

Prim's Algorithm

```
MST-Prim( $G, w, r$ )
```

```
   $Q = V[G];$ 
```

```
  for each  $u \in Q$ 
```

```
     $key[u] = \infty;$ 
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   $key[r] = 0;$ 
```

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   $p[r] = \text{NULL};$ 
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  while ( $Q$  not empty)
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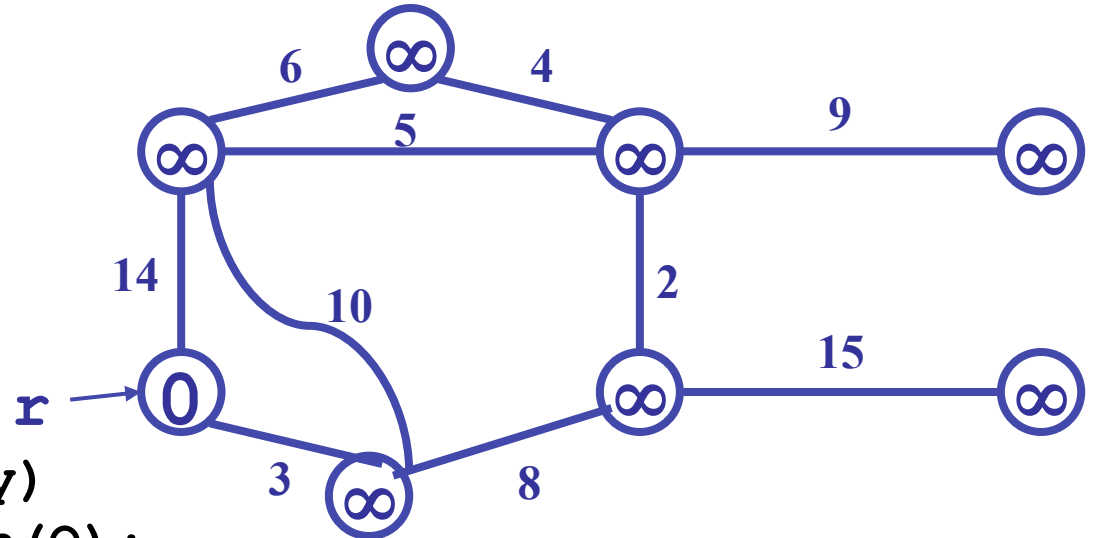
```
     $u = \text{ExtractMin}(Q);$ 
```

```
    for each  $v \in \text{Adj}[u]$ 
```

```
      if ( $v \in Q$  and  $w(u, v) < key[v]$ )
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         $p[v] = u;$ 
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Pick a start vertex r

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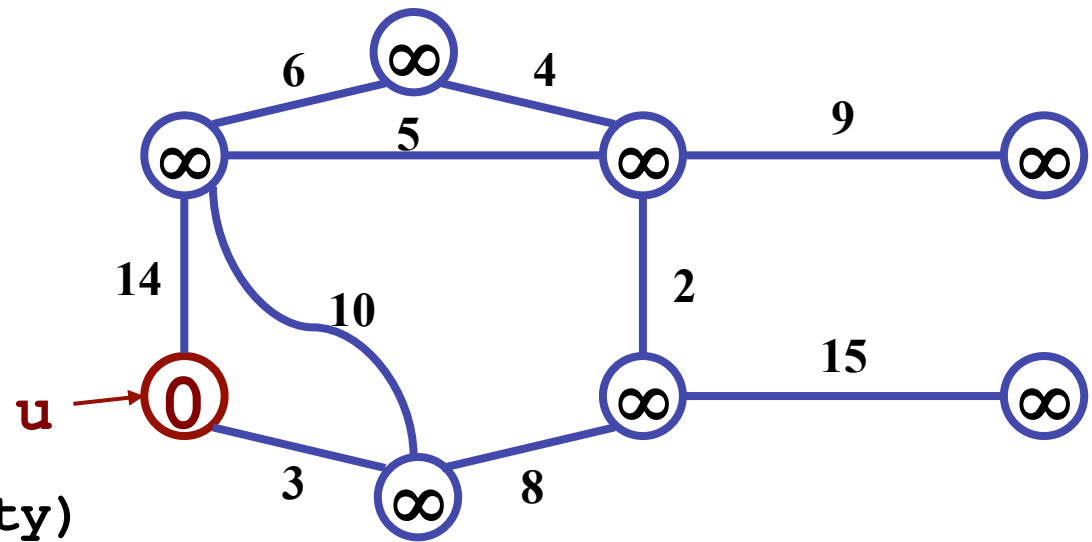
