CS483 - Practice Problems 2 (due September 29th)

Jana Košecká

Graph, Greedy algorithms

1. (5) When an adjacency-matrix representation is used, most graph algorithms require time $\Omega(V^2)$, but there are some exceptions. Show that determining whether a directed graph G contains a universal sink a vertex with in-degree $|V| - 1$ and out-degree 0 can be determined in time $O(V)$, given an adjacency matrix for G.

2. (3) We covered have two routines for graph traversal - DFS(G) and BFS(G,s) - where G is a graph and s is any node in G. These two procedures will create a DFS tree and a BFS tree respectively. If G = (V,E) is a connected, undirected graph then the height of DFS(G) tree is always larger than or equal to the height of any of the BFS trees created by BFS(G, s).

3. (9) For the following problem, use the directed unweighted graph given by the following adjacency list. Be sure to consider the edges in the given order.

   A: C E B
   B: E D
   C: E
   D: C F E
   E: F
   F:

   (a) For the source vertex s = A what is the order in which the vertices are visited by BFS (breadth first search)? Also, show the breadth-first search tree that you obtain.
   (b) What is the order in which the vertices are visited by DFS (depth first search)? You should assume that the top-level DFS procedure visits the vertices in alphabetical order. Set up a global counter which gets incremented everytime when the vertex if first explored or is finished being explored. For each vertex give the discovery and finishing time.
   (c) Suppose that this graph is a precedence graph. Using your work above either give a valid order in which to perform the tasks (call them task A, task B, . . ., task F) or prove that there is no valid order.

4. (5pt) Chapter 3.2 (discuss the solution using one the graph traversal algorithms BFS or DFS).

5. Given a set $x_1 \leq x_2 \leq ... \leq x_n$ of points on the real line, give an algorithm to determine the smallest set of unit-length closed intervals that contains all of the points. A closed interval includes both its endpoints; for example, the interval $[1.25, 2.25]$ includes all x_i such that $1.25 \leq x_i \leq 2.25$.

   Solution

6. (5pt) You are given a sequence of n songs, where the i-th song is l_i minutes long. You want to place all of the songs onto a collection of CDs, each of which can hold m minutes. Furthermore, (i) The songs must be recorded in the order given: song 1, song 2, . . . , song n. (ii) All songs must be included. (iii) No song may be split across CDs.
   Give an algorithm to place songs on CDs so as to minimize the number of CDs needed.