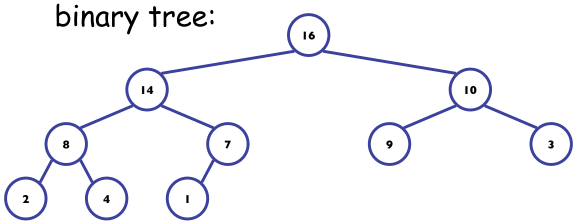
Heaps

- A *heap* can be seen as a complete binary tree:

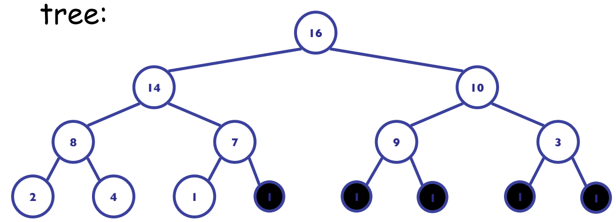


What makes a binary tree complete?

Is the example above complete?

Heaps

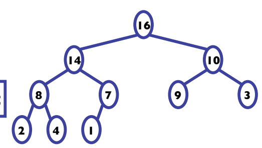
- A *heap* can be seen as a complete binary tree:



Heaps

- In practice, heaps are usually implemented as arrays:

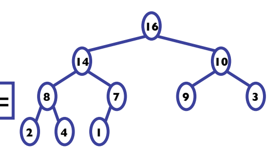
A =	16	14	10	8	7	9	3	2	4	1	=



Heaps

- To represent a complete binary tree as an array:
  - The root node is A[1]
  - Node *i* is A[*i*]
  - The parent of node *i* is A[*i*/2] (note: integer divide)
  - The left child of node *i* is A[2*i*]
  - The right child of node *i* is A[2*i* + 1]

A =	16	14	10	8	7	9	3	2	4	1	=



Referencing Heap Elements

- So…

Parent(i) { return ⌊i/2⌋; }

Left(i) { return 2*i; }

right(i) { return 2*i + 1; }

- An aside: *How would you implement this most efficiently?*
- Another aside: *Really?*

The Heap Property

- Heaps also satisfy the *heap property*:

  $A[Parent(i)] \geq A[i]$ for all nodes $i > 1$
  - In other words, the value of a node is at most the value of its parent
  - *Where is the largest element in a heap stored?*
- Definitions:
  - The *height* of a node in the tree = the number of edges on the longest downward path to a leaf
  - The height of a tree = the height of its root

Heap Height

- *What is the height of an n-element heap? Why?*
- This is nice: basic heap operations take at most time proportional to the height of the heap

Heap Height

- **`Heapify`**
- **`Build-heap`**
- **`Heapsort`**

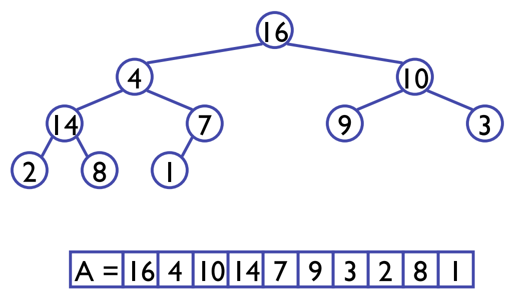
Heap Operations: Heapify()

- `Heapify()`: maintain the heap property
  - Given: a node *i* in the heap with children *l* and *r*
  - Given: two subtrees rooted at *l* and *r*, assumed to be heaps
  - Problem: The subtree rooted at *i* may violate the heap property (*How?*)
  - Action: let the value of the parent node "float down" so subtree at *i* satisfies the heap property
    - *What do you suppose will be the basic operation between i, l, and r?*

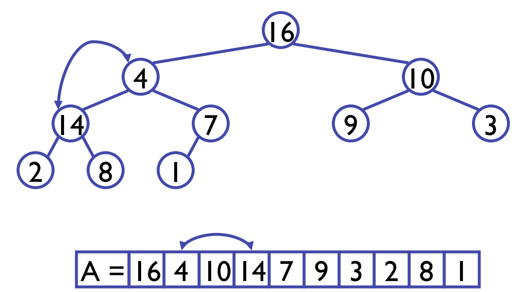
Heap Operations: Heapify()

```
Heapify(A, i)
{
   l = Left(i); r = Right(i);
   if (l <= heap_size(A) && A[l] > A[i])
     largest = l;
   else
     largest = i;
   if (r <= heap_size(A) && A[r] > A[largest])
     largest = r;
   if (largest != i)
     Swap(A, i, largest);
     Heapify(A, largest);
}
```

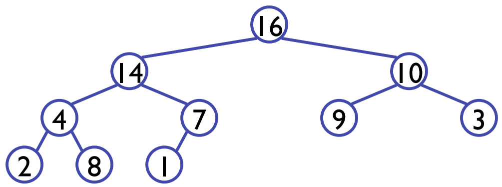
Heapify(A,2) Example



Heapify(A,2) Example

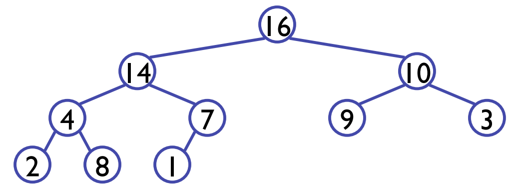


Heapify(A,2) Example



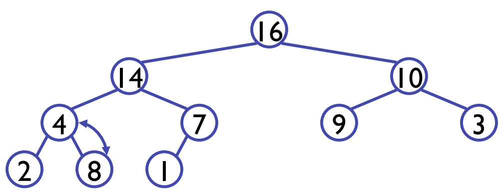
A = | 16 | 14 | 10 | 4 | 7 | 9 | 3 | 2 | 8 | 1 |

Heapify(A,2) Example



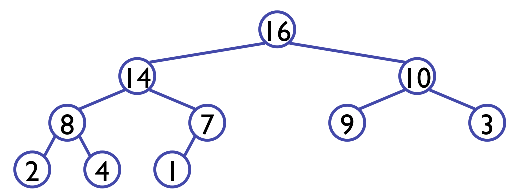
A = | 16 | 14 | 10 | 4 | 7 | 9 | 3 | 2 | 8 | 1 |

Heapify(A,4) Example



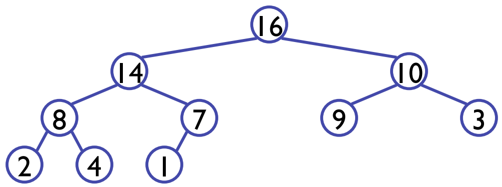
A = | 16 | 14 | 10 | 4 | 7 | 9 | 3 | 2 | 8 | 1 |

Heapify(A,4) Example



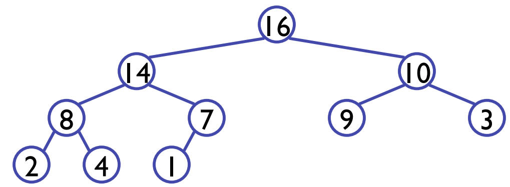
A = | 16 | 14 | 10 | 8 | 7 | 9 | 3 | 2 | 4 | 1 |

Heapify(A,4) Example



A =	16	14	10	8	7	9	3	2	4	1

Heapify(A,9) Example



A =	16	14	10	8	7	9	3	2	4	1

Analyzing Heapify(): Informal

- Aside from the recursive call, what is the running time of **`Heapify()`** ?
- How many times can **`Heapify()`** recursively call itself?
- What is the worst-case running time of **`Heapify()`** on a heap of size n?