Probabilistic Robotics

Bayes Filter Implementations

Gaussian filters

1

Kalman Filter Algorithm

1. Algorithm **Kalman_filter**(μ_{t-1}, Σ_{t-1}, u_t, z_t):

2. Prediction:
3. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

5. Correction:
6. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
8. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

9. Return μ_t, Σ_t

3

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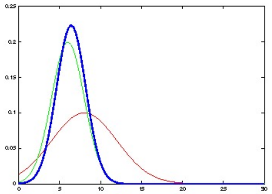
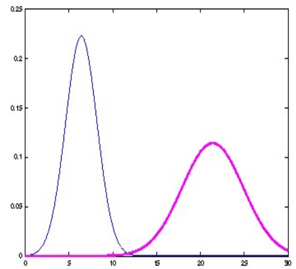
The Prediction-Correction-Cycle

Prediction



$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = a_t \mu_{t-1} + b_t u_t \\ \bar{\sigma}_t^2 = a_t^2 \sigma_t^2 + \sigma_{act,t}^2 \end{cases}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$



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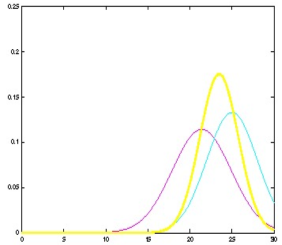
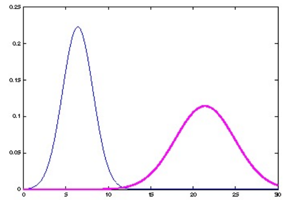
The Prediction-Correction-Cycle



$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - \bar{\mu}_t) \\ \sigma_t^2 = (1 - K_t)\bar{\sigma}_t^2 \end{cases}, K_t = \frac{\bar{\sigma}_t^2}{\bar{\sigma}_t^2 + \bar{\sigma}_{obs,t}^2}$$

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - C_t \bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases}, K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$



Correction

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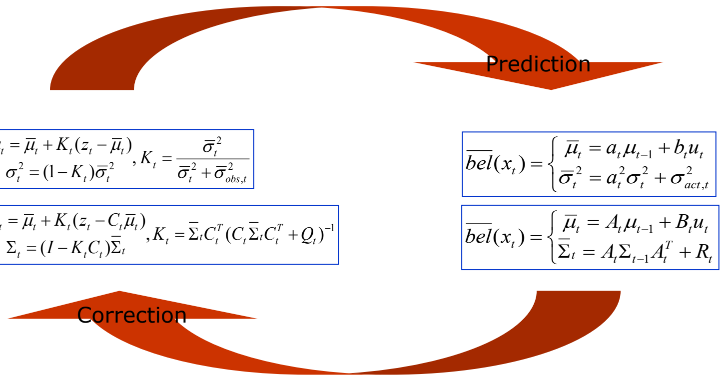
The Prediction-Correction-Cycle



$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = a_t \mu_{t-1} + b_t u_t \\ \bar{\sigma}_t^2 = a_t^2 \sigma_t^2 + \sigma_{act,t}^2 \end{cases}$$

$$\overline{bel}(x_t) = \begin{cases} \bar{\mu}_t = A_t \mu_{t-1} + B_t u_t \\ \bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t \end{cases}$$

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - \bar{\mu}_t) \\ \sigma_t^2 = (1 - K_t)\bar{\sigma}_t^2 \end{cases}, K_t = \frac{\bar{\sigma}_t^2}{\bar{\sigma}_t^2 + \bar{\sigma}_{obs,t}^2}$$

$$bel(x_t) = \begin{cases} \mu_t = \bar{\mu}_t + K_t(z_t - C_t \bar{\mu}_t) \\ \Sigma_t = (I - K_t C_t)\bar{\Sigma}_t \end{cases}, K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$$

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Kalman Filter Summary

- Highly efficient: Polynomial in measurement dimensionality *k* and state dimensionality *n*:
  $O(k^{2.376} + n^2)$
- Optimal for linear Gaussian systems!
- Most robotics systems are nonlinear!

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Nonlinear Dynamic Systems

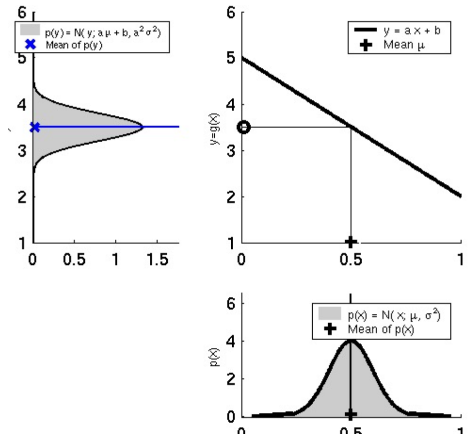
- Most realistic robotic problems involve nonlinear functions

