Rigid Body Motion

CS 685

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Linear Algebra Review
Rigid Body Motion in 2D
Rigid Body Motion in 3D

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Why do we need Linear Algebra?

- We will associate coordinates to
  - 3D points in the scene
  - 2D points in the CCD array
  - 2D points in the image
- Coordinates will be used to
  - Perform geometrical transformations
  - Associate 3D with 2D points
- Images are matrices of numbers
  - We will find properties of these numbers

2

Matrices

$$A_{n\times m} = \begin{bmatrix} a_{11} & a12 & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ & \dots & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{bmatrix}$$

Matrix Sum: $C_{n\times m} = A_{n\times m} + B_{n\times m}$

$c_{ij} = a_{ij} + b_{ij}$ A and B must have the same dimensions

Example:

$$\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 4 & 6 \end{bmatrix}$$

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Matrices

Product:

$C_{n\times p} = A_{n\times m} B_{m\times p}$ A and B must have compatible dimensions

$$c_{ij} = \sum_{k=1}^{m} a_{ik} b_{kj}$$

$$A_{n\times n} B_{n\times n} \neq B_{n\times n} A_{n\times n}$$

Examples:

$$\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 17 & 29 \\ 19 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 18 & 32 \\ 17 & 10 \end{bmatrix}$$

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Matrices

Transpose:

$$C_{m\times n} = A^T{}_{n\times m} \qquad (A+B)^T = A^T + B^T$$

$$c_{ij} = a_{ji} \qquad (AB)^T = B^T A^T$$

If $A^T = A$ A is symmetric

Examples:

$$\begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 \\ 2 & 5 \end{bmatrix} \qquad \begin{bmatrix} 6 & 2 \\ 1 & 5 \\ 3 & 8 \end{bmatrix}^T = \begin{bmatrix} 6 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix}$$

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Matrices

Determinant: A must be square

$$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

$$\det\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Example: $\det\begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix} = 2 - 15 = -13$

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Matrices

Inverse: A must be square

$$A_{n\times n}A^{-1}{}_{n\times n} = A^{-1}{}_{n\times n}A_{n\times n} = I$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{a_{11}a_{22} - a_{21}a_{12}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Example: $\begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix}^{-1} = \frac{1}{28}\begin{bmatrix} 5 & -2 \\ -1 & 6 \end{bmatrix}$

$$\begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} = \frac{1}{28}\begin{bmatrix} 5 & -2 \\ -1 & 6 \end{bmatrix} \cdot \begin{bmatrix} 6 & 2 \\ 1 & 5 \end{bmatrix} = \frac{1}{28}\begin{bmatrix} 28 & 0 \\ 0 & 28 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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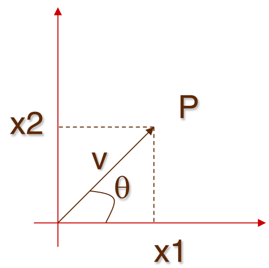
2D,3D Vectors



$$\mathbf{v} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \qquad \mathbf{v} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$$

Magnitude: $\|\mathbf{v}\| = \sqrt{x_1^2 + x_2^2}$

If $\|\mathbf{v}\| = 1$, $\mathbf{v}$ is a UNIT vector

$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{x_1}{\|\mathbf{v}\|}, \frac{x_2}{\|\mathbf{v}\|}\right)$ Is a unit vector

Orientation: $\theta = \tan^{-1}\left(\frac{x_2}{x_1}\right)$

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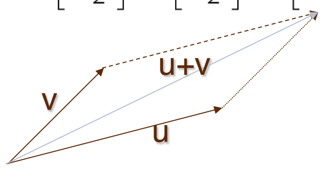
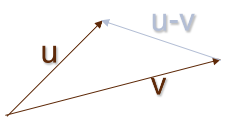
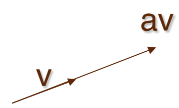
Vector Addition, Subtraction, Scalar Product

$$u+v=\begin{bmatrix}u_1\\u_2\end{bmatrix}+\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}u_1+v_1\\u_2+v_2\end{bmatrix}$$



$$u-v=\begin{bmatrix}u_1\\u_2\end{bmatrix}-\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}u_1-v_1\\u_2-v_2\end{bmatrix} \qquad av=a\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}av_1\\av_2\end{bmatrix}$$





