

1. Imagine that you work at Hershey's factory. Your job is to keep an eye on the conveyor belt, watching the Hershey's bars as they come out of the press one at a time. There are two types of bars: ones with almonds and ones without. For the first few problems, assume you can tell with 100% accuracy what the bars contains. In the control room, there is a lever that switches the almond control on and off. When the conveyor is turned on at the beginning of the day, there is a 50% chance that the almond lever is on, and a 50% chance that it is off. As soon as the conveyor belt is turned on, it starts making a piece of candy. Unfortunately, someone has let a monkey loose in the control room, and it has locked the door and started the conveyor belt. The lever cannot be moved while a piece of candy is being made. Between pieces, however, there is a 20% chance that the monkey switches the lever to the other position (i.e. turns almonds on if it was off, or off if it was on).
 - a. Draw a Markov Model that represents the situation and give the prior distribution on the state and the transition matrix.
 - b. Now assume that there is a coconut lever as well, so that there are four types of candy: Plain, Almond, Coconut, and Almond+Coconut. Again, there is to 50% chance of the lever being on at the beginning of the day, and the chance of the monkey switching the state of the second lever between candies is also 30%. Assume that the switching of the levers is independent of each other. Now Draw a model for production of all four types of chocolate.
 - c. What is the probability that the model will produce following chocolate bars , in order : { Plain, Almond, Almond Coconut, Almond }
 - d. Now assume that you cannot tell what the bar contains. You can only tell whether the bar is light or dark. If the color is dark with 0.8, 0.6, 0.2, 0.1 probability the bar is { Plain, Almond, Coconut or Almond-Coconut } otherwise it is light.

If the first two chocolates were dark and light, respectively, what is the smoothed distribution over the state at time 1 and the smoothed distribution over the state at time 2?

2. Optional problems not for grade (15.2 and 15.3)