

1. Describe an intersection of three planes in 4D space. Is it a line or a point or an empty set ?

$$\begin{aligned}u + v + w + z &= 6 \\u + w + z &= 4 \\u + w &= 2\end{aligned}\tag{1}$$

What is the intersection if the fourth plane $u = -1$ is included ?

2. Under what conditions on y_1, y_2 and y_3 for the points $(0, y_1), (1, y_2)$ and $(2, y_3)$ to lie on the straight line ?
3. It is impossible for a system of linear equations to have exactly two solutions. Argue why ?
4. Suppose you have matrix A which is 3×3 matrix and has $\text{row1} + \text{row2} = \text{row3}$. Show that A is not invertible.
- a) Explain why $Ax = (1, 0, 0)$ cannot have a solution.
- b) Which right hand sides (b_1, b_2, b_3) might allow solutions ?
5. If we add an extra column b to a matrix A , then the column space gets larger unless
Give an example in which the column space gets larger and an example in which it doesn't. Why is $Ax = b$ solvable exactly when the column space doesn't get larger by including b ?
6. Under what conditions on $b = [b_1, b_2]^T$ does $Ax = b$ have a solution ? Where

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 4 & 0 & 7 \end{bmatrix}$$

7. Upload the file `data1_hw3.mat`, which contains the values of controlled variables x and predicted variables y . Use linear regression to estimate the parameters of the model. Report the model parameters, their confidence intervals and coefficient of determination. Plot the data points and the resulting model. Submit also the plot of histogram of residuals and check/comment whether the residuals are normally distributed.
8. Upload the file `data2_hw3.mat` and fit the multilinear regression model to the data. Use multilinear regression to estimate the parameters of the model and report them.