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Red vertices have been removed from Q

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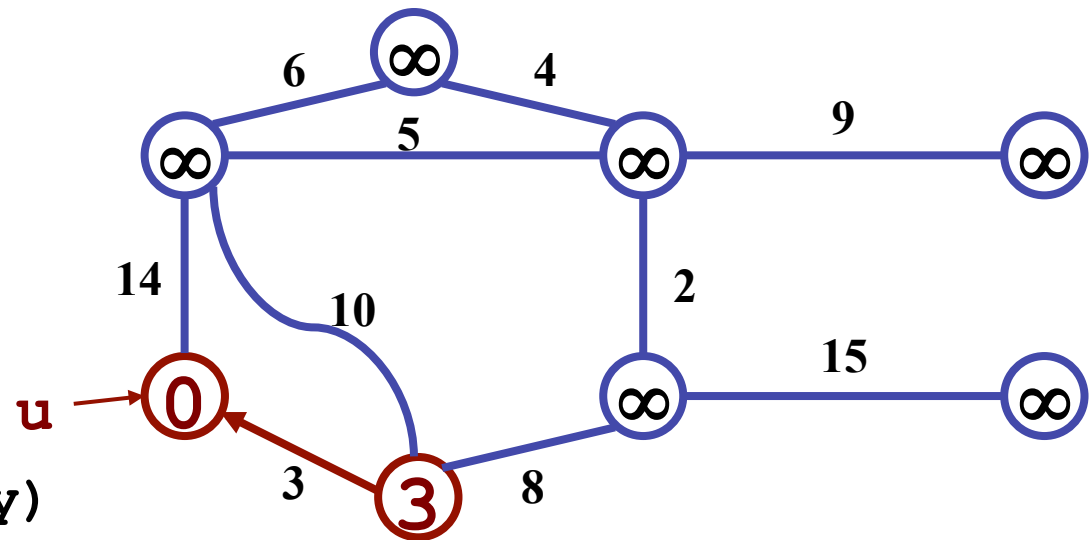
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    for each  $v \in \text{Adj}[u]$  Red arrows indicate parent pointers
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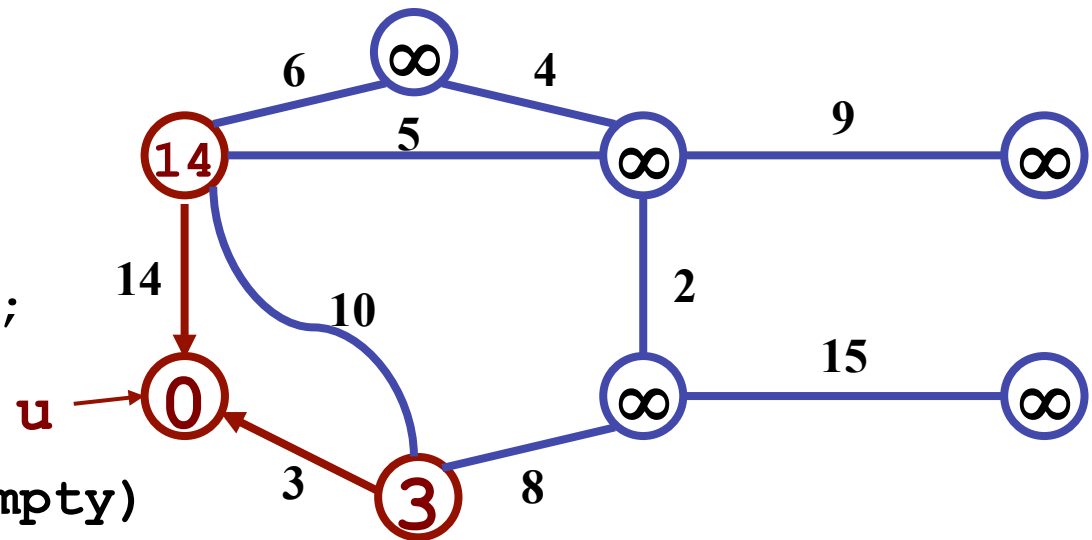
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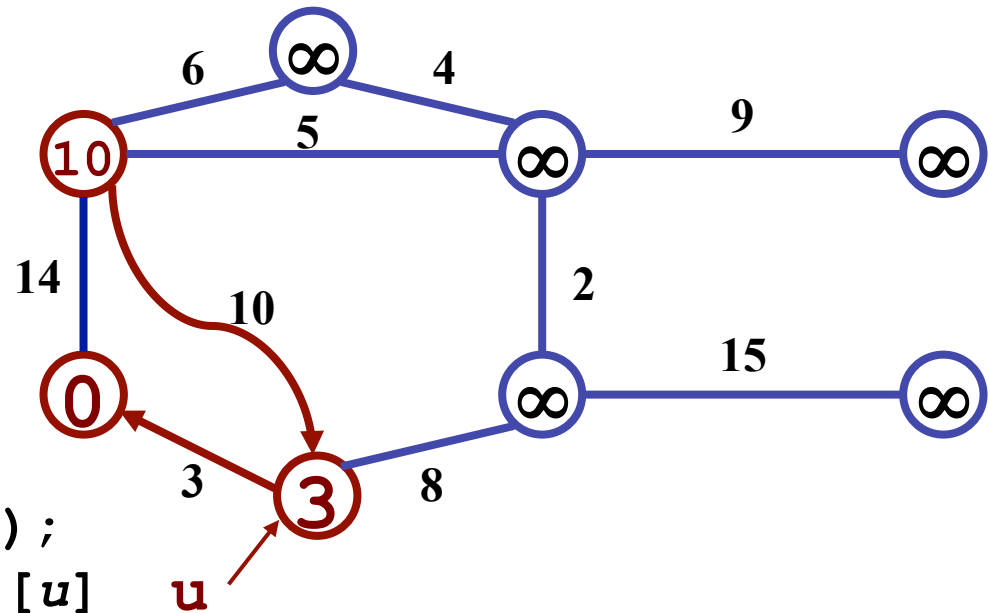
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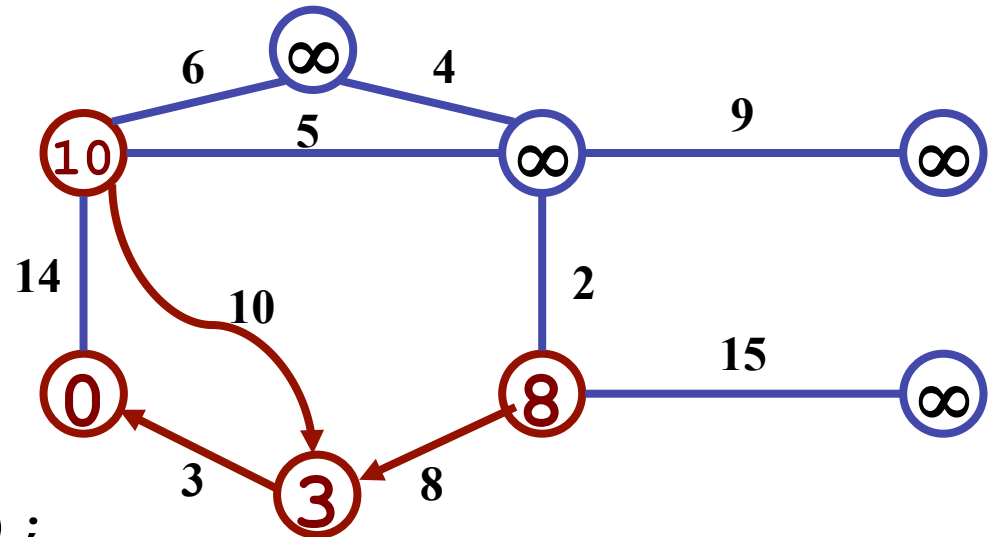
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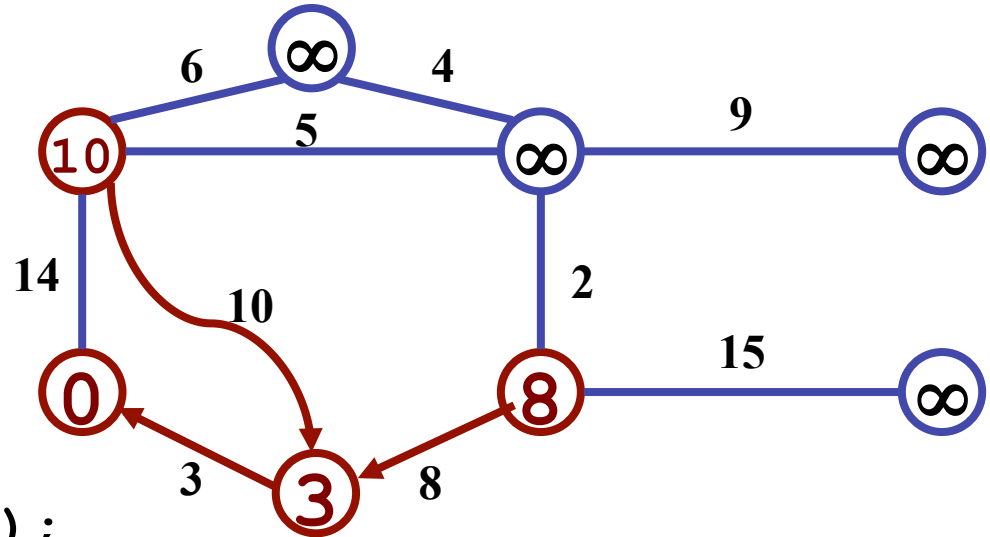
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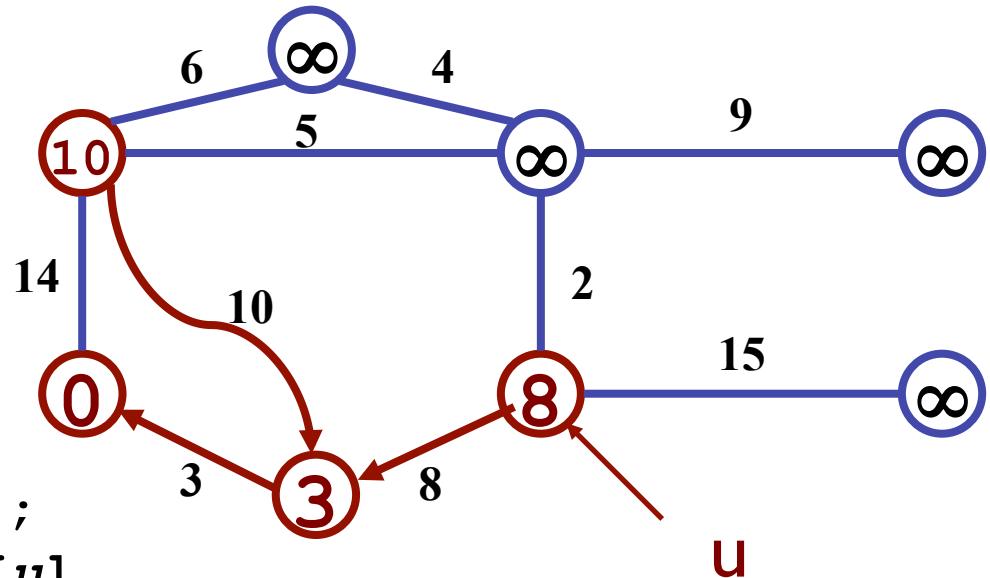
