CS483 Design and Analysis of Algorithms

Lecture 9 Decompositions of Graphs

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Office hours: STII, Room 443, Friday 4:00pm - 6:00pm or by appointments

Course web-site:

http://www.cs.gmu.edu/~lifei/teaching/cs483_fall08/

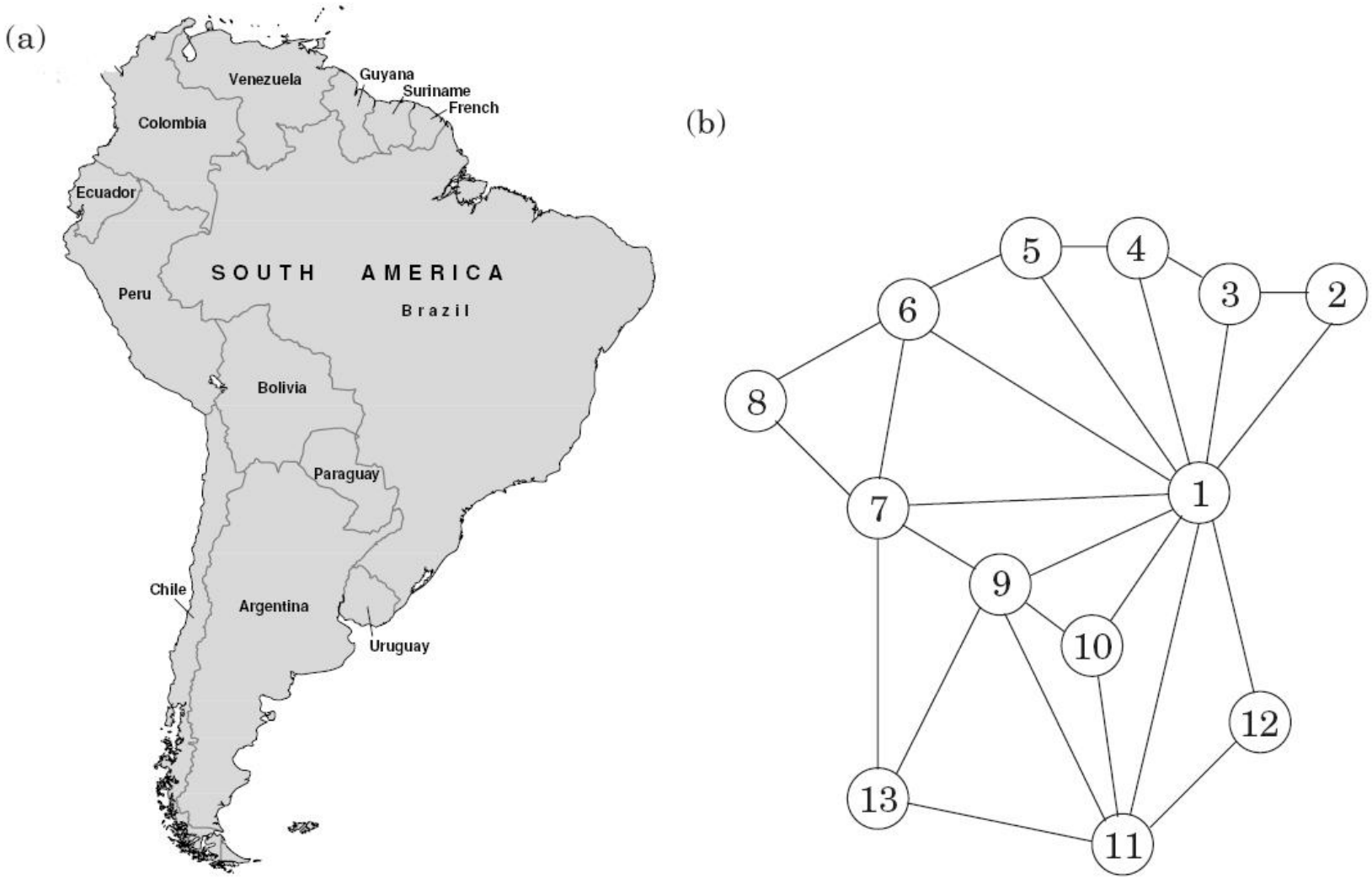
Figures unclaimed are from books "Algorithms" and "Introduction to Algorithms"