$$x_t = g(u_t, x_{t-1})$$

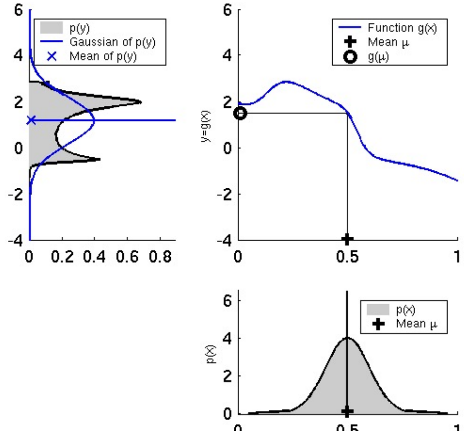
$$z_t = h(x_t)$$

- To be continued

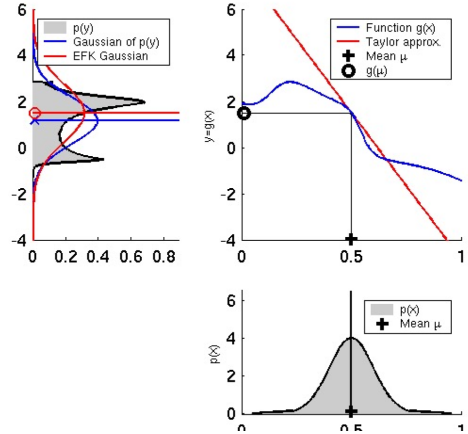
Linearity Assumption Revisited



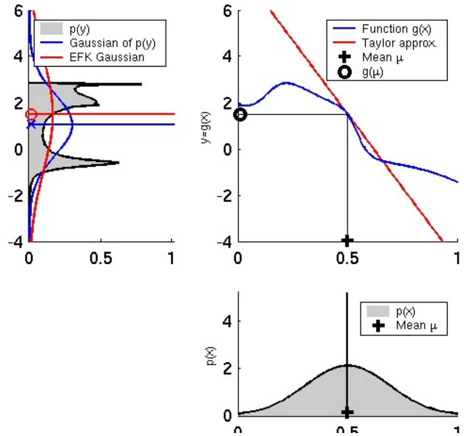
Non-linear Function



EKF Linearization (1)



EKF Linearization (2)



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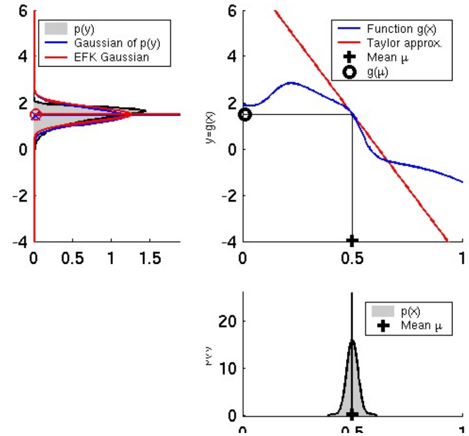
EKF Linearization (3)



Depends on uncertainty

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EKF Linearization: First Order Taylor Series Expansion

- Prediction:

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}(x_{t-1} - \mu_{t-1})$$

$$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + G_t\,(x_{t-1} - \mu_{t-1})$$

- Correction:

$$h(x_t) \approx h(\bar{\mu}_t) + \frac{\partial h(\bar{\mu}_t)}{\partial x_t}(x_t - \bar{\mu}_t)$$

$$h(x_t) \approx h(\bar{\mu}_t) + H_t\,(x_t - \bar{\mu}_t)$$

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EKF Algorithm

1. **Extended_Kalman_filter**($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$):
2. Prediction:
3. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ ⟵ $\bar{\mu}_t = A_t\mu_{t-1} + B_t u_t$
4. $\bar{\Sigma}_t = G_t\Sigma_{t-1}G_t^T + R_t$ ⟵ $\bar{\Sigma}_t = A_t\Sigma_{t-1}A_t^T + R_t$
5. Correction:
6. $K_t = \bar{\Sigma}_t H_t^T(H_t\bar{\Sigma}_t H_t^T + Q_t)^{-1}$ ⟵ $K_t = \bar{\Sigma}_t C_t^T(C_t\bar{\Sigma}_t C_t^T + Q_t)^{-1}$
7. $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$ ⟵ $\mu_t = \bar{\mu}_t + K_t(z_t - C_t\bar{\mu}_t)$
8. $\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t$ ⟵ $\Sigma_t = (I - K_t C_t)\bar{\Sigma}_t$
9. Return μ_t, Σ_t

$$H_t = \frac{\partial h(\bar{\mu}_t)}{\partial x_t} \qquad G_t = \frac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}}$$