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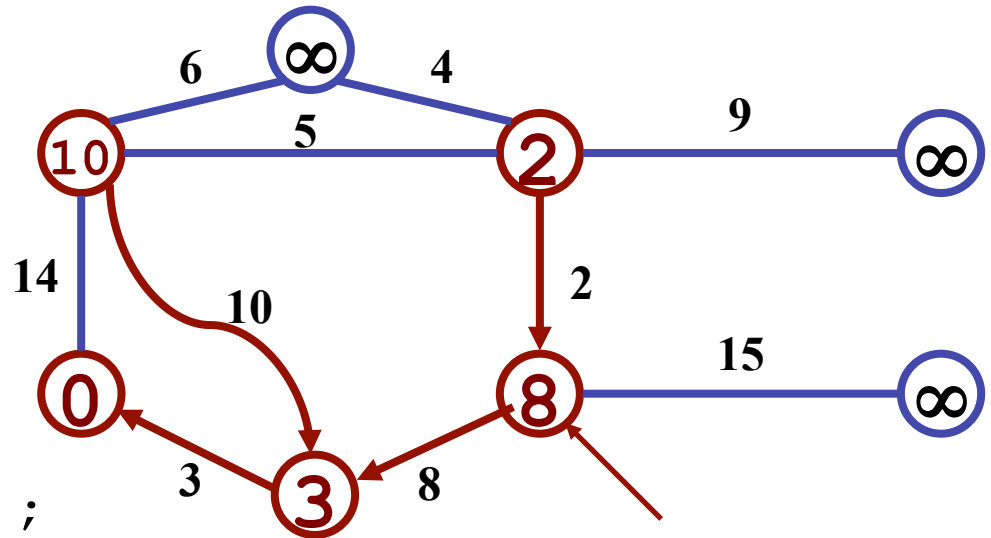
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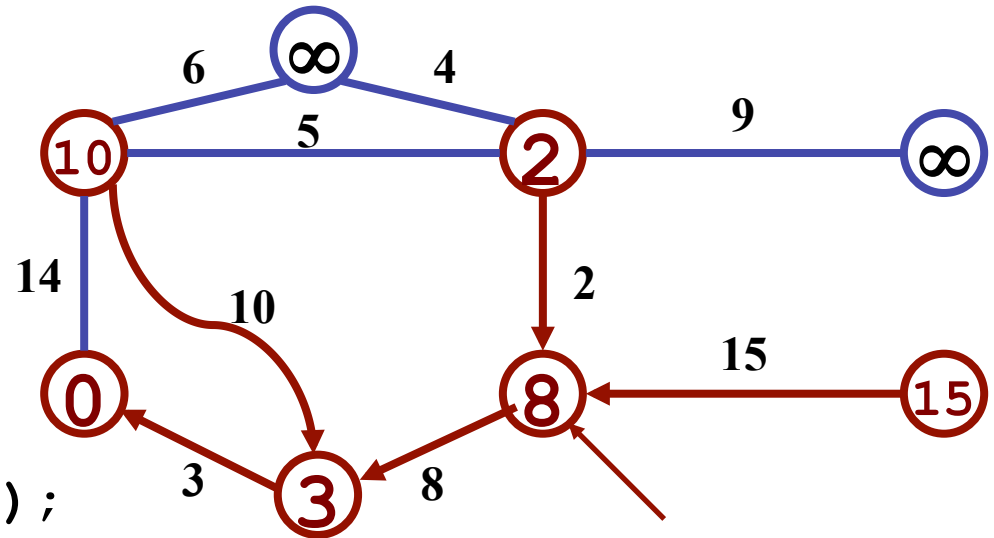
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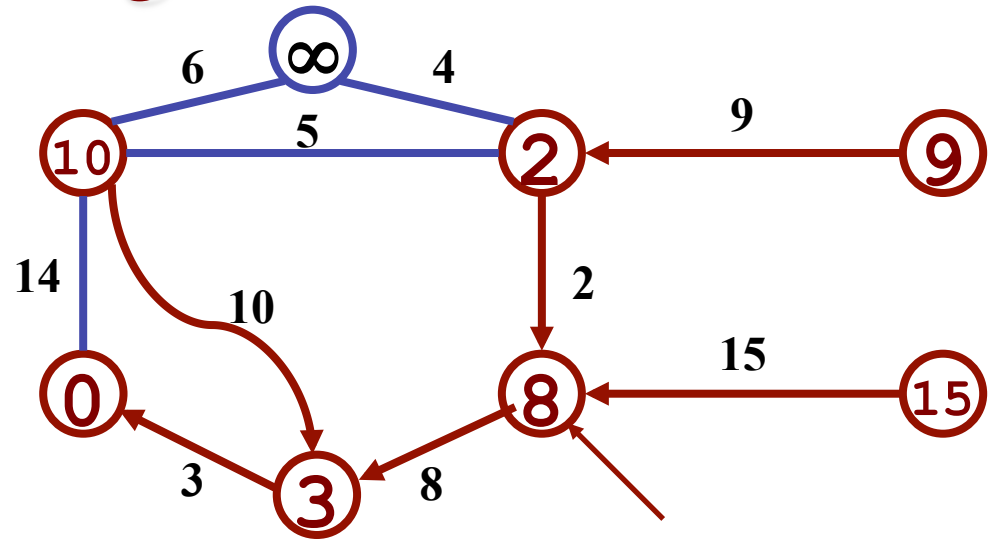
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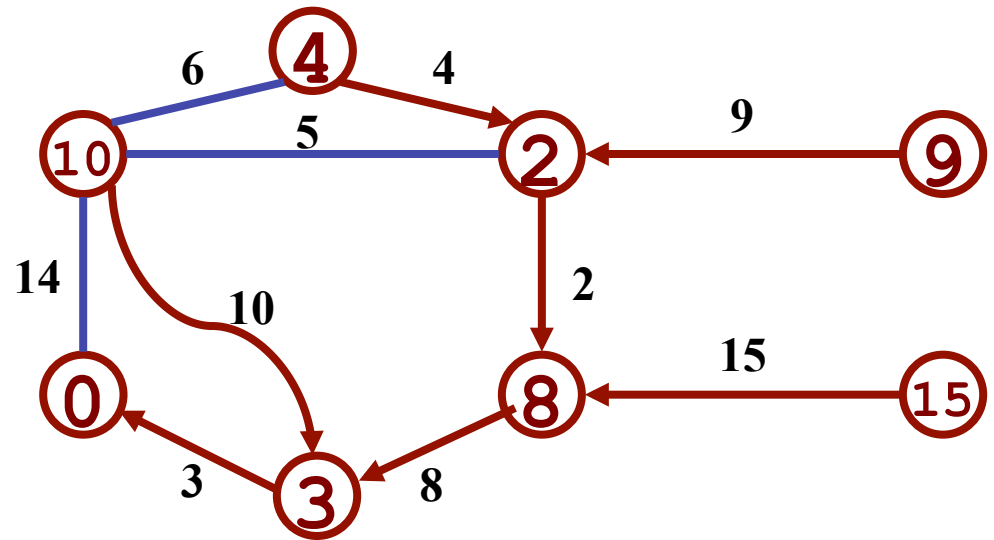
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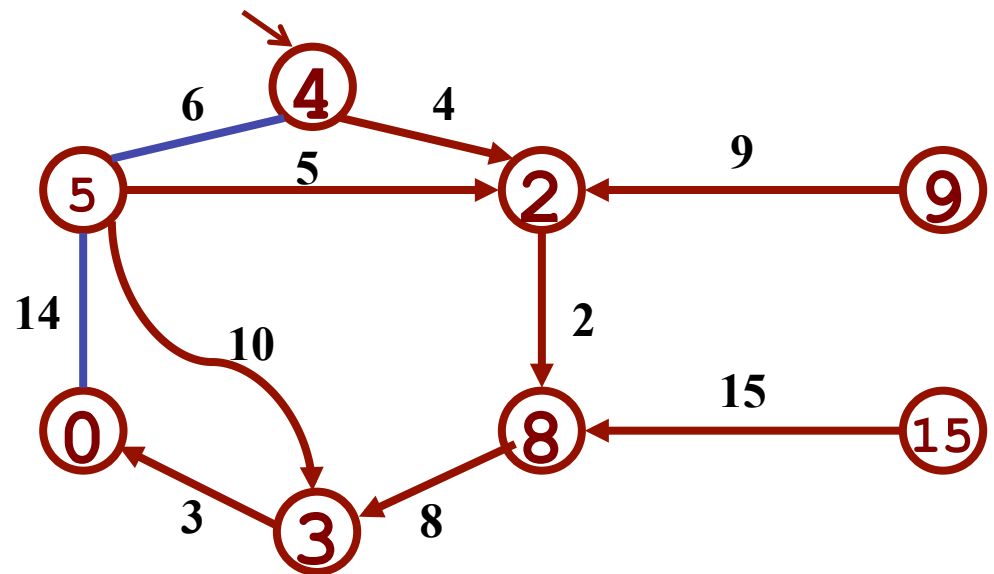
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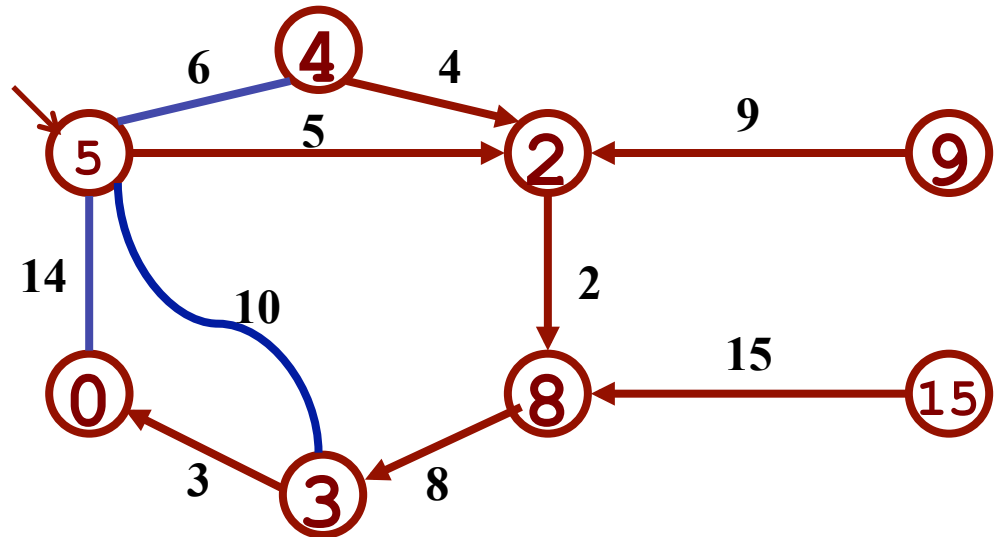
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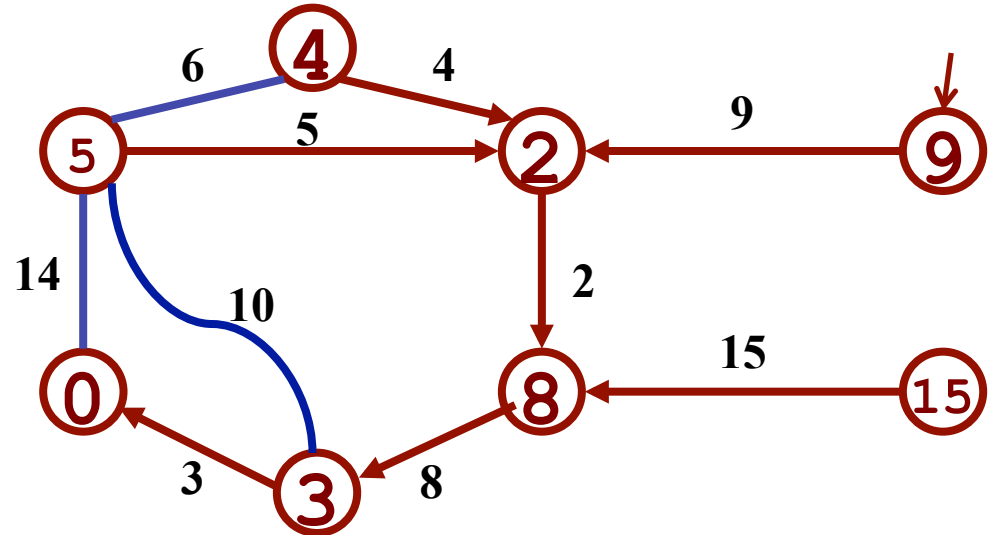
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```

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  key[r] = 0;
```

```
  p[r] = NULL;
```

```
  while (Q not empty)
```

