Controlled-Topology Filtering

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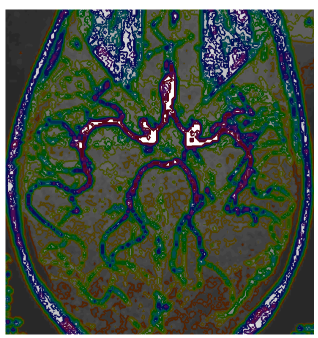
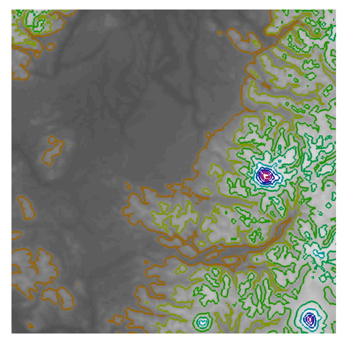
Motivation

Many applications require extraction of isolines & isosurfaces (contours) from scalar functions

- MRI, CT, terrain data, scientific computing



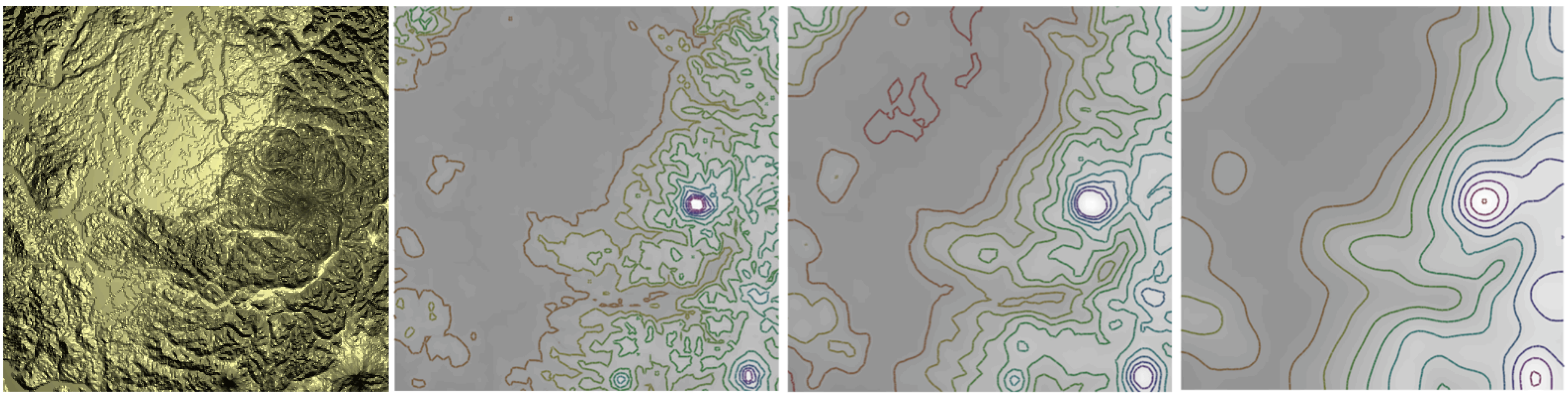
Cow CT Scan



Puget Sound

Motivation

Want to filter (smooth, sharpen) all contours at once



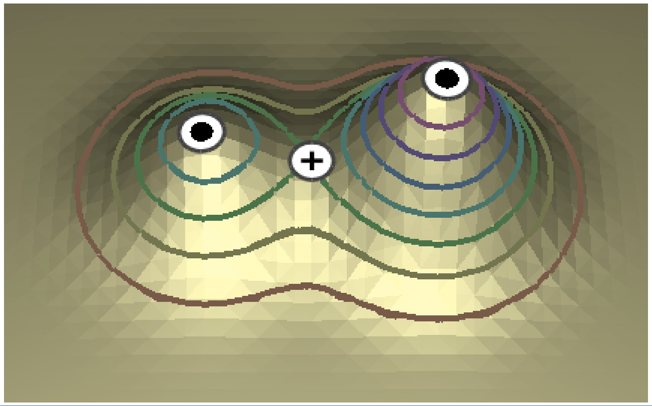
- Downsampling
- Noise reduction

Contour-line topology

Number of contours at all isovalues

- In 2D fields, value can be height

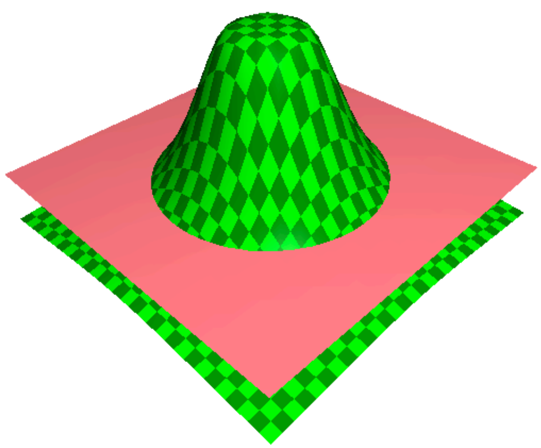
Topology changes occur when value equals value of a critical point (min, max, saddle)