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Robot Position Prediction

- In a first step, the robots position at time step k+1 is predicted based on its old location (time step k) and its movement due to the control input $u(k)$:

$$\hat{p}(k+1|k) = f(\hat{p}(k|k), u(k))$$

f: Odometry function

$$\Sigma_p(k+1|k) = \nabla_p f \cdot \Sigma_p(k|k) \cdot \nabla_p f^T + \nabla_u f \cdot \Sigma_u(k) \cdot \nabla_u f^T$$

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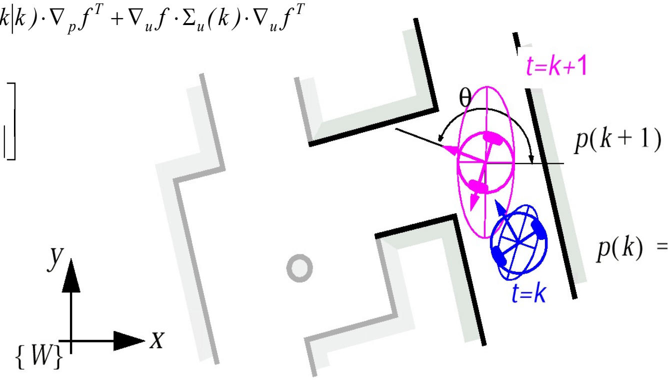
Robot Position Prediction: *Example*

$$\hat{p}(k+1|k) = \hat{p}(k|k) + u(k) = \begin{bmatrix} \hat{x}(k) \\ \hat{y}(k) \\ \hat{\theta}(k) \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_r + \Delta s_l}{2} \cos(\theta + \frac{\Delta s_r - \Delta s_l}{2b}) \\ \frac{\Delta s_r + \Delta s_l}{2} \sin(\theta + \frac{\Delta s_r - \Delta s_l}{2b}) \\ \frac{\Delta s_r - \Delta s_l}{b} \end{bmatrix}$$

Odometry

$$\Sigma_p(k+1|k) = \nabla_p f \cdot \Sigma_p(k|k) \cdot \nabla_p f^T + \nabla_u f \cdot \Sigma_u(k) \cdot \nabla_u f^T$$

$$\Sigma_u(k) = \begin{bmatrix} k_r|\Delta s_r| & 0 \\ 0 & k_l|\Delta s_l| \end{bmatrix}$$



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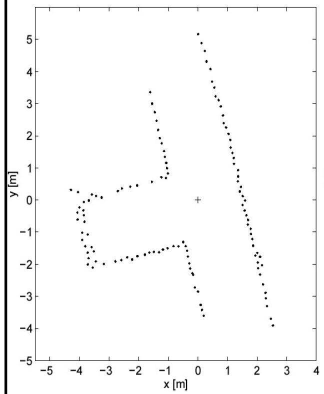
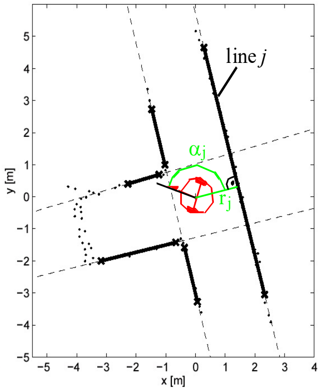
Kalman Filter for Mobile Robot Localization

Observation: *Example*

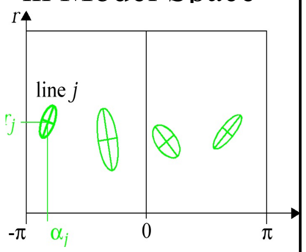
Raw Date of Laser Scanner



Extracted Lines



Extracted Lines in Model Space



$$z_j(k+1) = {}^{R}\begin{bmatrix} \alpha_j \\ r_j \end{bmatrix}$$

Sensor (robot) frame

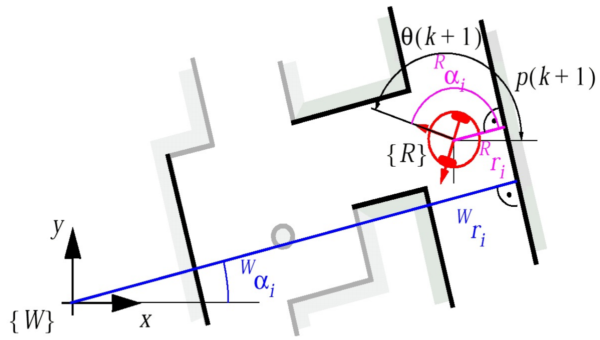
$$\Sigma_{R,j}(k+1) = \begin{bmatrix} \sigma_{\alpha\alpha} & \sigma_{\alpha r} \\ \sigma_{r\alpha} & \sigma_{rr} \end{bmatrix}_j$$

