Motion Planning

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- Potential Field Based Methods

Slides thanks to http://cs.cmu.edu/~motionplanning, Jyh-Ming Lien

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Potential Field Methods

- Construct a function over the extend of configuration space
- Environment is represented as a potential field

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Potential Field Methods

- Idea robot is a particle
- Environment is represented as a potential field (locally)
- Advantage – capability to generate on-line collision avoidance

Compute force acting on a robot – incremental path planning

$$F(q) = -\nabla U(q)$$

Example: Robot can translate freely , we can control independently
Environment represented by a potential function

$$U(x, y)$$

Force is proportional to the gradient of the potential function

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = -\nabla U(x, y)$$

Some slide thanks to http://cs.cmu.edu/~motionplanning

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Attractive Potential Field

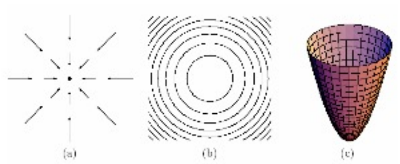
- Linear function of distance

$$U_a(q) = \xi \|q - q_{goal}\| \quad F_a(q) = -\nabla U_a(q) = -\xi \frac{(q - q_{goal})}{\|q - q_{goal}\|}$$

- Quadratic function of distance

$$U_a(q) = \xi \frac{1}{2} \|q - q_{goal}\|^2 \quad F_a(q) = -\nabla U_a(q) = -\xi (q - q_{goal})$$

Combination of two – far away use line
closer by use parabolic well



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Repulsive Potential Field

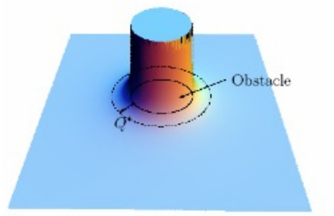
$$U_{rep} = \frac{1}{2}\nu \left(\frac{1}{\rho(q, q_{obst})} - \frac{1}{\rho_0} \right)^2 \quad \text{if} \quad \rho(q, q_{obst})\| \leq \rho_0$$

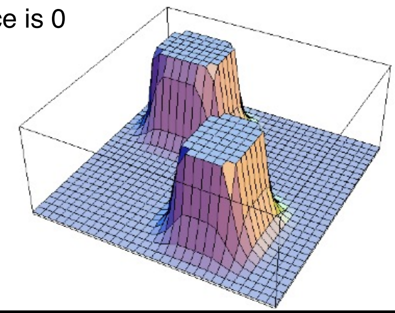
$$\text{else} \quad U_r(q) = 0$$

Minimal distance between the robot and the obstacle

$$F_{rep} = -\nabla U_{rep} = \nu \left(\frac{1}{\rho(q)} - \frac{1}{\rho_0} \right) \frac{1}{\rho(q)^2} \frac{q - q_{obs}}{\rho(q)}$$

Outside of sensitivity zone repulsive force is 0





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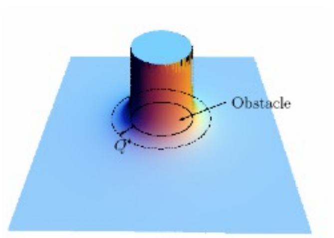
Repulsive Potential Field

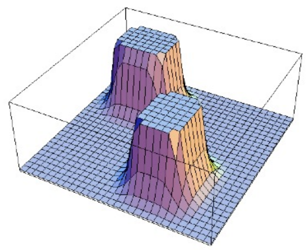
$$U_r(q) = \frac{1}{2}\nu \left(\frac{1}{\rho(q, q_{obst})} - \frac{1}{\rho_o} \right) \quad \text{if} \quad \rho(q, q_{obst})\| \leq \rho_0$$

$$\text{else} \quad U_r(q) = 0$$

Minimal distance between the robot and the obstacle

Previously – repulsive potential related to the square of the
Inverse distance – here just proportional to inverse distance
Note: need to compute gradient to get the force