Decomposition of Graphs

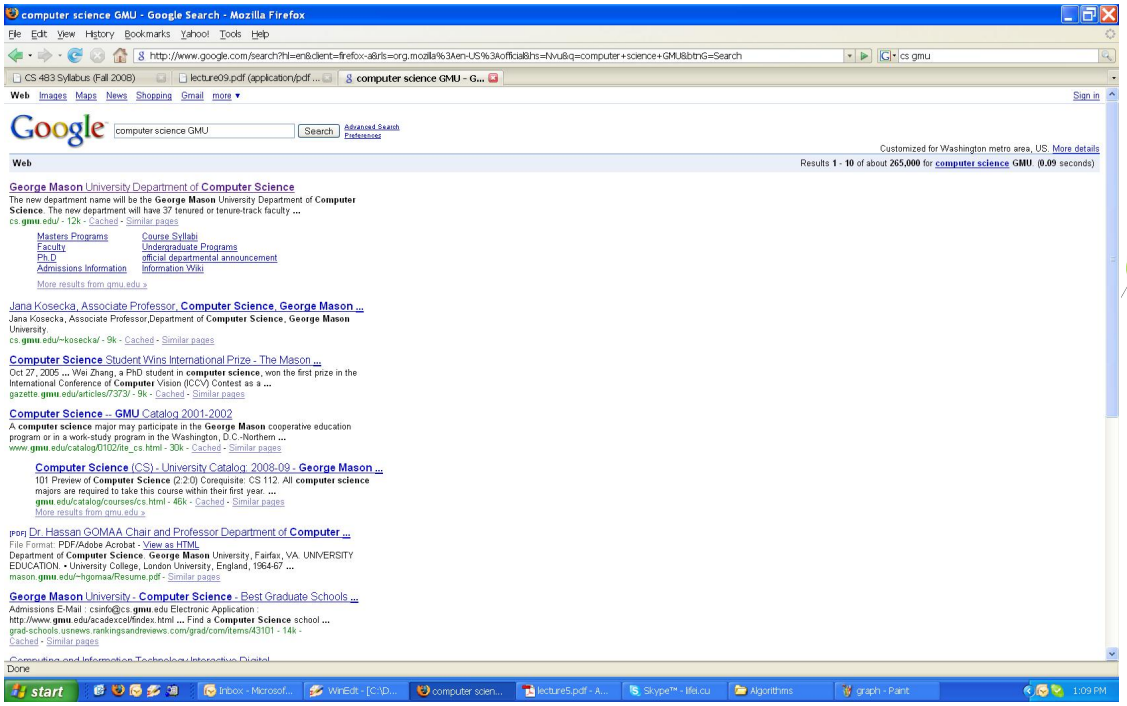
1. Why Graphs?
2. Depth-First Search
3. Topological Sorting
4. Strongly Connected Components (SCC)

Why Graphs?

Figure 3.1 (a) A map and (b) its graph.