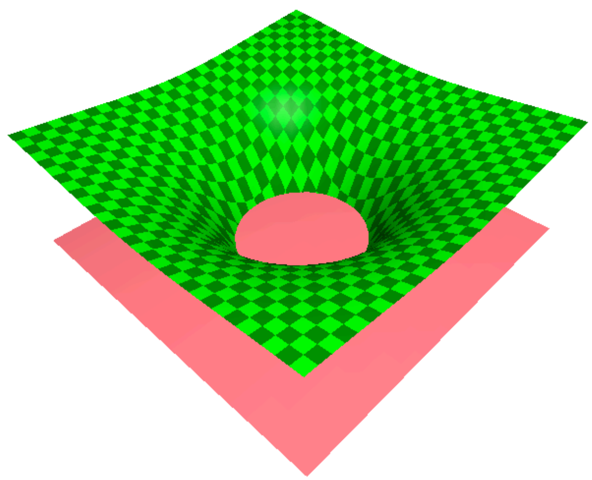


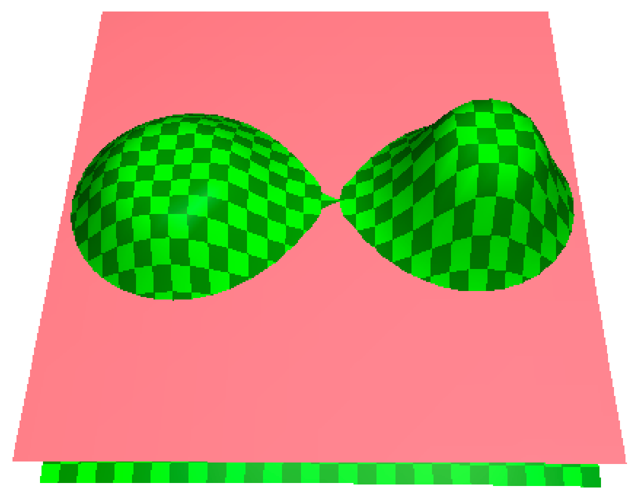
Critical Points

Maxima and minima correspond to hills and valleys

Saddles join two hills or valleys



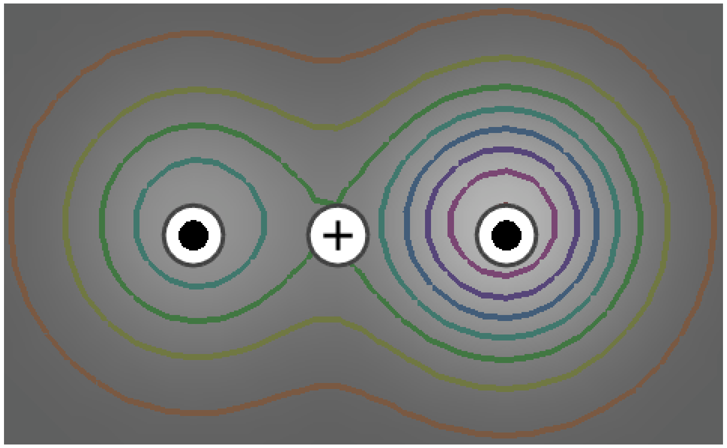




Features

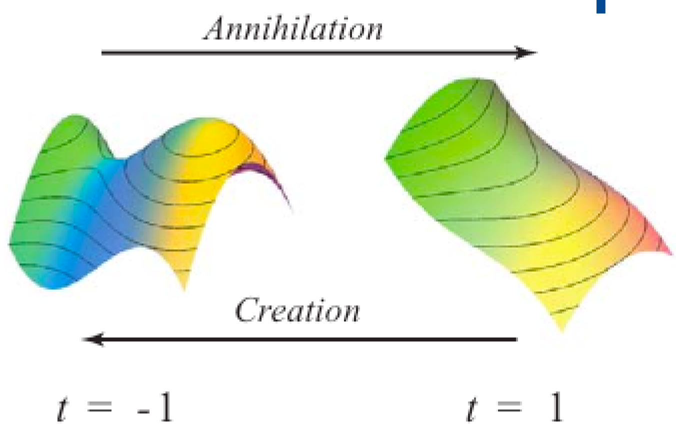
Correspond to critical contours passing through saddles

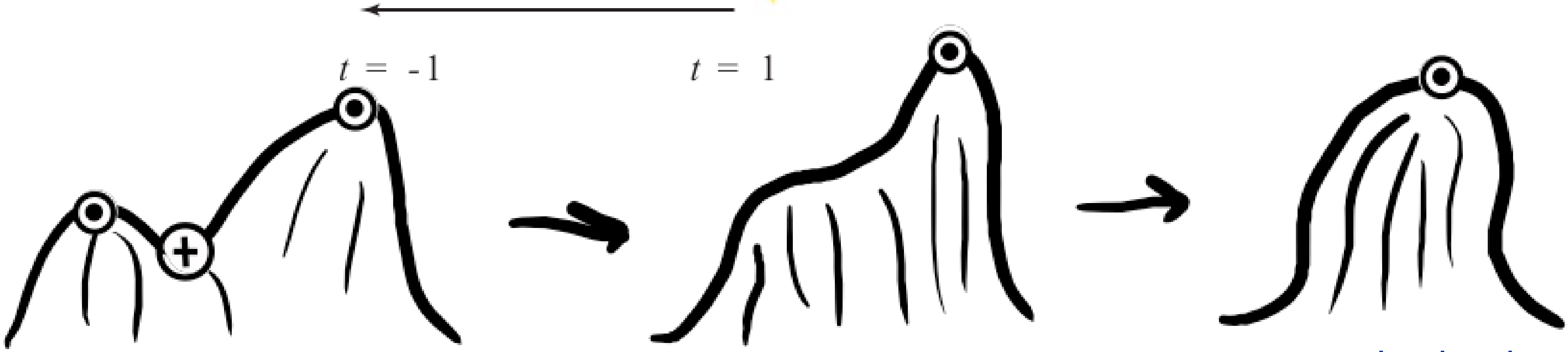
