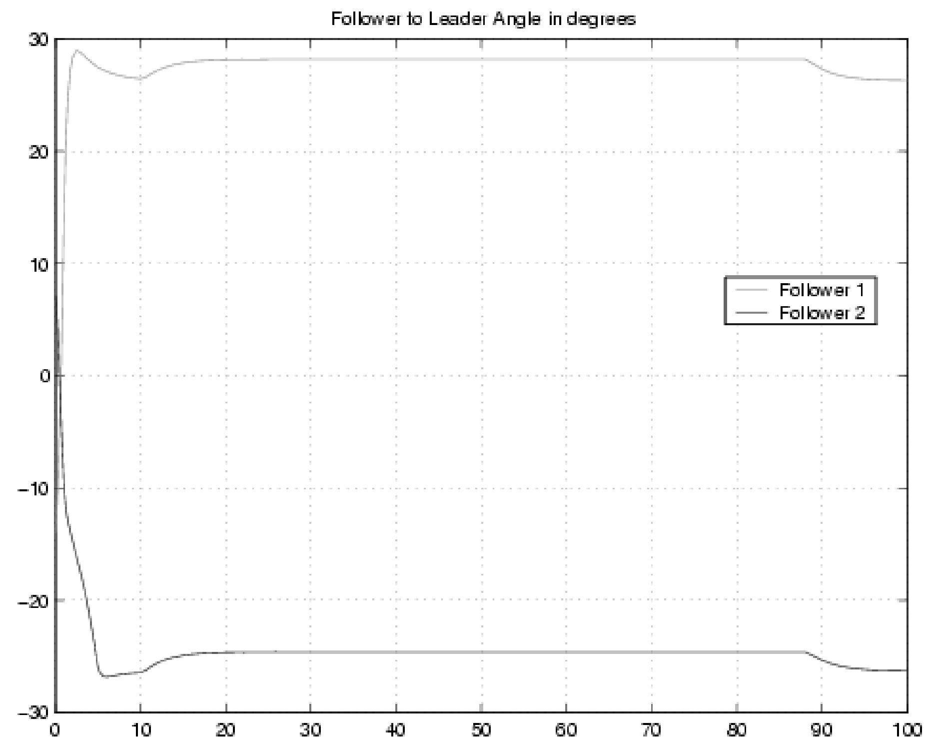




Wedge Formation



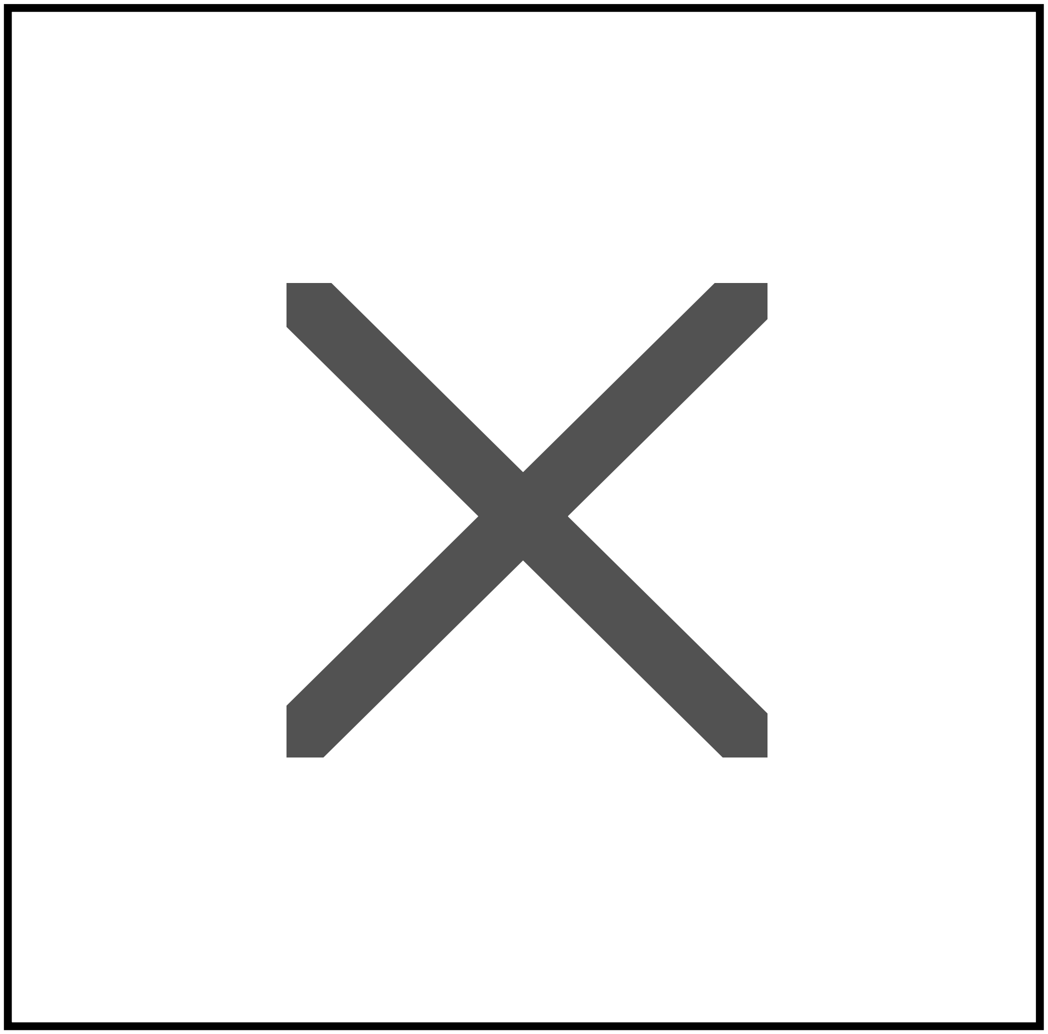
Distance to leader
(in pixels)

