1. (10) Find the shortest distances from the vertex S to all the other vertices. Show the intermediate distance values of all the nodes at each iteration of the algorithm.
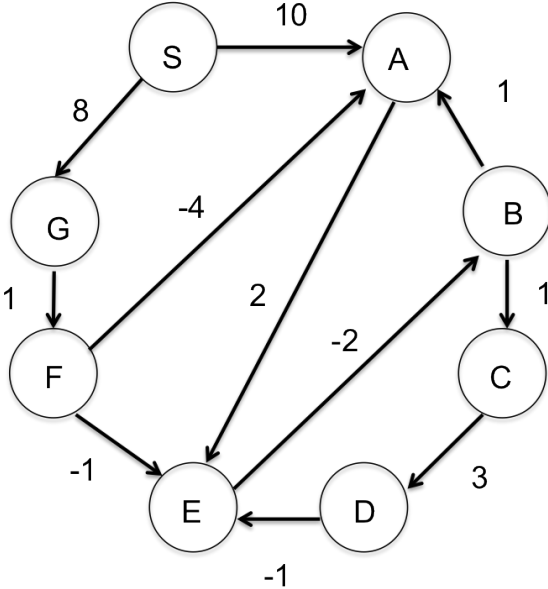
2. (10) Give a liner time algorithm that takes as input directed acyclic graph $G = (V, E)$ and two vertices $s$ and $t$ and returns the number of simple paths from $s$ to $t$. Your algorithm should just count the paths not list them. Simple path is a path with no repeated vertices or edges. The algorithm should be linear in $O(V + E)$ and exploit the fact that the graph is acyclic.

3. (10) A subsequence is palindromic if its the same whether read from left to right or right to left. For instance the sequence

A,C,G,T.G,T,C,A,A,A,A,T,C,G

has many palindromic subsequences including A,C,G,C,A and A,A,A,A. Devise an algorithm that takes the sequence $x[1, \ldots n]$ and returns the length of the longest palindromic subsequence. This problem can be solved using dynamic programming. Characterize recursively the solution of $x[i \ldots j]$ and describe the algorithm to solve the problem and characterize its running time.

4. (10) Several families go out to dinner. To increase the social interaction, they would like to sit at the tables so that no two members of the same family are at the same table. Show how to find a seating arrangement that meets this objective or prove that such arrangement does not exist using max-flow problem.

   Assume that there are $p$ families and that $i$ the family has $a_i$ members. Also assume that there are $q$ tables available and $j$ the table has capacity $b_j$.

5. (15) Are the following statements TRUE or FALSE. Justify in few sentences.

(3) All problems in P are also in NP .

TRUE

Problems which can be solved in polynomial time can also be verified in polynomial time.

(3) Every NP-complete problem is in NP.

TRUE, from definition.

(3) If a problem A is polynomially reducible to problem B and problem A is NP-complete then problem B is also NP-complete.

TRUE (see quiz)

(3) The approximate solutions to NP-complete problems can never give an optimal answer.

FALSE. This depends on the instance of the problem. Sometimes they can give optimal solutions.

(3) Suppose you have a graph $G$ and its Minimum Weight Spanning Tree $T$. Now replace each edge cost with $c_e$ with $c_e^2$. T must still be a minimum spanning tree for this new instance. If it is true give a short explanation, if it is false give a counter example.

TRUE. The order in which the edges will be chosen for MWST will be the same.

6. (10) Let 2-CNF-SAT be a set of satisfiable boolean formulas in CNF with exactly 2 literals per clause. Show that 2-CNF-SAT $\in$ P. Suggest an efficient algorithm. (Hint: Observe that $x \vee y$ is equivalent to $\neg x \rightarrow y$ and $\neg y \rightarrow x$). Reduce 2-CNF-SAT to a problem in a directed graph that is efficiently solvable.