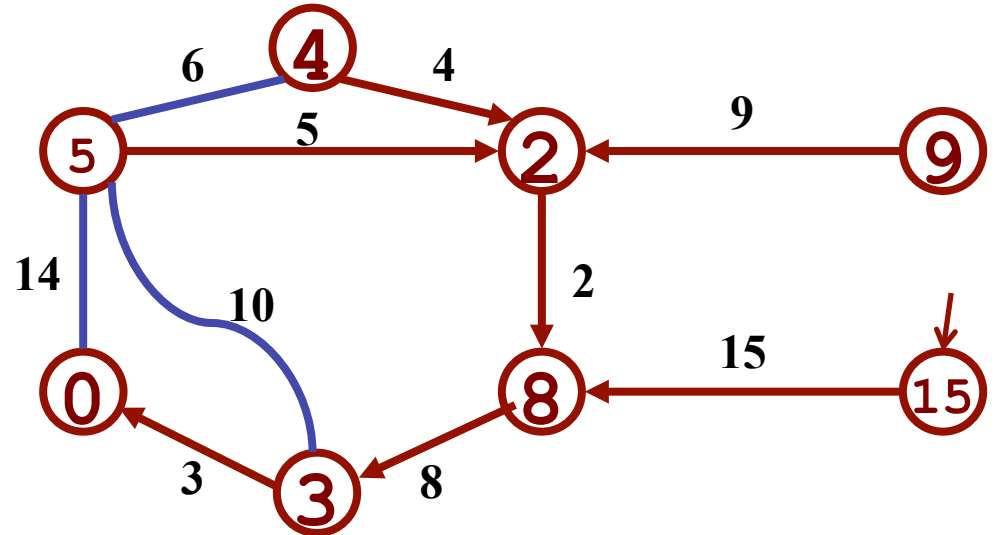
```
    u = ExtractMin(Q);
```

```
    for each  $v \in \text{Adj}[u]$ 
```

```
      if ( $v \in Q$  and  $w(u, v) < \text{key}[v]$ )
```

```
        p[v] = u;
```

```
        key[v] =  $w(u, v)$ ;
```



Review: Prim's Algorithm

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  for each  $u \in Q$ 
     $key[u] = \infty;$ 
   $key[r] = 0;$ 
   $p[r] = \text{NULL};$ 
  while ( $Q$  not empty)
     $u = \text{ExtractMin}(Q);$ 
    for each  $v \in \text{Adj}[u]$ 
      if ( $v \in Q$  and  $w(u, v) < key[v]$ )
         $p[v] = u;$ 
         $\text{DecreaseKey}(v, w(u, v));$ 
```

Kruskal's Algorithm

Run the algorithm:

```
Kruskal ()
```

```
{
```

```
  T =  $\emptyset$ ;
```

```
  for each  $v \in V$ 
```

```
    MakeSet(v);
```

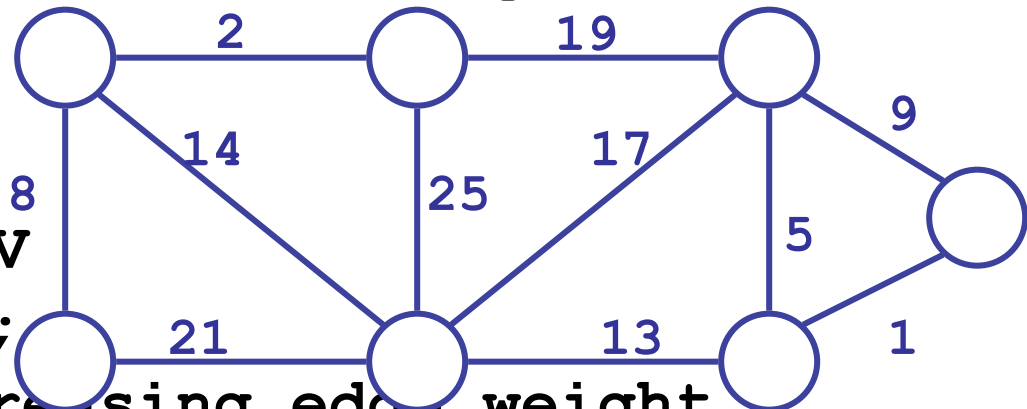