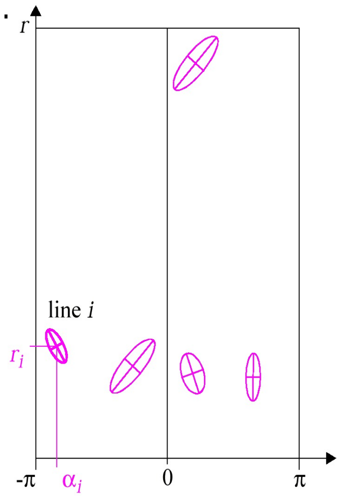
Set of discrete line features

Kalman Filter for Mobile Robot Localization

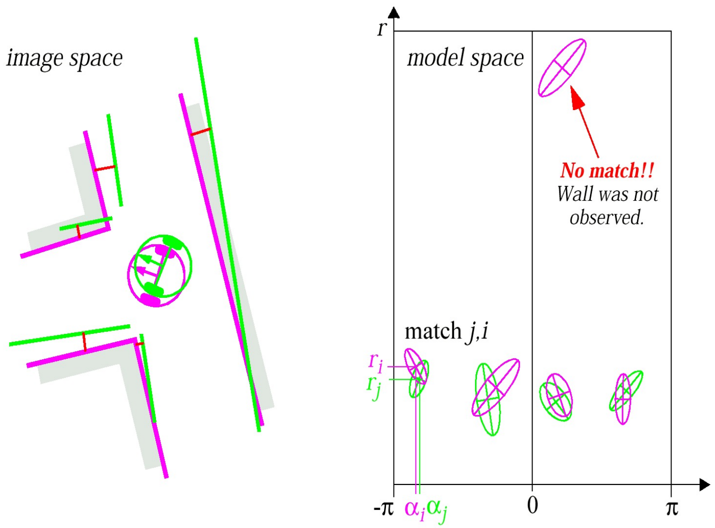
Measurement Prediction: *Example*

- For prediction, only the walls that are in the field of view of the robot are selected.
- This is done by linking the individual lines to the nodes of the path





Matching: *Example*



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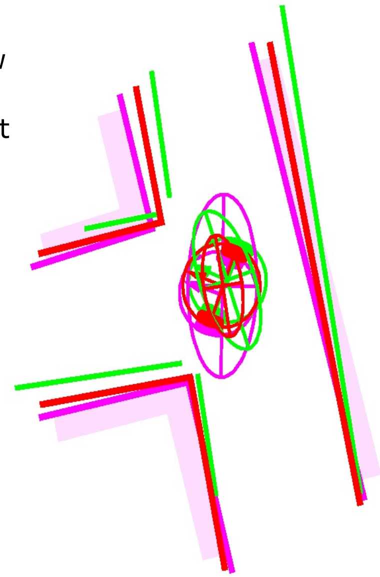
Kalman Filter for Mobile Robot Localization

Estimation: *Example*

- Kalman filter estimation of the new robot position $\hat{p}(k|k)$:
  - By fusing the prediction of robot position (magenta) with the innovation gained by the measurements (green) we get the updated estimate of the robot position (red)



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Localization

> "Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities." [Cox '91]

- **Given**
  - Map of the environment.
  - Sequence of sensor measurements.
- **Wanted**
  - Estimate of the robot's position.
- **Problem classes**
  - Position tracking
  - Global localization
  - Kidnapped robot problem (recovery)

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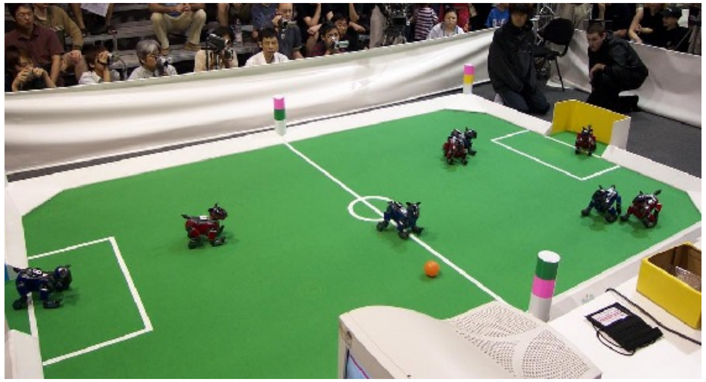
Landmark-based Localization

Chapter 7.4 Probabilistic robotics



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1. **EKF_localization** ($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, m$):

Prediction:

3. $G_t = \dfrac{\partial g(u_t, \mu_{t-1})}{\partial x_{t-1}} = \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix}$ Jacobian of *g* w.r.t location

5. $V_t = \dfrac{\partial g(u_t, \mu_{t-1})}{\partial u_t} = \begin{pmatrix} \frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\ \frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\ \frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t} \end{pmatrix}$ Jacobian of *g* w.r.t control

6. $M_t = \begin{pmatrix} (\alpha_1 |v_t| + \alpha_2 |\omega_t|)^2 & 0 \\ 0 & (\alpha_3 |v_t| + \alpha_4 |\omega_t|)^2 \end{pmatrix}$ Motion noise