Why Graphs?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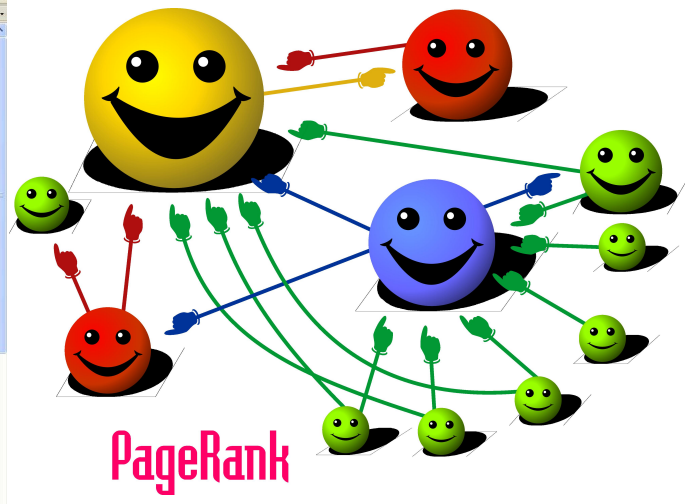
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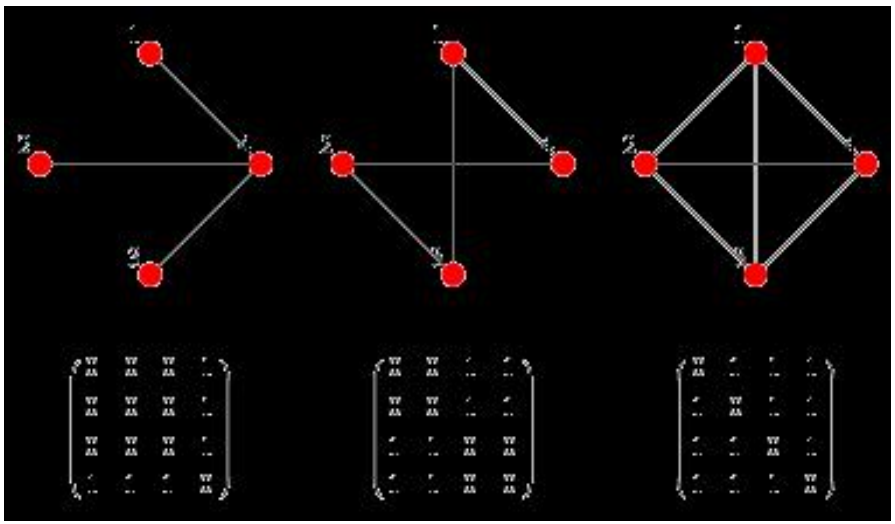
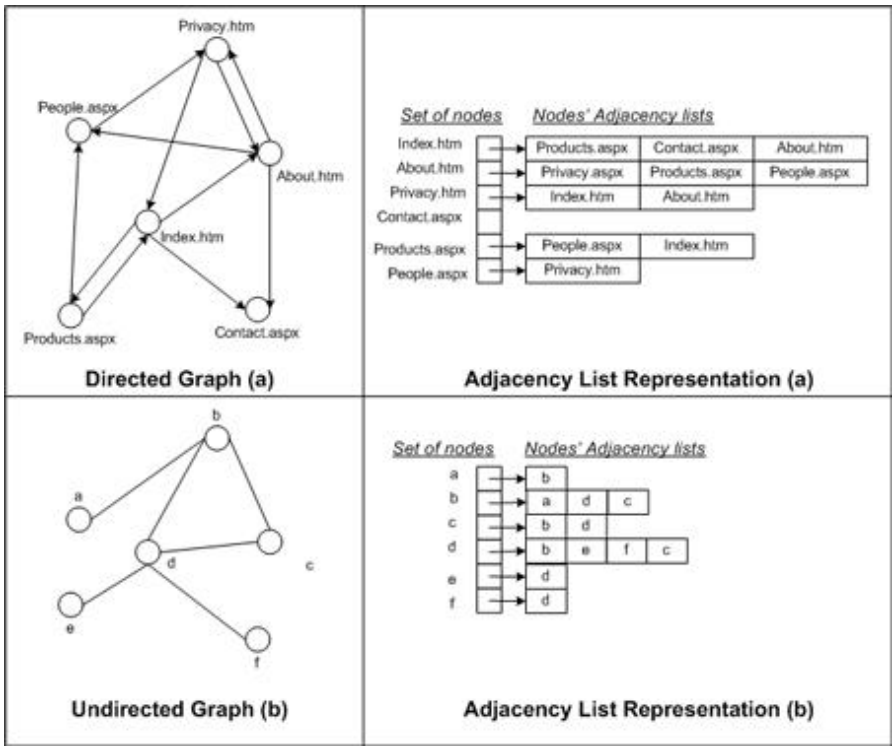
Graphs

1. A *graph* $G = (V, E)$ is specified by a set of *vertices (nodes)* V and *edges* E between selected pairs of vertices
2. Edges are symmetric $\longrightarrow$ *undirected graph*
3. Directions over edges $\longrightarrow$ *directed graph*
4. E.g., political maps, exam conflicts, World Wide Web, etc.