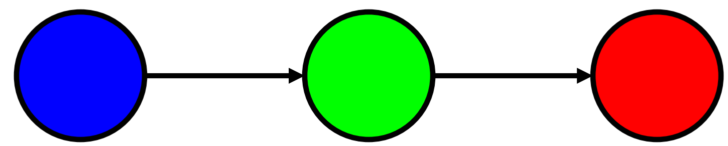


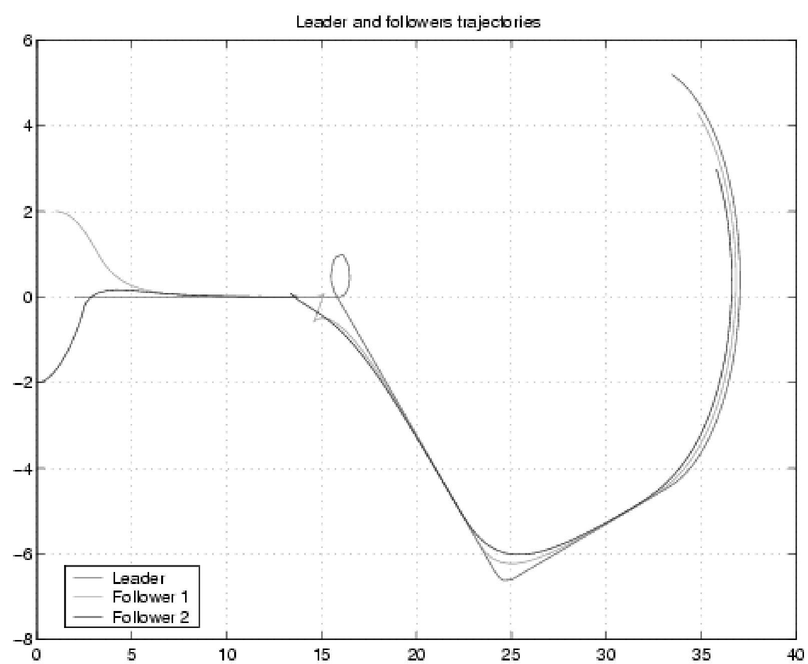
Angle to leader
(in degrees)