7. $\bar{\mu}_t = g(u_t, \mu_{t-1})$ Predicted mean

8. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + V_t M_t V_t^T$ Predicted covariance

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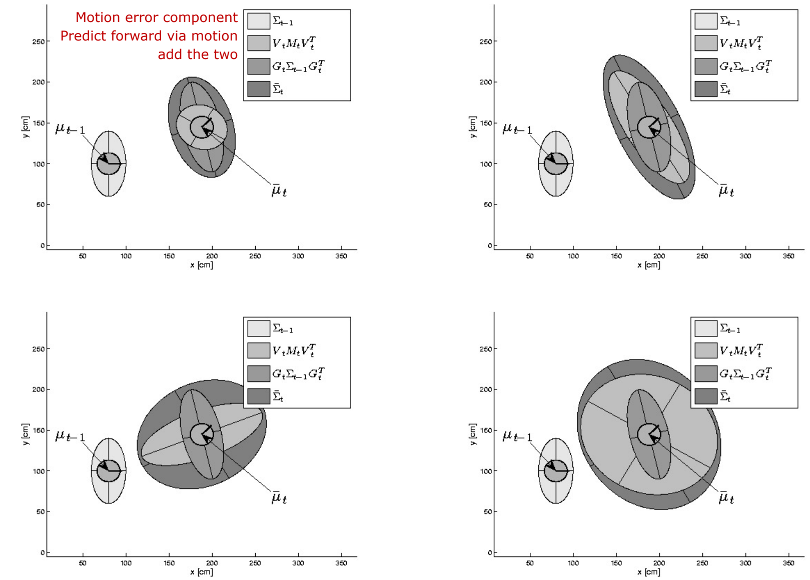
EKF Prediction Step

Prediction step for different motion noise parameters (10%,10%), (30%,10%),(10%,30%) (30%,30%) – for motion 10cm/sec, 5 degrees/sec for 9 seconds), previous mean (80,100,0)



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1. **EKF_localization** ($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, m$):

Correction:

3. $\hat{z}_t = \begin{pmatrix} \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2} \\ \operatorname{atan2}(m_y - \bar{\mu}_{t,y}, m_x - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix}$ Predicted measurement mean

5. $H_t = \frac{\partial h(\bar{\mu}_t, m)}{\partial x_t} = \begin{pmatrix} \frac{\partial r_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial \varphi_t}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix}$ Jacobian of *h* w.r.t location

6. $Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{pmatrix}$

7. $S_t = H_t \bar{\Sigma}_t H_t^T + Q_t$ Pred. measurement covariance

8. $K_t = \bar{\Sigma}_t H_t^T S_t^{-1}$ Kalman gain

9. $\mu_t = \bar{\mu}_t + K_t (z_t - \hat{z}_t)$ Updated mean

10. $\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$ Updated covariance

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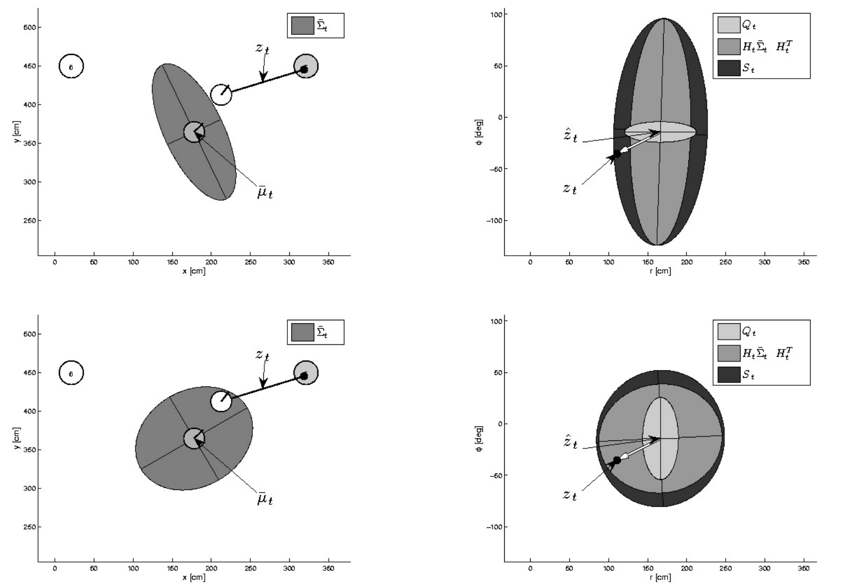
EKF Observation Prediction Step