Graph Representation



http://msdn2.microsoft.com/en-us/library/



Graph Traversal

Exploring a graph is rather like navigating a maze
Which parts of the graph are reachable from a given vertex?

http://www.northgacornmaze.com/images/maze2006.jpg

Depth-First Search

Input: Graph $G = (V, E)$, vertex $s \in V$
Output: All vertices u reachable from s

```
function DFS(G)

    for all v ∈ V
        visited(v) = false;

    for all v ∈ V
        if not visited(v)
            explore(v);
```

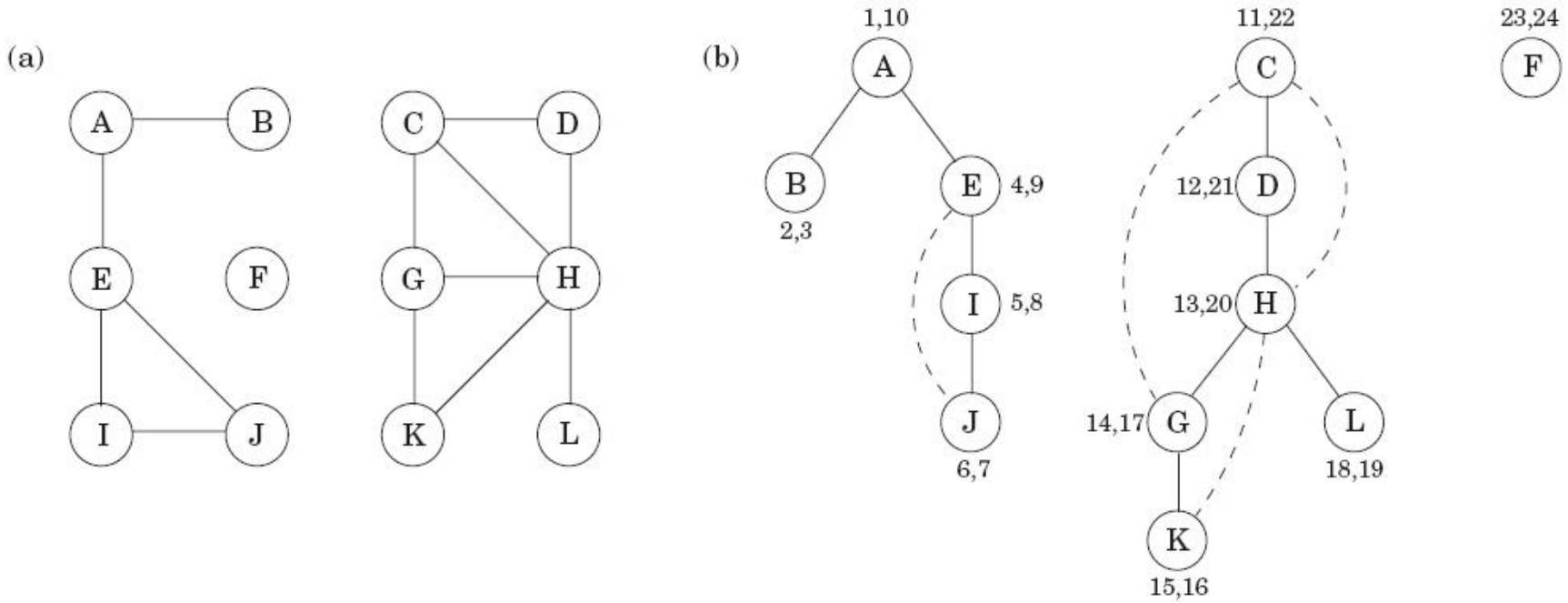
```
function explore(G, v)

    visited(v) = true;

    for each edge (v, u) ∈ E
        if not visited(u)
            explore(u);
```

Depth-First Search

Figure 3.6 (a) A 12-node graph. (b) DFS search forest.