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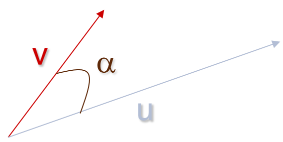
Inner (dot) Product



$$u^Tv=\begin{bmatrix}u_1 & u_2\end{bmatrix}\begin{bmatrix}v_1\\v_1\end{bmatrix}=u_1.v_1+u_2.v_2$$

The inner product is a SCALAR!

$$u^Tv=\begin{bmatrix}u_1 & u_2\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix}=\|u\|\|v\|\cos\alpha$$

$$u^Tv=0\leftrightarrow u\perp v$$

$$u=\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} \qquad v=\begin{bmatrix}v_1\\v_2\\v_3\end{bmatrix}$$

$$\langle u,v\rangle \doteq u^Tv=u_1v_1+u_2v_2+u_3v_3 \qquad \cos(\theta)=\frac{\langle u,v\rangle}{\|u\|\|v\|}$$

$$\|u\|\doteq\sqrt{u^Tu}=\sqrt{u_1^2+u_2^2+u_3^3}$$ norm of a vector

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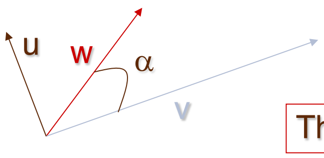
Vector (cross) Product



$$u = v \times w$$

The cross product is a VECTOR!

Magnitude: $\| u \| = \| v \times w \| = \| v \| \| w \| \sin\alpha$

Orientation:

$$\begin{aligned} u \perp v \rightarrow u^T v &= (u \times v)^T v = 0 \\ u \times v &= -v \times u \\ a(u \times v) &= au \times v = u \times av \\ u \parallel u \rightarrow (u \times v) &= 0 \end{aligned}$$

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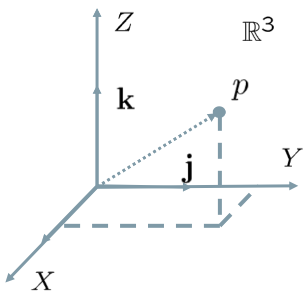
Orthonormal Basis in 3D

Standard base vectors:

$$\mathbf{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \mathbf{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$



Coordinates of a point p in space:

$$\boldsymbol{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \in \mathbb{R}^3 \qquad \boldsymbol{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = X.\mathbf{i} + Y.\mathbf{j} + Z.\mathbf{k}$$

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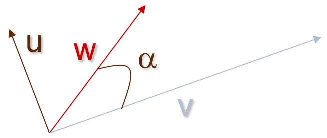
Vector (Cross) Product Computation

$$\mathbf{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \mathbf{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \mathbf{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

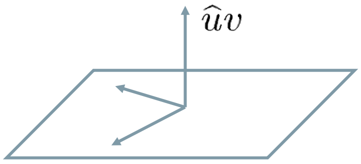
$$u \times v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$



$$= (u_2v_3 - u_3v_2)\mathbf{i} + (u_3v_1 - u_1v_3)\mathbf{j} + (u_1v_2 - u_2v_1)\mathbf{k}$$

$$u \times v \doteq \hat{u}v, \quad u, v \in \mathbb{R}^3$$

$$\hat{u} = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix} \in \mathbb{R}^{3\times3}$$



Skew symmetric matrix associated with vector

$$\hat{u} = -(\hat{u})^T$$

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2D Geometrical Transformations

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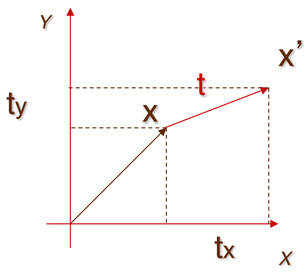
2D Translation Equation



$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\mathbf{t} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\mathbf{x}' = \mathbf{x} + t = \begin{bmatrix} \mathbf{x} + t_x \\ \mathbf{y} + t_y \end{bmatrix}$$

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Homogeneous Coordinates

Homogeneous coordinates:

$$\mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix} \quad \rightarrow \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \in \mathbb{R}^3 ,$$

Translation using matrices:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\mathbf{x}' = P_t \mathbf{x}$$