S_t predicted
and actual measurements
measurement covariance Q_t



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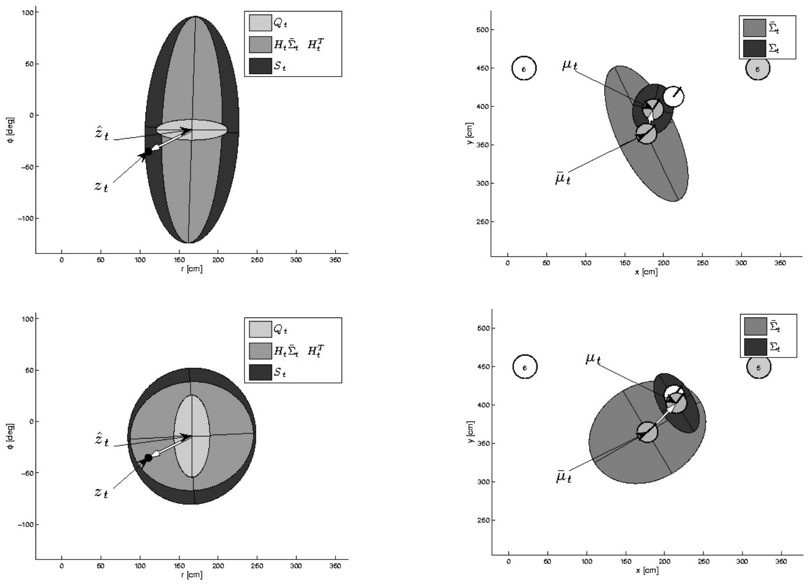
EKF Correction Step

Reduction in uncertainty
and different mean location



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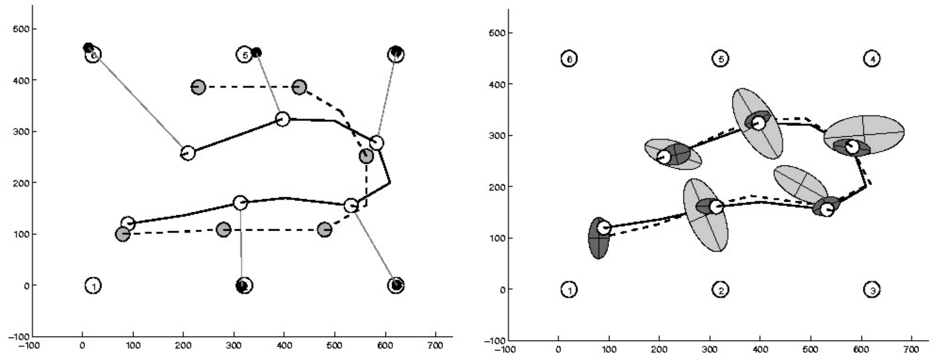
Estimation Sequence (1)



Errors with motion models, with landmark based localization

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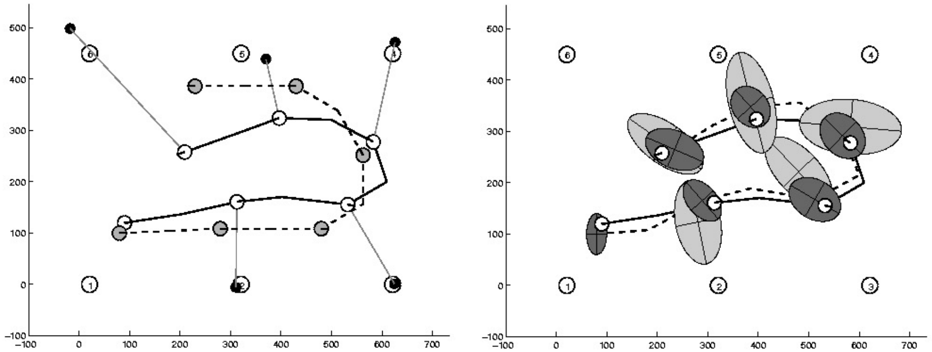
Estimation Sequence (2)



Errors with motion models, with landmark based localization with less accurate sensor

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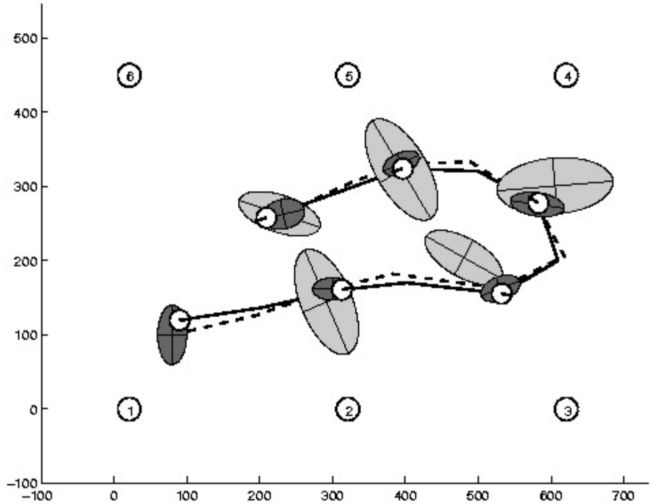
Comparison to GroundTruth



