**Game playing**

**Chapter 6**

### Outline

- Games
- Perfect play
  - minimax decisions
  - $\alpha$–$\beta$ pruning
- Resource limits and approximate evaluation
- Games of chance
- Games of imperfect information

### Games vs. search problems

“Unpredictable” opponent ⇒ solution is a strategy
specifying a move for every possible opponent reply

Time limits ⇒ unlikely to find goal, must approximate

Plan of attack:

- Computer considers possible lines of play (Babbage, 1846)
- Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
- Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
- First chess program (Turing, 1951)
- Machine learning to improve evaluation accuracy (Samuel, 1952–57)
- Pruning to allow deeper search (McCarthy, 1956)

### Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest minimax value
  = best achievable payoff against best play

E.g., 2-ply game:
Minimax algorithm

function $\text{MINMAX-DECISION}(state)$ returns an action
inputs: state, current state in game
return the $a$ in $\text{ACTIONS}(state)$ maximizing $\text{MIN-VALUE}(\text{RESULT}(a, state))$

function $\text{MAX-VALUE}(state)$ returns a utility value
if $\text{TERMINAL-TEST}(state)$ then return $\text{UTILITY}(state)$
$v \leftarrow -\infty$
for $a, s$ in $\text{SUCCESSORS}(state)$ do $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s))$
return $v$

function $\text{MIN-VALUE}(state)$ returns a utility value
if $\text{TERMINAL-TEST}(state)$ then return $\text{UTILITY}(state)$
$v \leftarrow \infty$
for $a, s$ in $\text{SUCCESSORS}(state)$ do $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(s))$
return $v$

Properties of minimax

Complete?? Yes, if tree is finite (chess has specific rules for this)
Optimal?? Yes, against an optimal opponent. Otherwise??
Time complexity??

For chess, $b \approx 35$, $m \approx 100$ for “reasonable” games
$\Rightarrow$ exact solution completely infeasible
But do we need to explore every path?
\[ \alpha - \beta \text{ pruning example} \]

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\begin{array}{c}
\text{MAX} \\
\text{MIN}
\end{array}
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3 12 8

\[ \alpha - \beta \text{ pruning example} \]

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Why is it called \( \alpha - \beta \)?

\( \alpha \) is the best value (to \( \text{MAX} \)) found so far off the current path

If \( V \) is worse than \( \alpha \), \( \text{MAX} \) will avoid it \( \Rightarrow \) prune that branch

Define \( \beta \) similarly for \( \text{MIN} \)
The α–β algorithm

function Alpha-Beta-Decision(state) returns an action
return the a in Actions(state) maximizing Min-Value(Result(a, state))

function Max-Value(state, α, β) returns a utility value
inputs: state, current state in game
        α, the value of the best alternative for MAX along the path to state
        β, the value of the best alternative for MIN along the path to state
if Terminal-Test(state) then return Utility(state)
v ← −∞
for a, s in Successors(state) do
  v ← Max(v, Min-Value(s, α, β))
if v ≥ β then return v
α ← Max(α, v)
return v

function Min-Value(state, α, β) returns a utility value
same as Max-Value but with roles of α, β reversed

Properties of α–β

Pruning does not affect final result
Good move ordering improves effectiveness of pruning
With “perfect ordering,” time complexity = \(O(b^{m/2})\)
\(\Rightarrow\) doubles solvable depth
A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)
Unfortunately, \(35^{50}\) is still impossible!

Evaluation functions

Black to move
White slightly better

White to move
Black winning

For chess, typically linear weighted sum of features
\[\text{Eval}(s) = w_1f_1(s) + w_2f_2(s) + \ldots + w_nf_n(s)\]
e.g., \(w_1 = 9\) with \(f_1(s) = \text{(number of white queens)} - \text{(number of black queens)}\), etc.

Digression: Exact values don’t matter

MAX

MIN

Behaviour is preserved under any monotonic transformation of Eval
Only the order matters:
  payoff in deterministic games acts as an ordinal utility function

Deterministic games in practice

Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.


Othello: human champions refuse to compete against computers, who are too good.