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  sort E by increasing edge weight w  
  for each  $(u,v) \in E$  (in sorted order)
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```
    if FindSet(u)  $\neq$  FindSet(v)
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```
      T = T  $\cup$  {{u,v}};
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      Union(FindSet(u), FindSet(v));
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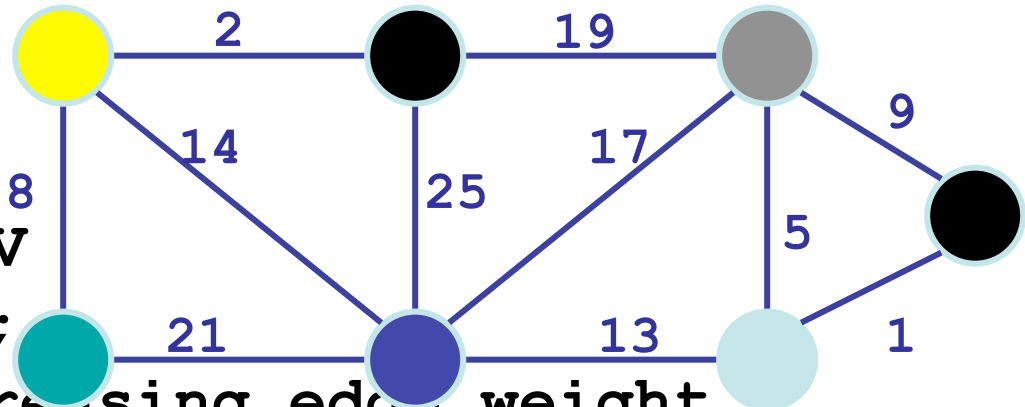
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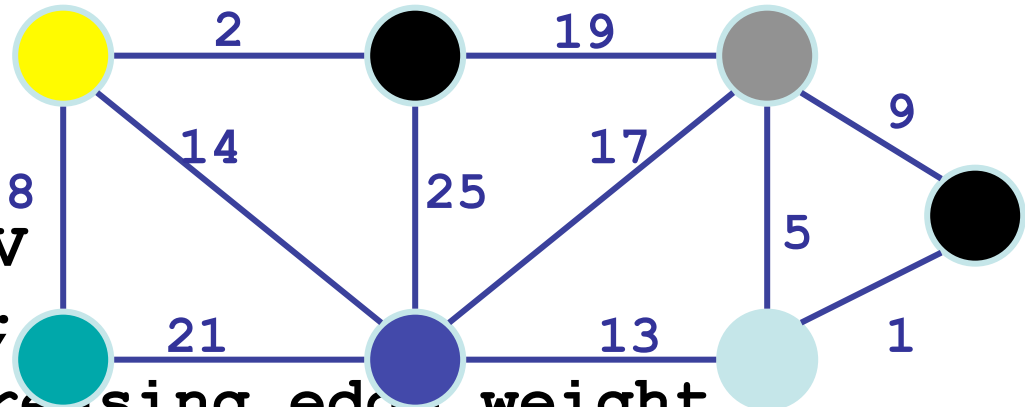
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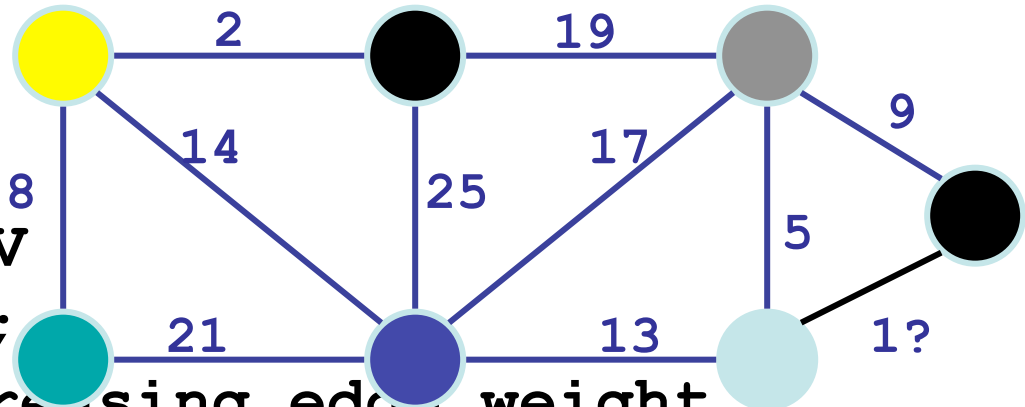
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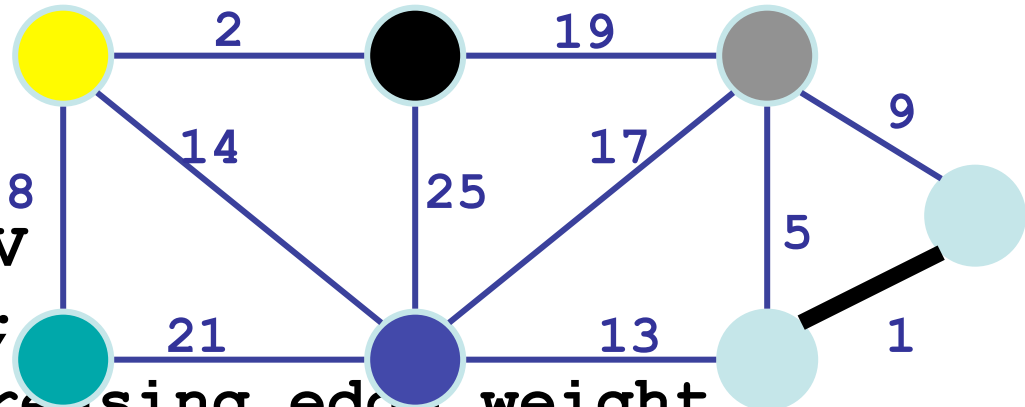