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EKF Summary

- Highly efficient: Polynomial in measurement dimensionality k and state dimensionality n:

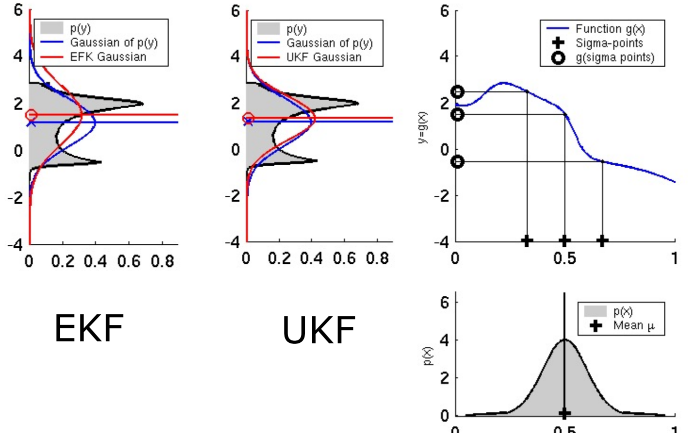
$$O(k^{2.376} + n^2)$$

- Not optimal!
- Can diverge if nonlinearities are large!
- Works surprisingly well even when all assumptions are violated!

- Unscented EKF

Linearization via Unscented Transform

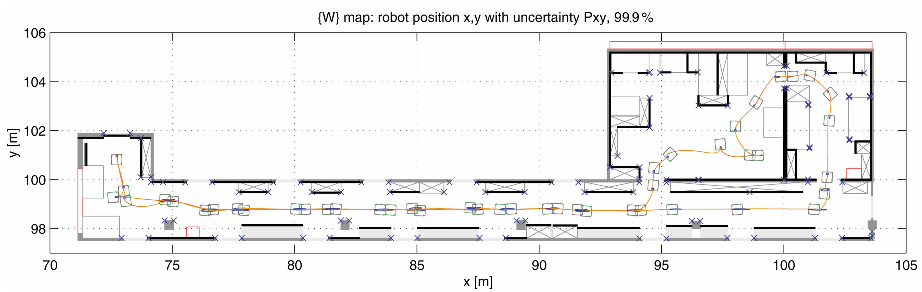
Chapter 3.4 Probabilistic robotics



Idea – better approximation of the mean and covariance Transform (sigma points of the original gaussian, to better Approximate the final distribution

Kalman Filter-based System

- [Arras et al. 98]:
  - Laser range-finder and vision
  - High precision (<1cm accuracy)



[Courtesy of Kai Arras]

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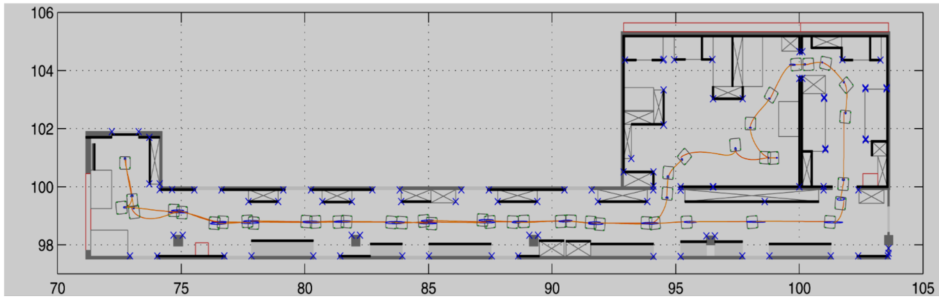
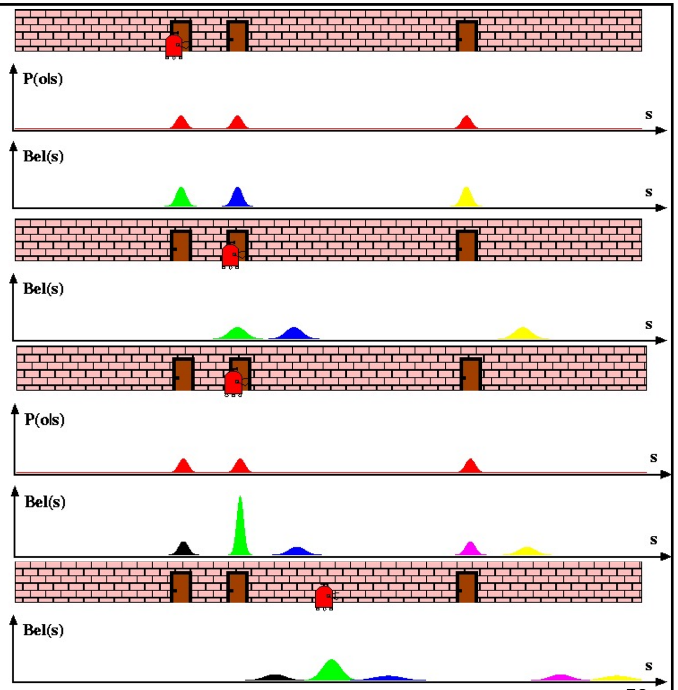
Autonomous Indoor Navigation (Pyg