Analysis of DFS

Theorem

All nodes reachable from s can be found via `DFS`

Proof.

?

□

Theorem

The overall running time of `DFS` *is* $O(|V| + |E|)$

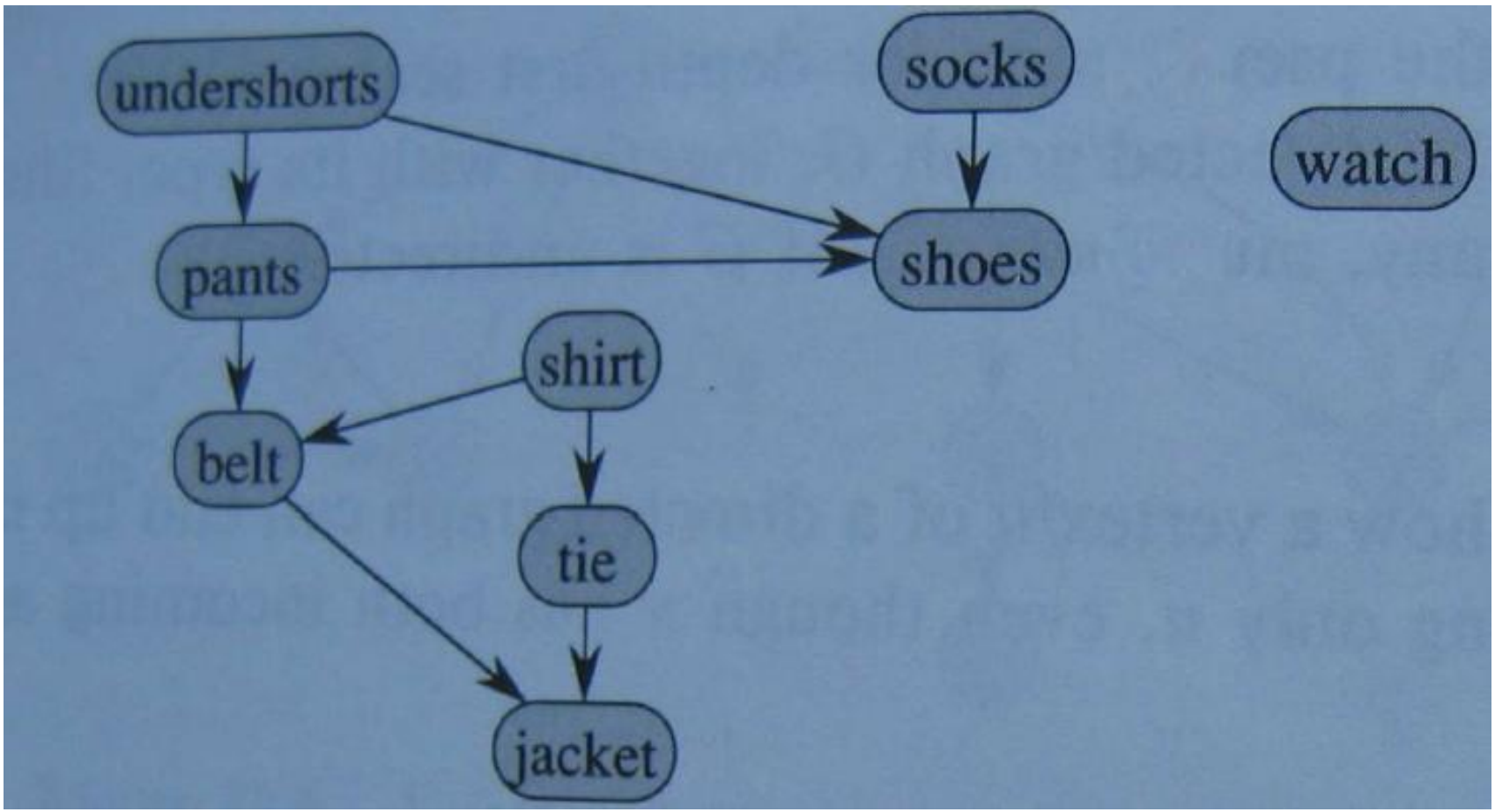
Proof.

