Fighting Children [4 points]
Professor Adam has two children who, unfortunately, dislike each other. The problem is so severe that not only do they refuse to walk to school together, but in fact each one refuses to walk on any block that the other child has stepped on that day. The children have no problem with their paths crossing at a corner. Fortunately both the professors house and the school are on corners, but beyond that he is not sure if it is going to be possible to send both of his children to the same school. The professor has a map of his town.
Show how to formulate the problem of determining whether both his children can go to the same school as a maximum-flow problem.

Variations [4 points]
There are many common variations of the maximum flow problem. Try to solve the following two subproblems.

1. (2 points.) There are many sources and many sinks, and we wish to maximize the total flow from all sources to all sinks.
2. (2 points.) Suppose that, in addition to edge capacities, a flow network has vertex capacities. That is each vertex v has a limit $l(v)$ on how much flow can pass though v. Show how to transform a flow network $G = (V, E)$ with vertex capacities into an equivalent flow network $G' = (V', E')$ without vertex capacities, such that a maximum flow in G' has the same value as a maximum flow in G. How many vertices and edges does G' have?