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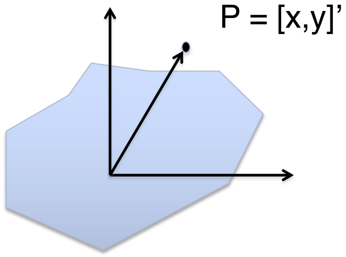
Coordinate frames

- In order to specify a position of a rigid body In 2D space, we need to attach a coordinate frame to it

- Frame defines a coordinate system
- Coordinates of any point on the body can be expressed in that coordinate system



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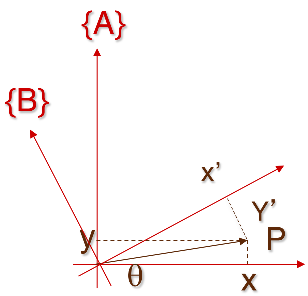
Rotation Matrix

- Counter-clockwise rotation of a coordinate frame by an angle θ



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Counter-clockwise rotation of a coordinate frame attached to a rigid body by an angle θ

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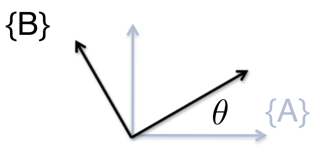
Rotation Matrix

Interpretations of the rotation matrix R_{AB}



$$R_{AB} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Columns of R_{AB} are the unit vectors of the axes of frame B expressed in coordinate frame A. Such rotation matrix transforms coordinates of points in frame B to points in frame A

Use of the rotation matrix as transformation R_{AB}

$$\mathbf{X}_A = R_{AB}\mathbf{X}_B$$

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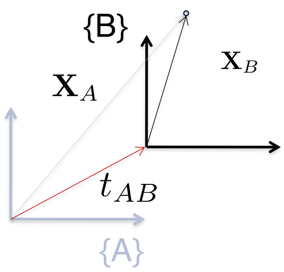
Rigid Body Transform

Translation only, t_{AB} is the origin of the frame B expressed in the Frame A

$$\mathbf{X}_A = \mathbf{X}_B + t_{AB}$$

Composite transformation:

$$\mathbf{X}_A = R_{AB}\mathbf{X}_B + t_{AB}$$



Transformation: $T = (R_{AB}, t_{AB})$

Homogeneous coordinates

$$\mathbf{X}_A = \begin{bmatrix} R_{AB} & t_{AB} \\ 0 & 1 \end{bmatrix} \mathbf{X}_B$$

The points from frame A to frame B are transformed by the inverse of $T = (R_{AB}, t_{AB})$ (see example next slide)

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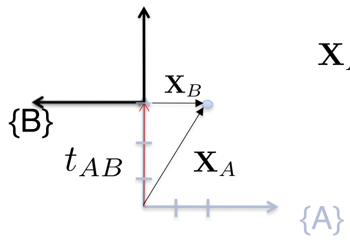
Example

$$\mathbf{X}_A = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix} \mathbf{X}_B$$

In homogeneous coordinates:

$$\mathbf{X}_A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{X}_B \quad \text{for} \quad \theta = 90^o, t_{AB} = [0, 3]^T$$



$$\mathbf{X}_A = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \quad \mathbf{X}_B = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

Verify that the inverse of the above transform
Transforms coordinates in frame {A} to frame {B}

Degrees of Freedom

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

R is 2x2 ⇨ 4 elements

BUT! There is only 1 degree of freedom: θ

The 4 elements must satisfy the following constraints:

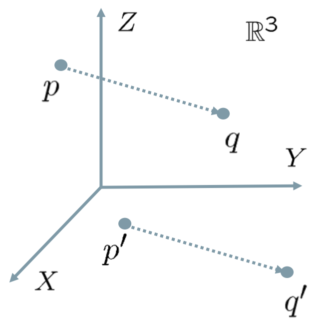
$$R.R^T = I$$ Rows and columns are orthogonal and of unit length

$$det(R) = 1$$ Matrix is orientation preserving

3-D Euclidean Space - Vectors

A "free" vector is defined by a pair of points (p, q)

$$\boldsymbol{X}_p = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \in \mathbb{R}^3, \ \boldsymbol{X}_q = \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \in \mathbb{R}^3,$$



Coordinates of the vector :