1. The time initializing each vertex is $O(|V|)$
2. Each edge $(u, v) \in E$ is examined twice, once exploring u and once exploring v. Therefore takes $O(|E|)$ time

□

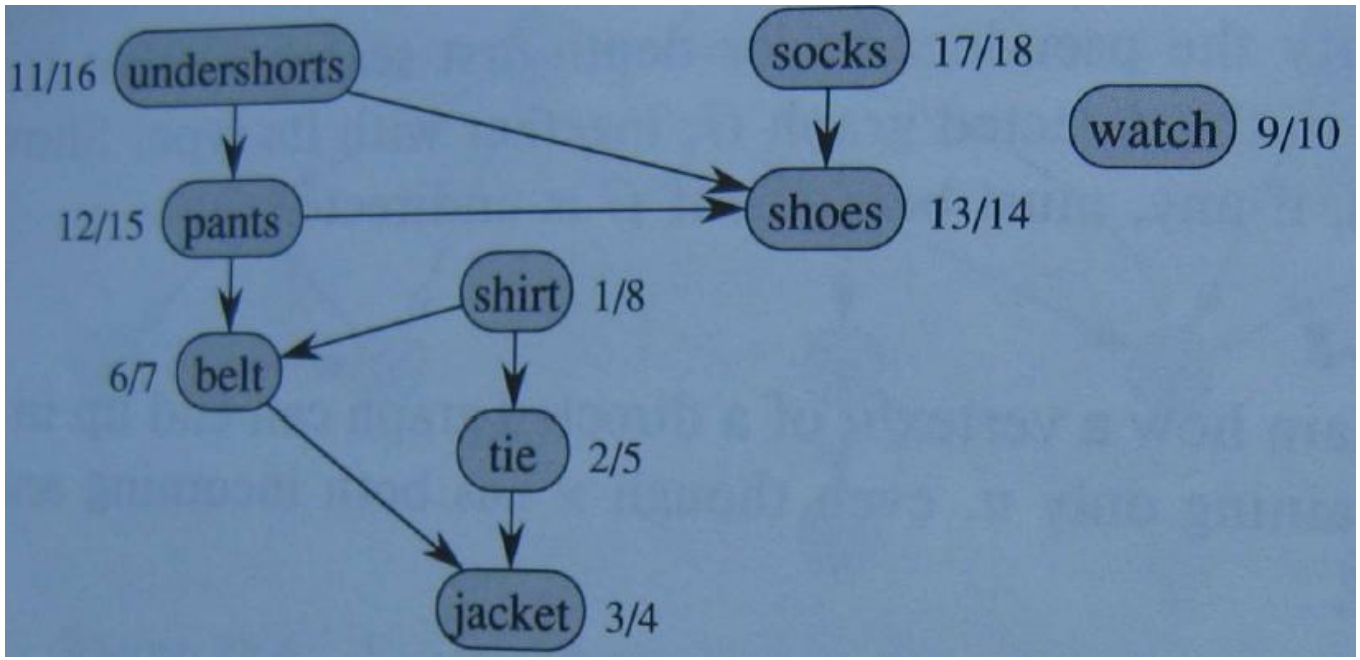
Topological Sorting — An Application of DFS

Input: a directed acyclic graph (DAG) G
output: A linear ordering of all its vertices, such that if G contains an edge $(u,\ v)$, then, u appears before v in the ordering