- A saddle divides a contour in two
- The interior of a contour containing an unpaired extremum is a feature



Topological Events

Features appear/disappear in saddle-min/max pairs





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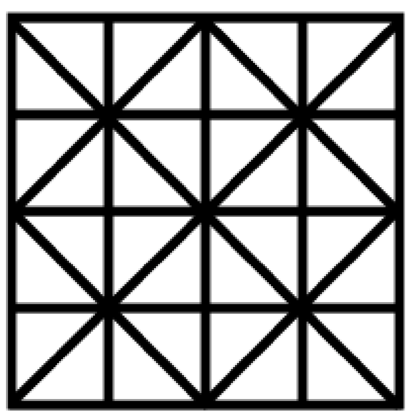
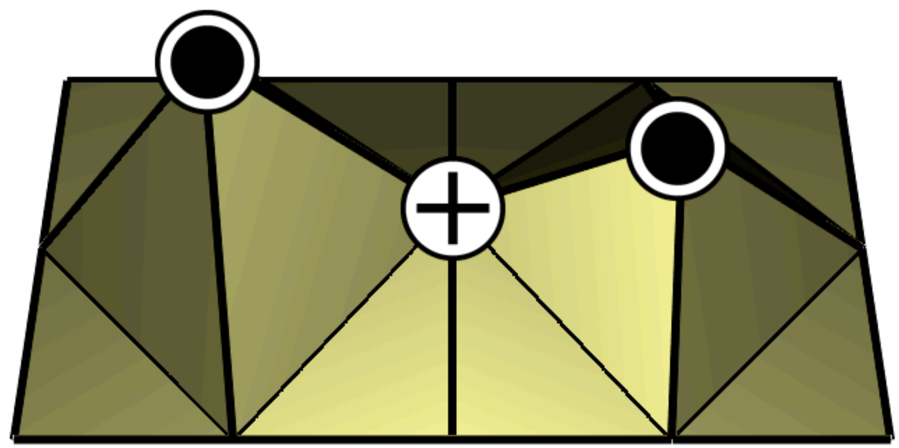
Piecewise linear data