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} X_2 - X_1 \\ Y_2 - Y_1 \\ Z_2 - Z_1 \end{bmatrix} \in \mathbb{R}^3$$

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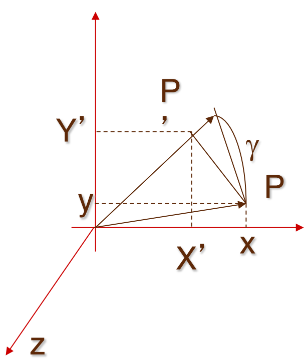
3D Rotation of Points – Euler angles

Rotation around the coordinate axes, counter-clockwise:

$$\begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$



$$R = R_z(\gamma) R_y(\beta) R_x(\alpha)$$

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Rotation Matrix

- Euler theorem – any rotation can be expressed as a sequence of rotations around different coordinate axes
- Different order of rotations yields different final rotation
- Rotation multiplication is not commutative

- Different ways how to obtain final rotation – rotation around 3 axes no successive rotations around same axes
- XYX, XZX, YXY, YZX, ZXZ, ZYZ – Eulerian involves repetition
- Cardanian – no repetitions XYZ, XZY, YZX, YXZ, ZXY, ZYX.
- Another widely used convention roll-pitch-yaw

$$R = R_z(\alpha)R_y(\beta)R_x(\gamma)$$

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Rotation Matrices in 3D

- 3 by 3 matrices
- 9 parameters – only three degrees of freedom
- Representations – either three Euler angles
- or axis and angle representation

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

- Properties of rotation matrices (constraints between the elements)

$$RR^T = I$$

$$det(R) = I$$

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Rotation Matrix

- Problem with 3 angle representations: singularities
- The mapping between angles and Rotation matrix is unique
- i.e. given the rotation matrix, compute ϕ, θ, ψ

$$\begin{bmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\phi\sin\phi + \cos\theta\cos\psi\sin\phi & \sin\psi\sin\theta \\ \sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{bmatrix}$$

- The inverse mapping between rotation matrix and the angles sometimes cannot be computed or is not unique

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Rotation Matrix

- Problem with 3 angle representations: singularities
- The mapping between angles and Rotation matrix is unique
- i.e. given the rotation matrix, compute ϕ, θ, ψ

$$\begin{bmatrix} \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi & \cos\phi\sin\phi + \cos\theta\cos\psi\sin\phi & \sin\psi\sin\theta \\ \sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi & -\sin\psi\sin\phi + cos\theta\cos\phi\cos\psi & \cos\psi\sin\theta \\ \sin\theta\sin\phi & -\sin\theta\cos\phi & \cos\theta \end{bmatrix}$$

- The inverse mapping between rotation matrix and the angles sometimes cannot be computed or is not unique

Angle Axis Representation

- Two coordinates frames of arbitrary orientations can be related by a single rotation about 'some' axis in space and an angle

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Rotation Matrices in 3D

- 3 by 3 matrices
- 9 parameters – only three degrees of freedom
- Representations – either three Euler angles
- or axis and angle representation

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

- Properties of rotation matrices (constraints between the elements)

$$R.R^T = I \qquad r_i^T r_j = \delta_{ij} \doteq \begin{cases} 1 & \text{for} \quad i = j, \\ 0 & \text{for} \quad i \neq j, \end{cases} \qquad \forall i, j \in \{1, 2, 3\}.$$

$det(R) = I$ Columns are orthonormal

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Rotation Matrix

- Problem with 3 angle representations: singularities
- The mapping between angles and Rotation matrix is unique
- The inverse mapping between Rotation matrix and the angles sometimes cannot be computed or is not unique

Angle Axis Representation:

- Two coordinates frames of arbitrary orientations can be related by a single rotation about 'some' axis in space and an angle

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Canonical Coordinates for Rotation

Property of R $R(t)R^T(t) = I$

Taking derivative

$$\dot{R}(t)R^T(t) + R(t)\dot{R}^T(t) = 0 \quad \Rightarrow \quad \dot{R}(t)R^T(t) = -(\dot{R}(t)R^T(t))^T$$