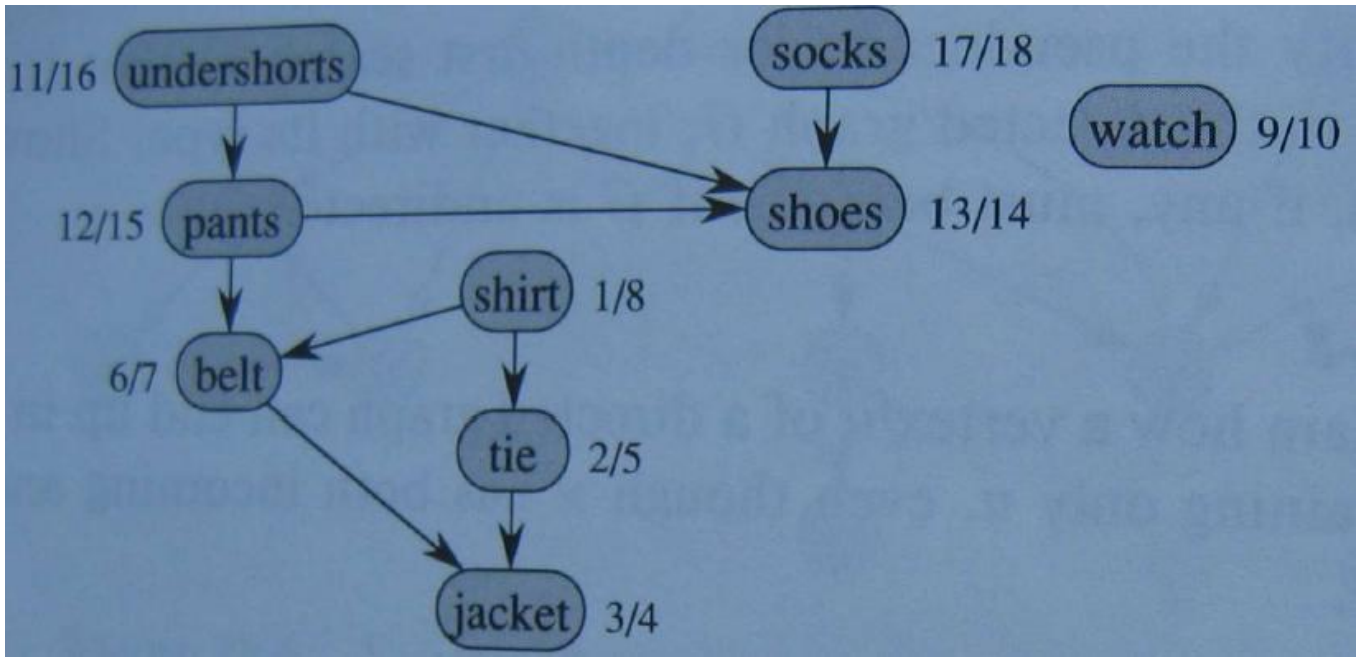
Topological Sorting — An Application of DFS

1. Run DFS to get (`startingtime`, `finishingtime`)

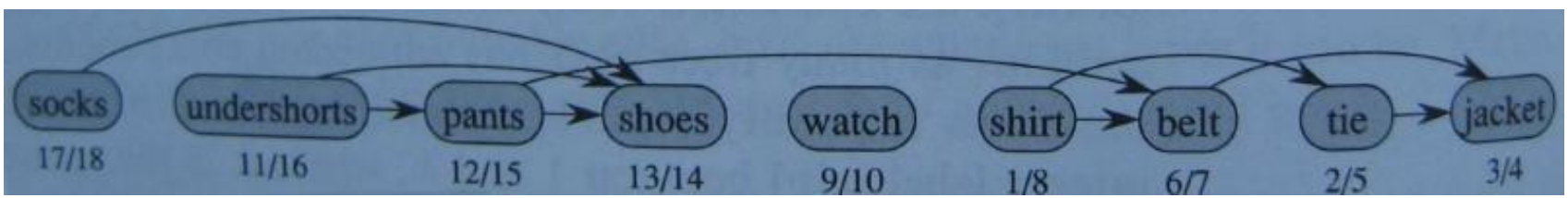


Topological Sorting — An Application of DFS

1. Run DFS to get (`startingtime`, `finishingtime`)



2. List nodes in reverse order of their finishing time



Announcements

1. Assignment 4 is due on this Wednesday.
2. There is a review class on October 8th (this Wednesday): Chapters 0 – 4.3 of DPV, Chapter 5 of CLSR.
   Covers: Solutions of assignment 4 and a few selected problems with their solutions.
   Later submission of assignment 4 will not be accepted.
3. Next Monday (October 13th) is a holiday. (Monday class meets Tuesday).
4. Midterm is scheduled on October 14th (next TUESDAY), 1:30pm - 2:45pm, Innovation Hall 136.
5. The class on October 15th (next Wednesday) is canceled.