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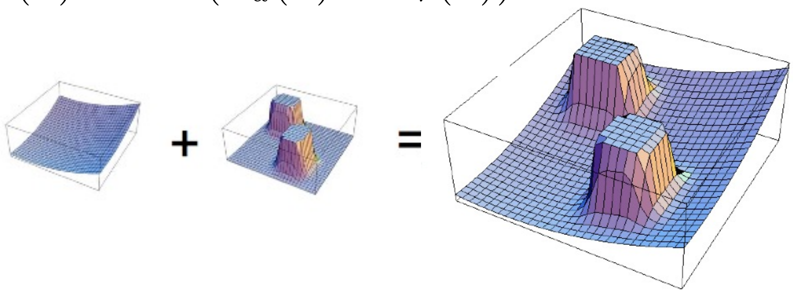
Potential Function

Resulting force $F(\mathbf{x}) = -\nabla(U_a(\mathbf{x}) + U_r(\mathbf{x}))$

$F(q) = -\nabla(U_a(q) + U_r(q))$

Iterative gradient descent planning $q_{i+1} = q_i + \delta_i \frac{F(q)}{\|F(q)\|}$

$$F(\mathbf{x}) = -\nabla(U_a(\mathbf{x}) + U_r(\mathbf{x}))$$



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Potential Fields

- Simple way to get to the bottom, follow the gradient
- Make the speed proportional to the gradient

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = -\nabla U(x, y) \qquad \mathbf{v} \propto -\nabla(U_a(\mathbf{x}) + U_r(\mathbf{x}))$$

- Gradient descent strategy

$$\dot{q} = -\nabla U(q) \qquad \nabla U(q) = 0$$

- A critical, stationary point is such that
- Equation is stationary at the critical point
- To check whether critical point is a minimum – look at the second order derivatives (Hessian for m -> n function)

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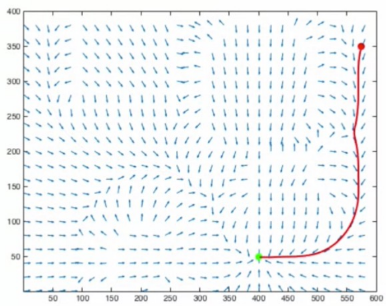
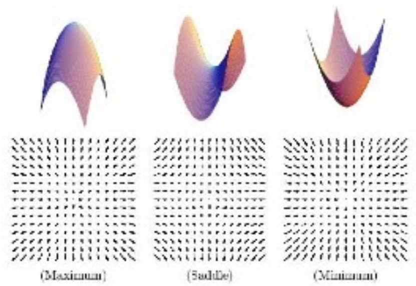
Following gradient of the potential field

$$\mathbf{v} \propto -\nabla(U_a(\mathbf{x}) + U_r(\mathbf{x}))$$





- Attractive approach towards control
- Potential function can be instantiated from local sensing data

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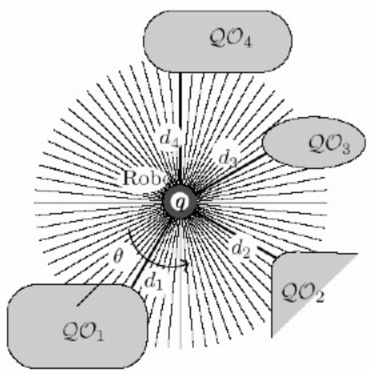
Computing Distances

- In practice the distances are computed using sensors
- Obstacles are not circular
- Consider a range sensor which returns the closest distance to the obstacle



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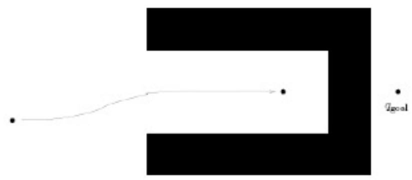
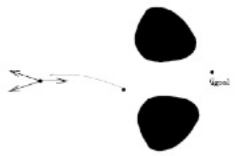
Potential Functions

- How do we know we have single global minimum ?





