HONOR SYSTEM: This examination is strictly individual. You are not allowed to talk, discuss, exchange solutions, etc., with other fellow students. Furthermore, you are only allowed to use the book, slides and your class notes. You should not use Internet. You may only ask questions to the class instructor. Any violation of the honor system, or any of the ethic regulations, will be immediately reported according to George Mason University honor court.
1. (20) Edge detection. This problem is a pencil and paper exercise.

(a) Compute gradient magnitude of the above image at all points. The derivatives are computed by convolution with a simple \([-1, 0, 1]\) filter.

(b) Compute gradient direction at these points.

(c) Show the image after applying non-maximum suppression step on the gradient magnitude image.

(d) Discuss the idea behind hysteresis thresholding in edge detection used in Canny edge detector.
2. (15) An important parameter of the imaging system is the field of view (FOV). Field of view is twice the angle between the optical axis (z-axis) and the end of the retinal plane (CCD array). Imagine that you have a camera system with focal length 16mm, and retinal plane (CCD array) is (16mm × 12mm) and that your digitizing process samples the imaging surface into 640 × 480 pixels in each direction.

a) Compute the FOV (horizontal and vertical)

b) Show how the matrix on intrinsic parameters $K$ looks like for this camera and write down the relationship between the image coordinate and a point in 3D world expressed in the camera coordinate system.

d) Describe how is the size of FOV related to the focal length and how it affects the resolution in the image.

e) Given the horizontal FOV you computed, how many images do you need to create 360 degree panorama, assuming that you will need 50% overlap between neighboring views.
3. (15) **Motion recovery.** Consider set of corresponding points \( \mathbf{x}_1 \) and \( \mathbf{x}_2 \) in retinal coordinates in two views, which are related by planar motion in the plane perpendicular to the image plane (i.e. if the optical axis of the camera is \( z - \text{axis} \) the camera moves is in \( x - z \) plane and rotates around \( y \) axis.

(a) Assume that the intrinsic camera parameters are known and write down a simplified version of the epipolar constraint for this planar motion case.

(b) Describe a linear least squares algorithm for estimation of matrix \( E_{\text{planar}} \). What is the minimal number of corresponding points needed in order to solve for \( E_{\text{planar}} \)?

(c) Suppose now that you know only partial intrinsic calibration of the camera, i.e. the center of the projection is known, but the focal length of the camera is unknown and you can measure only pixel coordinates points \( \mathbf{x}_1' \) and \( \mathbf{x}_2' \). Can the still recover the motion between the two views?
4. (10) Describe an extension of the Hough Transform discussed for line to circles. Circles are parametric primitives which are characterized by center of the circle and radius such that \((x - x_c)^2 + (y - y_c)^2 = r^2\) and \(x = r \cos(\theta)\) and \(y = r \sin(\theta)\). Write down the algorithm in the pseudo-code.
5. (10) **Image Motion.** Consider the camera moving along optical axis observing a planar pattern in the plane parallel to image plane. The image motion model (related to affine and translational models considered in the class) which properly models the following local changes of image appearance, is a a simple isotropic expansion modelled by a parameter \( a \). Derive a method for estimating the unknown parameters of this model and discuss conditions when such method would fail.
6. (20) Answer the following questions briefly.

- What is an image of a circle?

- What is the aperture problem?

- Why is it recommended to smooth the image before computing derivatives?

- Given two lines $l_1$ and $l_2$ in the image denoted by their projective coordinates (as plane normals) compute an intersection point of these two lines.

- How is the rotational invariance enforced in SIFT features?

- What is an epipole?

- Does cylindrical projection used in mosaicing preserve straight lines?