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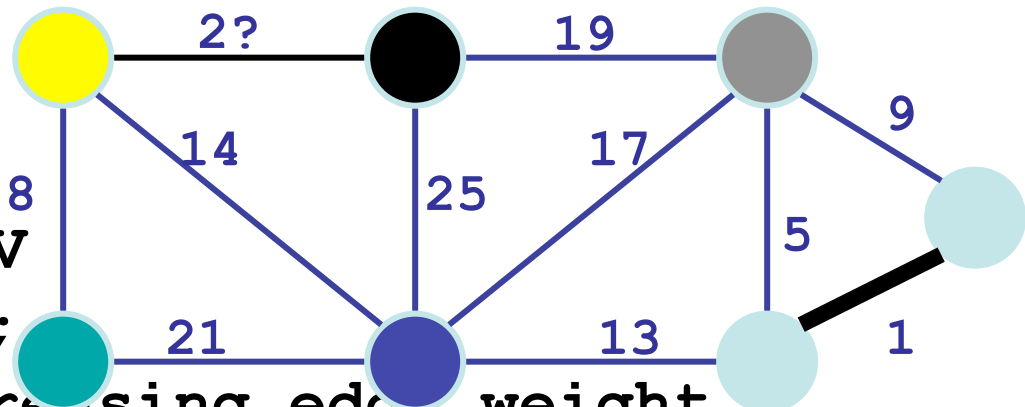
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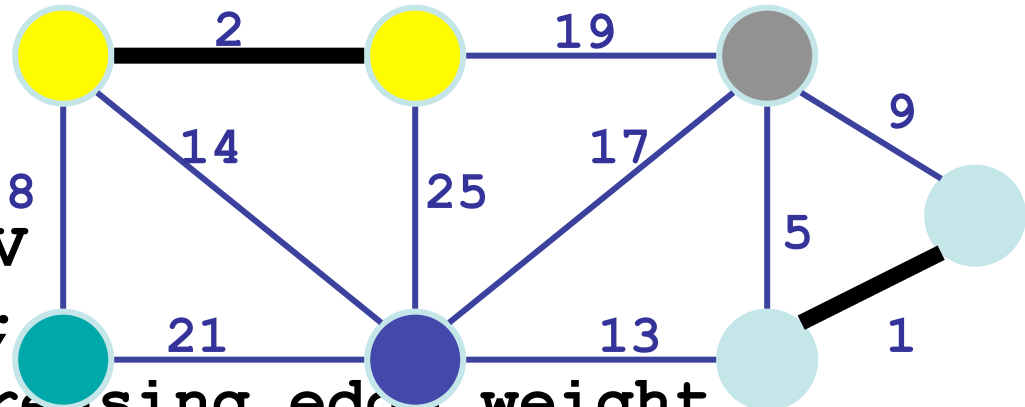
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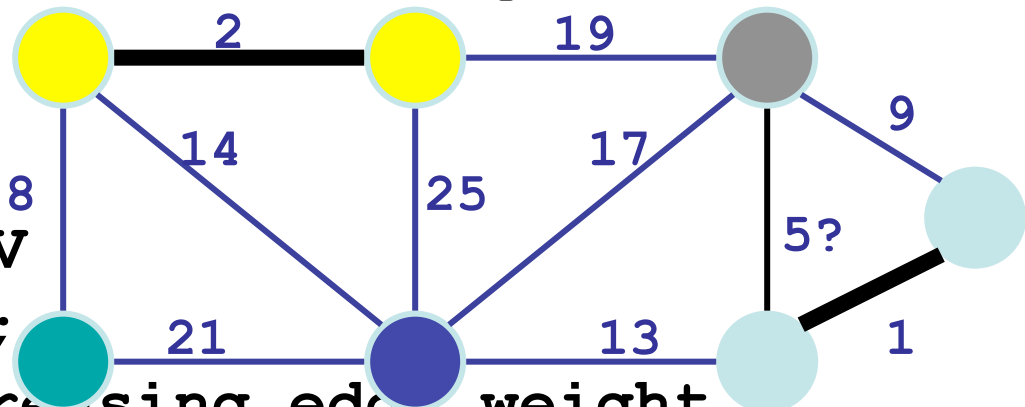
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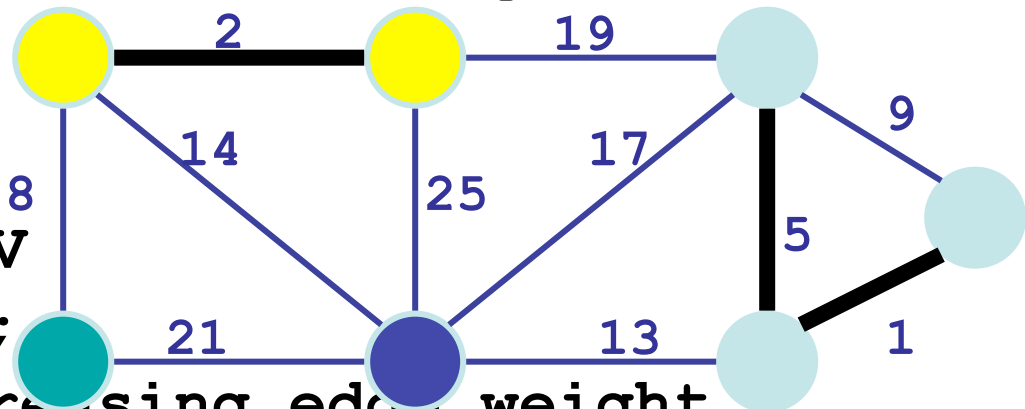
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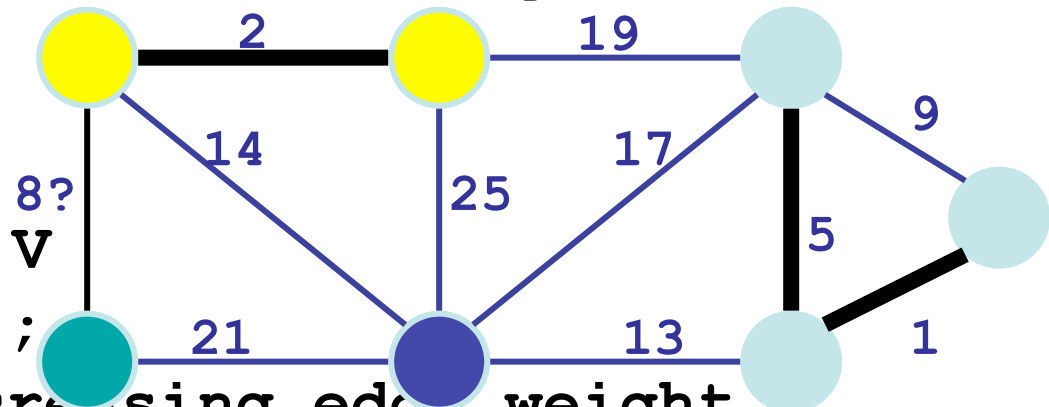
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Kruskal's Algorithm

Run the algorithm:

```
Kruskal ()
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```
  T =  $\emptyset$ ;
```

```
  for each  $v \in V$ 
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```
    MakeSet(v);
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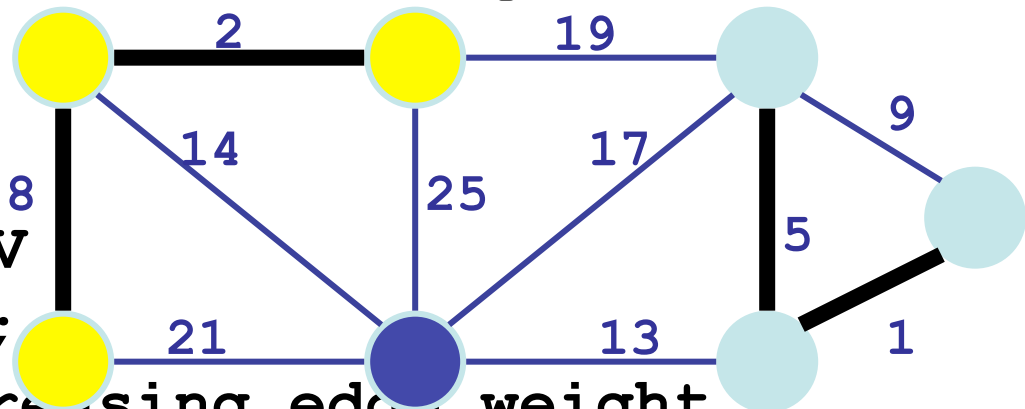
```