- very robust on-the-fly localization
- one of the first systems with probabilistic s
- 47 steps,78 meter length, realistic office er
- conducted 16 times > 1km overall distance
- partially difficult surfaces (laser), partially f... edges (vision)



Multi-hypothesis Tracking



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Localization With MHT

- Belief is represented by multiple hypotheses
- Each hypothesis is tracked by a Kalman filter
- **Additional problems**:
  - Data association: Which observation corresponds to which hypothesis?
  - Hypothesis management: When to add / delete hypotheses?
- Huge body of literature on target tracking, motion correspondence etc.

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MHT: Implemented System (1)

- Hypotheses are extracted from LRF scans
- Each hypothesis has probability of being the correct one:

$$H_i = \{\hat{x}_i, \Sigma_i, P(H_i)\}$$

- Hypothesis probability is computed using Bayes' rule

$$P(H_i \mid s) = \frac{P(s \mid H_i) P(H_i)}{P(s)}$$

- Hypotheses with low probability are deleted.
- New candidates are extracted from LRF scans.

$$C_j = \{z_j, R_j\}$$

[Jensfelt et al. '00]

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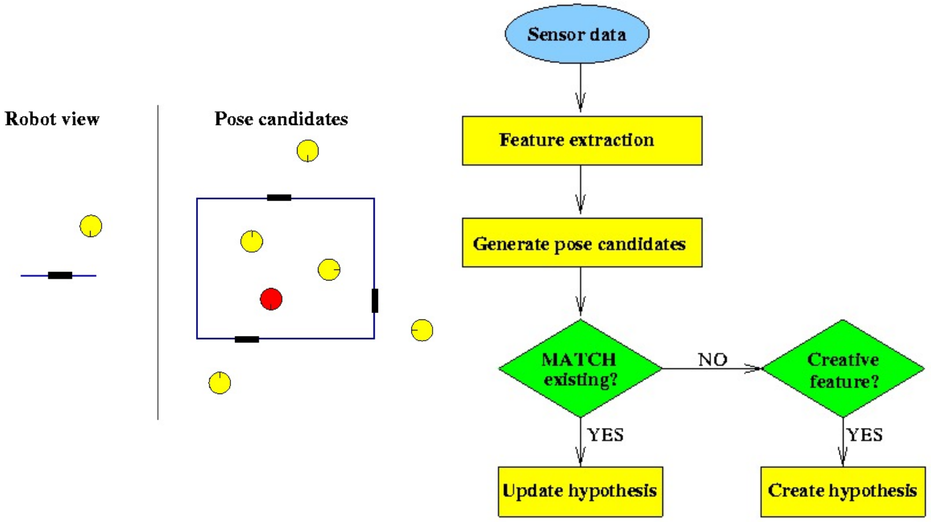
MHT: Implemented System (2)



Courtesy of P. Jensfelt and S. Kristensen

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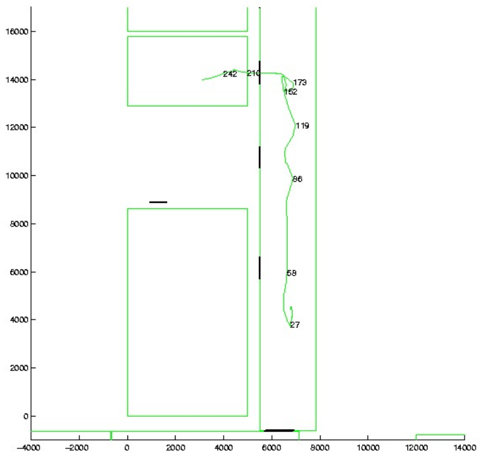
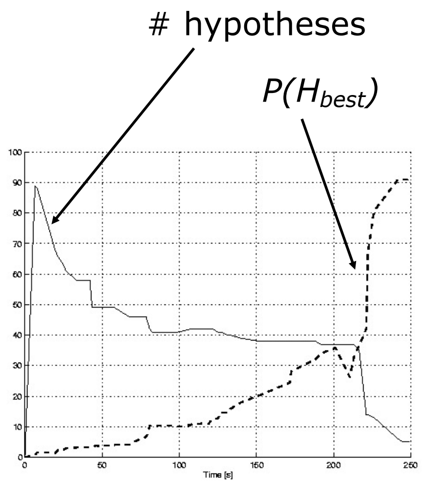
MHT: Implemented System (3)
Example run



Map and trajectory



#hypotheses vs. time

Courtesy of P. Jensfelt and S. Kristensen