Strongly Connected Components in Directed Graphs

Definition

Two nodes u and v of a directed graph are **connected** if there is a path from u to v and a path from v to u.

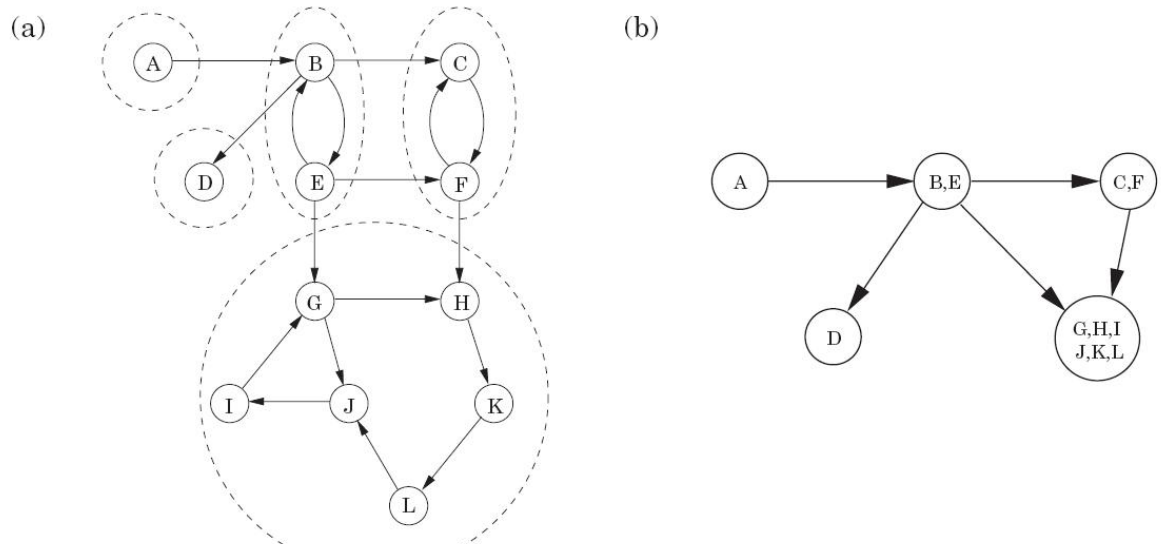
Remark

Connectivity in undirected graphs is straightforward.

Remark

Every directed graph is a directed acyclic graph (DAG) of its strongly connected components (disjoint sets of V)

Figure 3.9 (a) A directed graph and its strongly connected components. (b) The meta-graph.



How to Find SCC?

Remark

If the `explore` *subroutine starts at node u, then it will terminate precisely when all nodes reachable from u have been visited. If we call* `explore` *on a node that lies somewhere in a* sink *strongly connected component, then we will retrieve exactly that component*

What can we learn?

1. How do we find a node that we know for sure lies in a sink strongly connected component?
2. How do we continue once this first component has been discovered?

Remark

The node that receives the highest `ending time` *(i.e.,* `post` *number) in a depth-first search must lie in a source strongly connected component.*

How to Find SCC?

Remark

If C and C' are strongly connected components, and there is an edge from a node in C to a node in C', the the highest `post` *number in C is larger than the highest* `post` *number in C'*

What can we learn?

Remark

The strongly connected components can be linearized by arranging them in decreasing order of their highest `post` *numbers.*

Remark

The reverse graph G^R, the same as G but with all edges reversed, has the same SCC as G.

Strongly Connected Components

1. Run depth-first search on G^R
2. Run the undirected connected components algorithm on G, and during the depth-first search, process the vertices in decreasing order of their *finishing time* from step 1