  sort E by increasing edge weight w  
  for each  $(u,v) \in E$  (in sorted order)
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    if FindSet(u)  $\neq$  FindSet(v)
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      T = T U {{u,v}};
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      Union(FindSet(u), FindSet(v));
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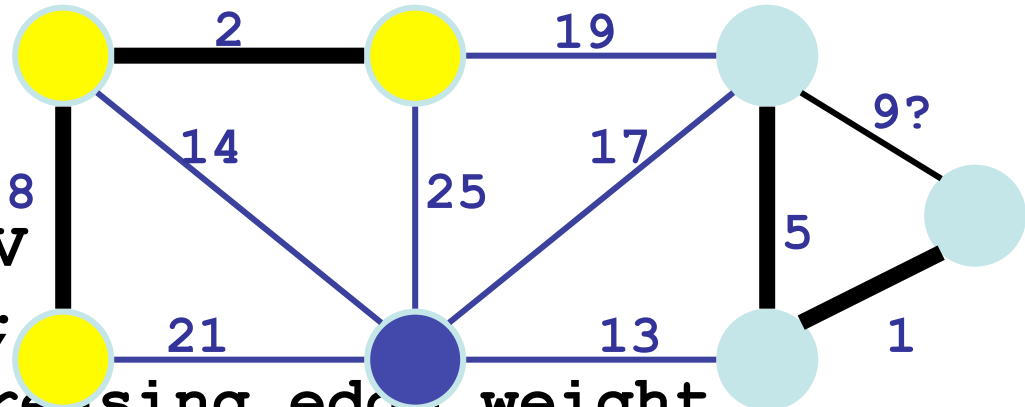
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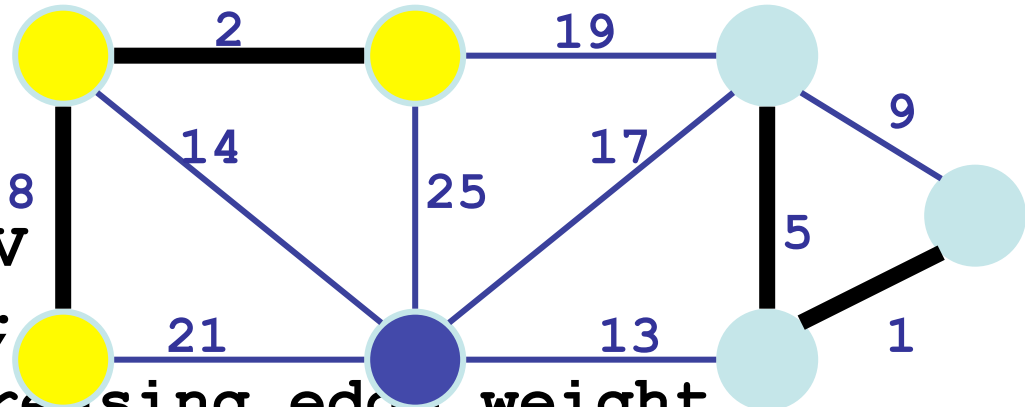
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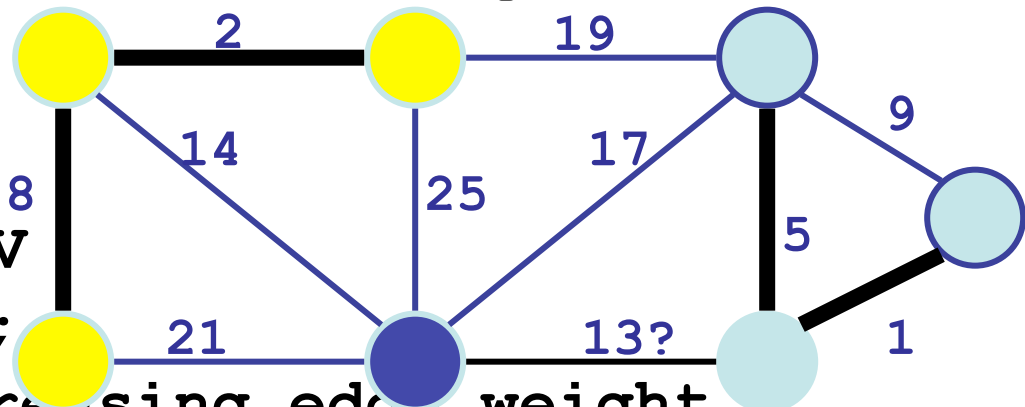
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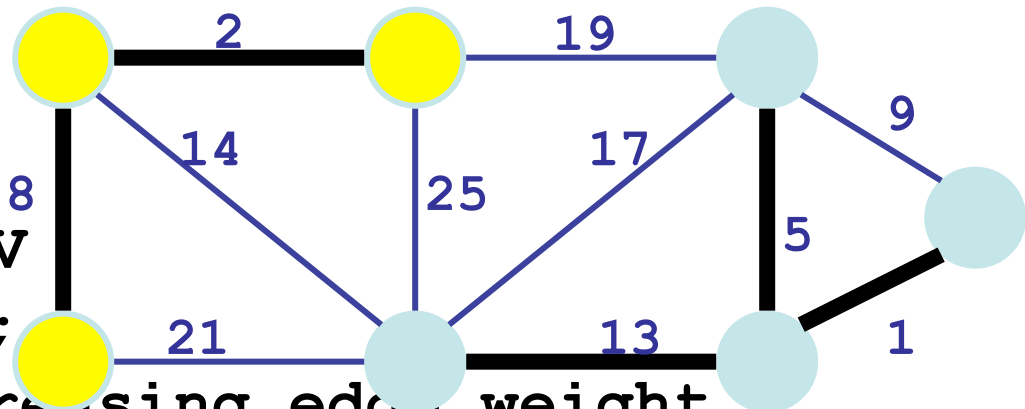
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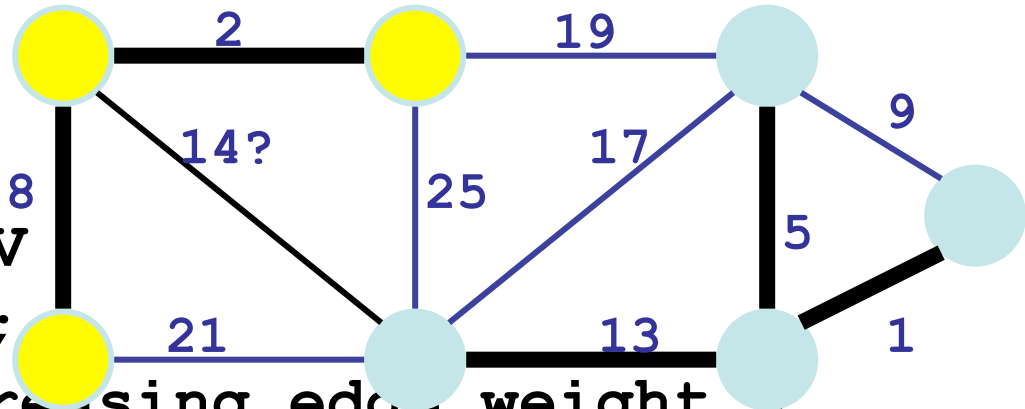