Skew symmetric matrix property

$$\dot{R}(t)R^T(t) = \widehat{\omega}(t)$$

By algebra

$$\dot{R}(t) = \widehat{\omega}R(t)$$

By solution to ODE

$$R(t) = e^{\widehat{\omega}t}$$

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3D Rotation (axis & angle)

Solution to the ODE

$$R(t) = e^{\widehat{\omega}t}$$

$$R = I + \widehat{\omega} sin(\theta) + \widehat{\omega}^2(1 - cos(\theta))$$

with $\|\omega\| = 1$ $\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \in \mathbb{R}^3$

or

$$R = I + \frac{\widehat{\omega}}{\|\omega\|}\sin(\|\omega\|) + \frac{\widehat{\omega}^2}{\|\omega\|^2}(1 - \cos(\|\omega\|))$$

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Rotation Matrices

Given

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix},$$

How to compute angle and axis

$$\|\omega\| = \cos^{-1}\left(\frac{\text{trace}(R) - 1}{2}\right), \quad \frac{\omega}{\|\omega\|} = \frac{1}{2\sin(\|\omega\|)} \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}.$$

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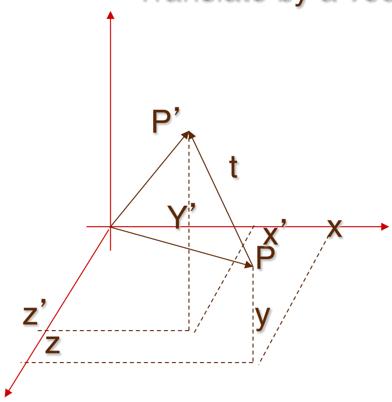
3D Translation of Points

Translate by a vector $t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \in \mathbb{R}^3$



$$\mathbf{x}' = x + t = \begin{bmatrix} X + t_x \\ Y + t_y \\ Z + t_z \end{bmatrix}$$

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Rigid Body Motion – Homogeneous Coordinates

3-D coordinates are related by: $\boldsymbol{X}_c = R\boldsymbol{X}_w + T,$

Homogeneous coordinates:

$$\boldsymbol{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \rightarrow \quad \boldsymbol{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \in \mathbb{R}^4,$$

Homogeneous coordinates are related by:

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

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Rigid Body Motion – Homogeneous Coordinates

3-D coordinates are related by: $\boldsymbol{X}_c = R\boldsymbol{X}_w + T,$

Homogeneous coordinates:

$$\boldsymbol{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \rightarrow \quad \boldsymbol{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \in \mathbb{R}^4,$$

Homogeneous coordinates are related by:

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix} = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

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Properties of Rigid Body Motions

Rigid body motion composition

$$\bar{g}_1\bar{g}_2 = \begin{bmatrix} R_1 & T_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_2 & T_2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1R_2 & R_1T_2 + T_1 \\ 0 & 1 \end{bmatrix} \quad \in SE(3)$$

Rigid body motion inverse

$$\bar{g}^{-1} = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} R^T & -R^T T \\ 0 & 1 \end{bmatrix} \quad \in SE(3).$$

Rigid body motion acting on vectors

Vectors are only affected by rotation – 4^{th} homogeneous coordinate is zero

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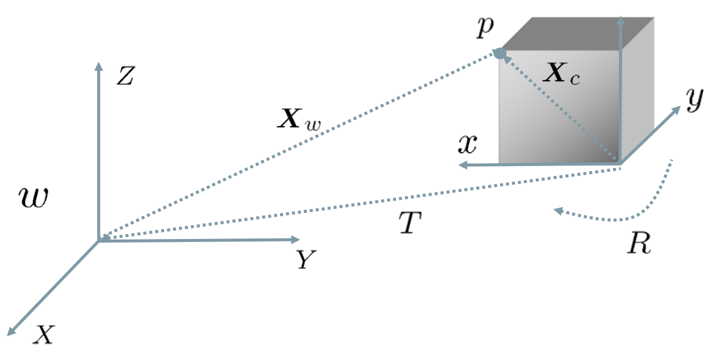
Rigid Body Transformation



Coordinates are related by: $X_c = RX_w + T,$

Camera pose is specified by: $g = (R, T) \in SE(3)$

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Rigid Body Motion

- Shown how to describe positions and orientations of coordinate frames (poses) with respect to the origin world frame
- Relative pose (R,T) – relationship between two consecutive poses