Go: human champions refuse to compete against computers, who are too bad. In go, \(b > 300\), so most programs use pattern knowledge bases to suggest plausible moves.
Nondeterministic games in general

In nondeterministic games, chance introduced by dice, card-shuffling

Simplified example with coin-flipping:

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}
\text{MIN} & 2 & 4 & 0 & -2 & 2 & 4 & 0 & -2 & 2 & 4 & 0 \\
\text{CHANCE} & 3 & & & & & & & & & & \\
\text{MAX} & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\
\end{array}
\]

Algorithm for nondeterministic games

\text{\textsc{Expectiminimax}} gives perfect play

Just like \text{\textsc{Minimax}}, except we must also handle chance nodes:

\[
\begin{align*}
\text{if } \text{state} \text{ is a } \text{MAX node then} & \quad \text{return the highest } \text{Expectiminimax-Value of Successors(\text{state})} \\
\text{if } \text{state} \text{ is a } \text{MIN node then} & \quad \text{return the lowest } \text{Expectiminimax-Value of Successors(\text{state})} \\
\text{if } \text{state} \text{ is a chance node then} & \quad \text{return average of } \text{Expectiminimax-Value of Successors(\text{state})}
\end{align*}
\]

Nondeterministic games in practice

Dice rolls increase to: 21 possible rolls with 2 dice

Backgammon \approx 20 legal moves (can be 6,000 with 1-1 roll)

\[
\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9
\]

As depth increases, probability of reaching a given node shrinks

\Rightarrow \text{value of lookahead is diminished}

\alpha-\beta \text{ pruning is much less effective}

TDGammon uses depth-2 search + very good \text{Eval}

\approx \text{world-champion level}

Digression: Exact values DO matter

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}
\text{DICE} & 2 & 2 & 3 & 3 & 1 & 1 & 4 & 4 & 2 & 2 & 3 \\
\text{MIN} & 2 & 2 & 3 & 3 & 1 & 1 & 4 & 4 & 2 & 2 & 3 \\
\text{MAX} & 2.1 & 1.3 & 21 & 40.9 & 20 & 30 & 1 & 400 & 20 & 30 & 1 \\
\end{array}
\]

Behaviour is preserved only by positive linear transformation of \text{Eval}

Hence \text{Eval} should be proportional to the expected payoff

Games of imperfect information

E.g., card games, where opponent’s initial cards are unknown

Typically we can calculate a probability for each possible deal

Seems just like having one big dice roll at the beginning of the game

\text{Idea: compute the minimax value of each action in each deal, then choose the action with highest expected value over all deals}

\text{Special case: if an action is optimal for all deals, it’s optimal.}

\text{GIB, current best bridge program, approximates this idea by}

1) generating 100 deals consistent with bidding information

2) picking the action that wins most tricks on average
Example

Four-card bridge/whist/hearts hand, MAX to play first

MAX
MIN

0

Commonsense example

Road A leads to a small heap of gold pieces
Road B leads to a fork:
- take the left fork and you’ll find a mound of jewels;
- take the right fork and you’ll be run over by a bus.

Example

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Example

Four-card bridge/whist/hearts hand, MAX to play first

MAX
MIN

-0.5

Commonsense example

Road A leads to a small heap of gold pieces
Road B leads to a fork:
- guess correctly and you’ll find a mound of jewels;
- guess incorrectly and you’ll be run over by a bus.

Example

Four-card bridge/whist/hearts hand, MAX to play first

MAX
MIN

-0.5

Commonsense example

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MIN

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Four-card bridge/whist/hearts hand, MAX to play first

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MIN

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- guess correctly and you’ll find a mound of jewels;
- guess incorrectly and you’ll be run over by a bus.
Proper analysis

Intuition that the value of an action is the average of its values in all actual states is **WRONG**

With partial observability, value of an action depends on the information state or belief state the agent is in.

Can generate and search a tree of information states.

Leads to rational behaviors such as:

- Acting to obtain information
- Signalling to one’s partner
- Acting randomly to minimize information disclosure

Summary

Games are fun to work on! (and dangerous)

They illustrate several important points about AI:

- Perfection is unattainable ⇒ must approximate
- Good idea to think about what to think about
- Uncertainty constrains the assignment of values to states
- Optimal decisions depend on information state, not real state

Games are to AI as grand prix racing is to automobile design.