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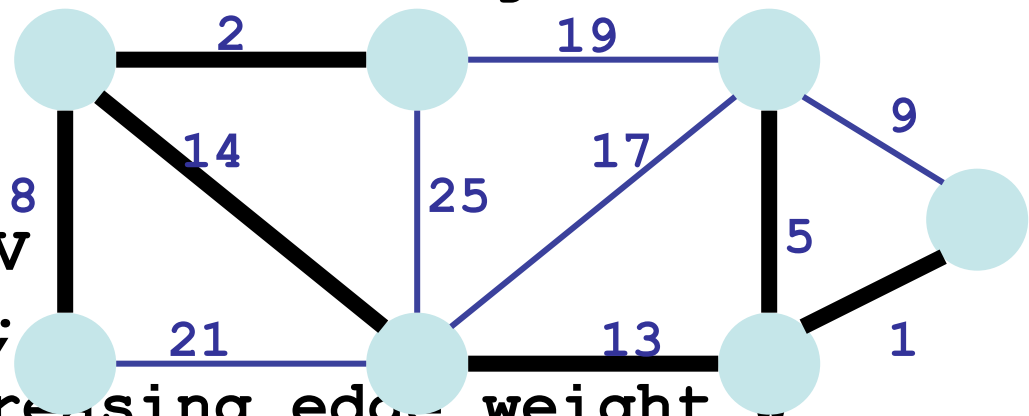
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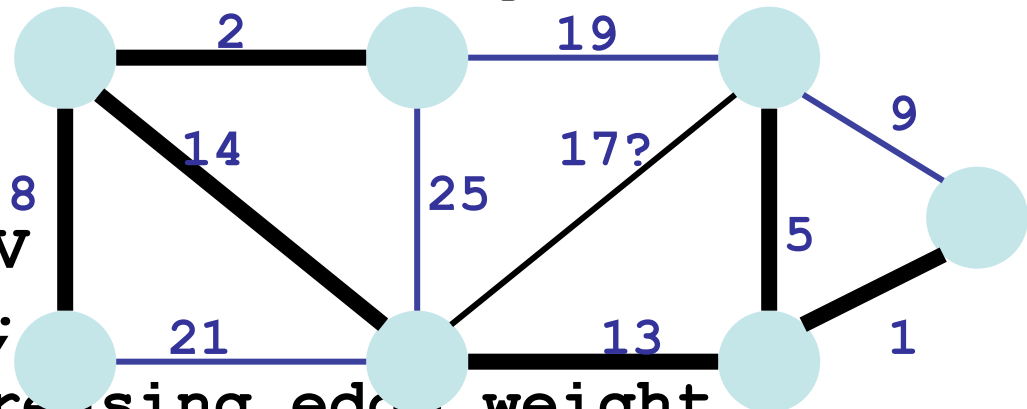
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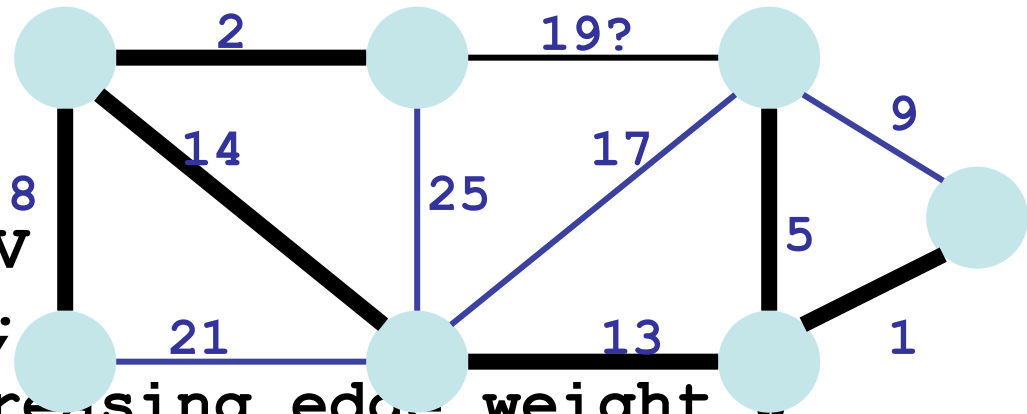
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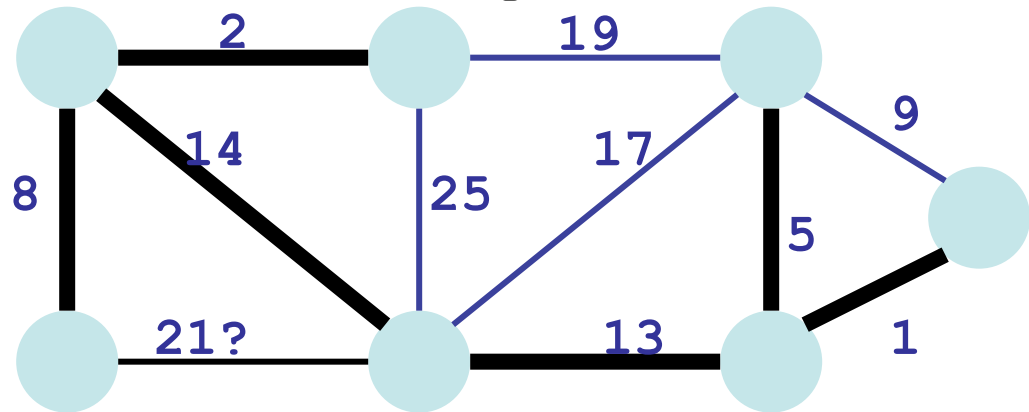
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  for each (u,v)  $\in$  E (in sorted order)
```

```
    if FindSet(u)  $\neq$  FindSet(v)
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```
      T = T  $\cup$  {(u,v)};
```

```
      Union(FindSet(u), FindSet(v));
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```
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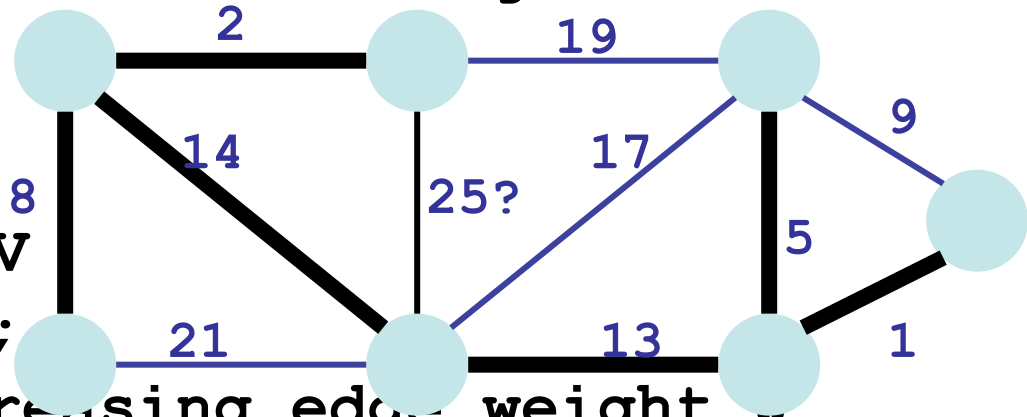
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