Assignment 3, Due on September 24th, Wednesday

1. (1.5 **points.**) Page 40 of DPV, 1.29. You do not need to answer how many bits are needed to choose a function from the family.
2. (1 **point.**) (CLSR page 98, 5.2-4) Use indicator random variable to solve the following problem, which is known as the **hat-check problem**. Each of n customers gives a hat to a hat-checker persona at a restaurant. The hat-checker person gives the hats back to the customer in a random order. What is the expected number of customers that get back their own hat?
3. (1.5 **points.**) (CLSR page 98, 5.2-5) Let $A[1, \ldots, n]$ be an array of n distinct numbers. If $i < j$ and $A[i] < A[j]$, then the pair $(i,\ j)$ is called an **inversion** of A.
   - 3.1 (0.5 **point.**) What array with elements from the set $\{1, 2, \ldots, n\}$ has the most inversions? How many does it have?
   - 3.2 (1 **point.**) Suppose that the elements of A form a uniform random permutation of $< 1, 2, \ldots, n >$. Use indicator random variables to compute the expected number of inversions.
4. (1 **point**.) (CLSR page 105, 5.3-5) Prove that in the array P in procedure `permute-by-sorting`, the probability that all elements are unique is at least $1 - 1/n$.