String Formation

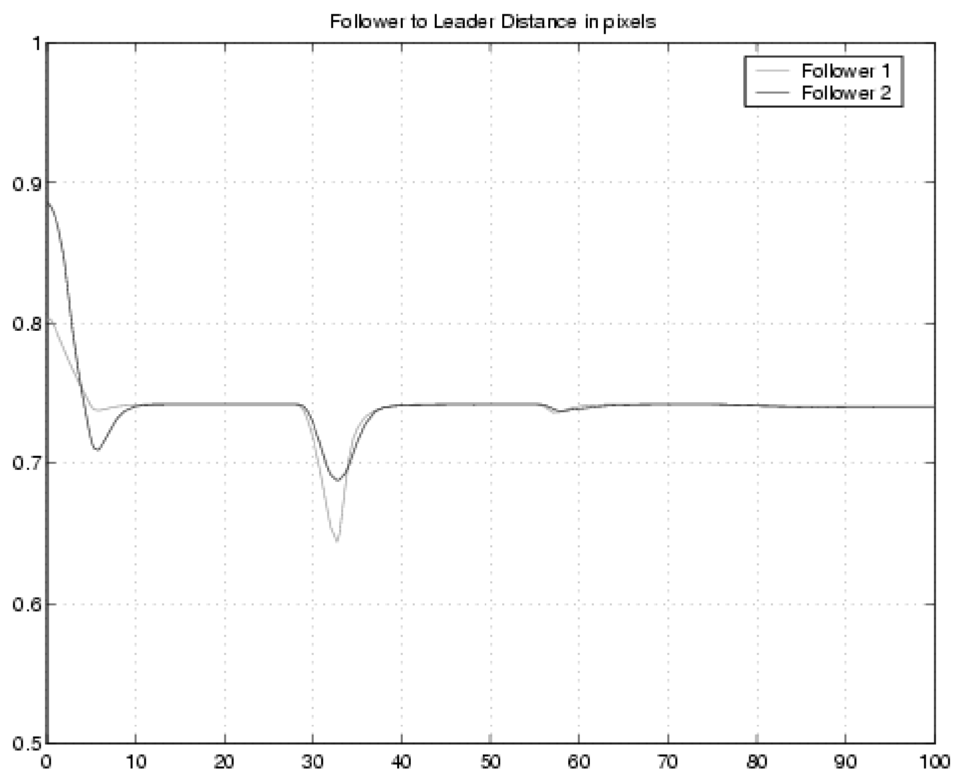
- Green follows red $r_d = 1/\sqrt{2}, \qquad \theta_d = 0$
- Blue follows green $r_d = 1/\sqrt{2}, \qquad \theta_d = 0$



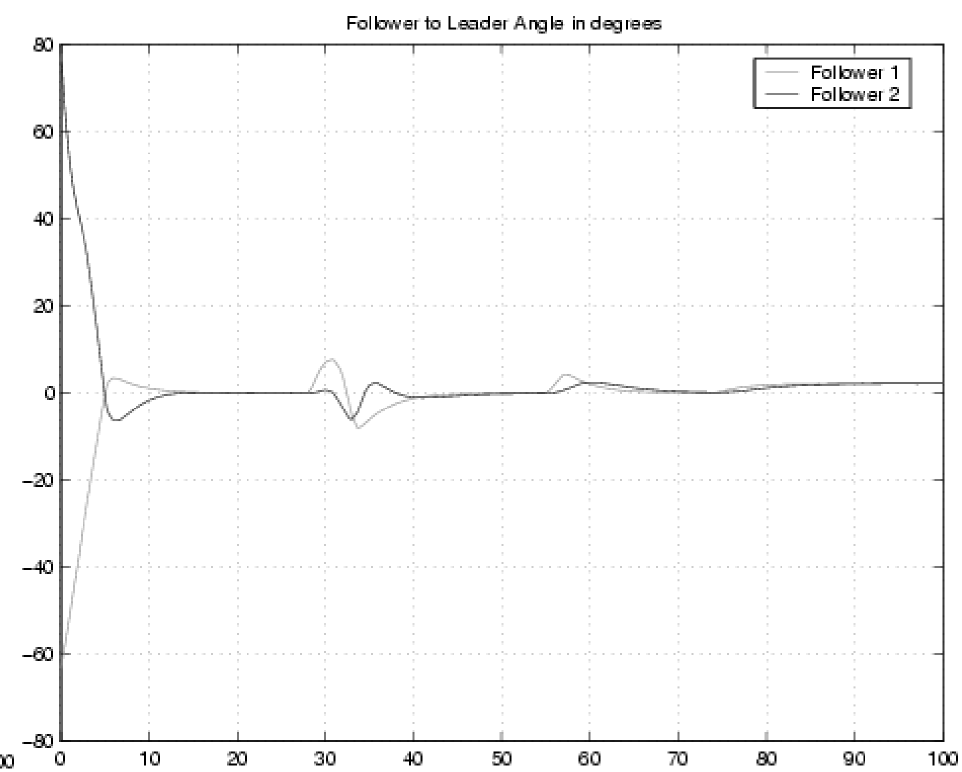




String Formation



Distance to leader
(in pixels)



Angle to leader
(in degrees)

Conclusions

- A framework for distributed formation control in the omni-directional image plane
- An algorithm for multi-body motion segmentation in omni-directional images

- Future work
  - Generalize formation control to UAV dynamics
  - Hybrid theoretic formation switching control
  - Implement on BEAR fleet of UGVs and UAVs