- If global minimum is not guaranteed, need to do something else then gradient descent
- Design functions in such a way that global minimum can be guaranteed

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Potential function

- Local minima – hard to eliminate
- Strategies for escaping the local minima
- Can be used in local and global context
- Numerical techniques, Random walk methods

- Navigation functions (Rimon & Kodistchek, 92)
- Navigations in sphere worlds and worlds diffeomorphic to them

- Potential fields – useful heuristics

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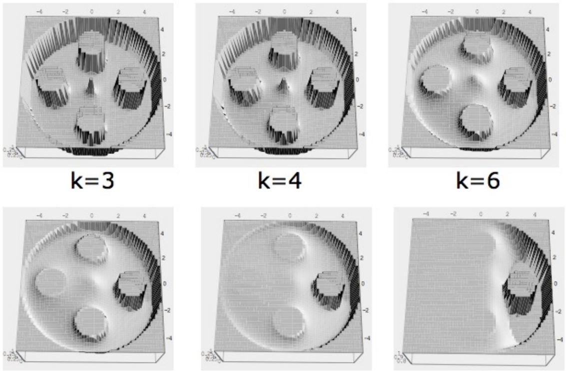
Navigation functions

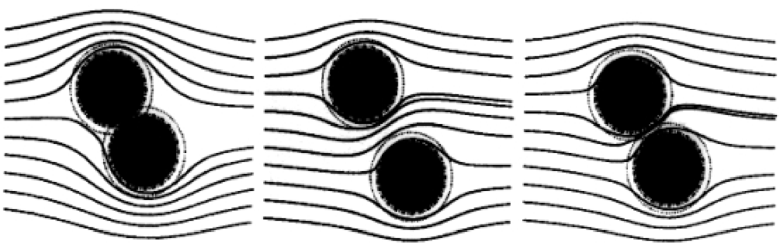
$$\phi(q) = -\frac{d^2(q, q_{goal})}{[d(q, q_{goal})^{2k} + \beta(q)]^{1/k}} \qquad \beta(q) \quad \text{Obstacle term}$$

- For sufficiently large k – this is a navigation function [Rimon-Koditschek, 92]



Potential Field Path Planning: Using Harmonic Potentials

- Hydrodynamics analogy
  - robot is moving similar to a fluid particle following its stream



- Note:
  - Complicated, only simulation shown

Generalized Potential fields

- So far robot was considered a point- gradient of the potential function – force acting on a point
- For robots with many degrees of freed – consider set of control points distributed over surface of robot, position of control points is computed as function of configuration space parameters $P_1(\mathbf{x})$
- For each control point construct potential function $f_i(P_i(\mathbf{x}))$
- Distances computed in the workspace
- Final function $f(\mathbf{x}) = \sum_i f_i(P_i(\mathbf{x}))$
- Control

$$\mathbf{v} \propto -\nabla f(\mathbf{x}) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \dots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

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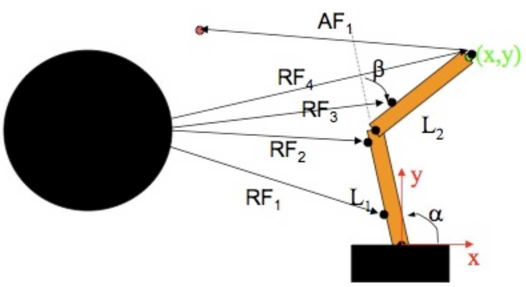
Potential fields for Rigid Bodies

- How to generalize to manipulators of objects ?
- Idea – forces acting on objects – forces acting on multiple points of the object (black board)
- http://www.cs.cmu.edu/~motionplanning/
- For robots, pick enough control points to pin down the robot – define forces in workspace – map them to configuration space



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