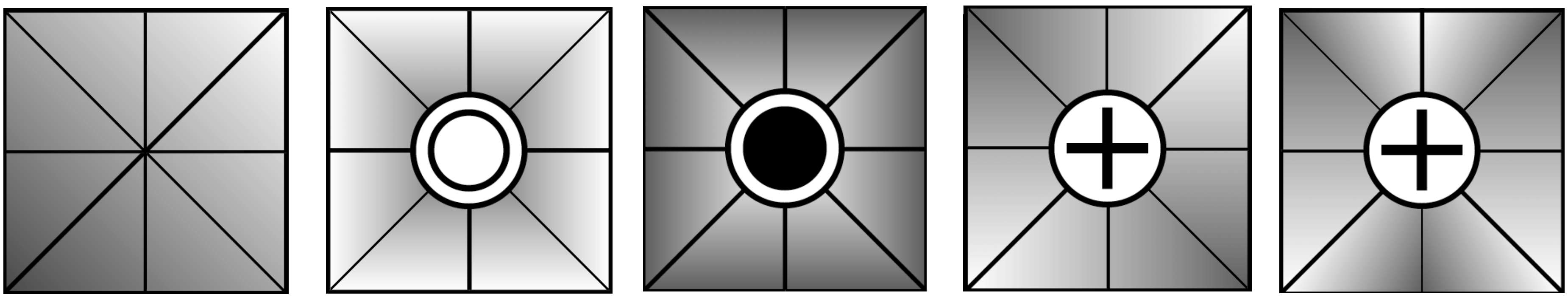
- Scalar values defined on a regular grid
- Simplicial mesh guarantees critical points only at vertices
- Some complex critical points can be stable





Piecewise linear data

Find critical points by comparing value with neighbors



regular **min** **max** **saddle (simple)** **saddle (monkey)**

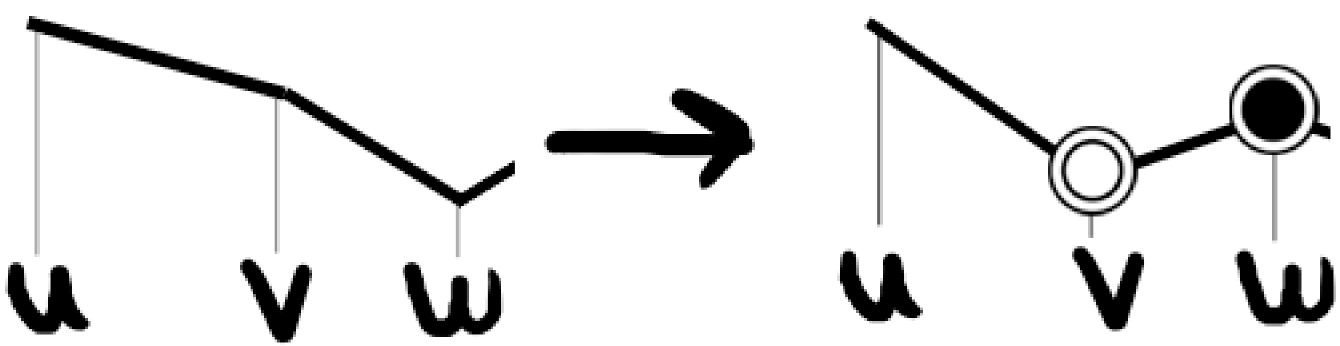
- Break ties with arbitrary, consistent perturbation

PL topological events

Only happens when 2 adjacent vertices change relative height

- The edge *flips* relative to equality

1D

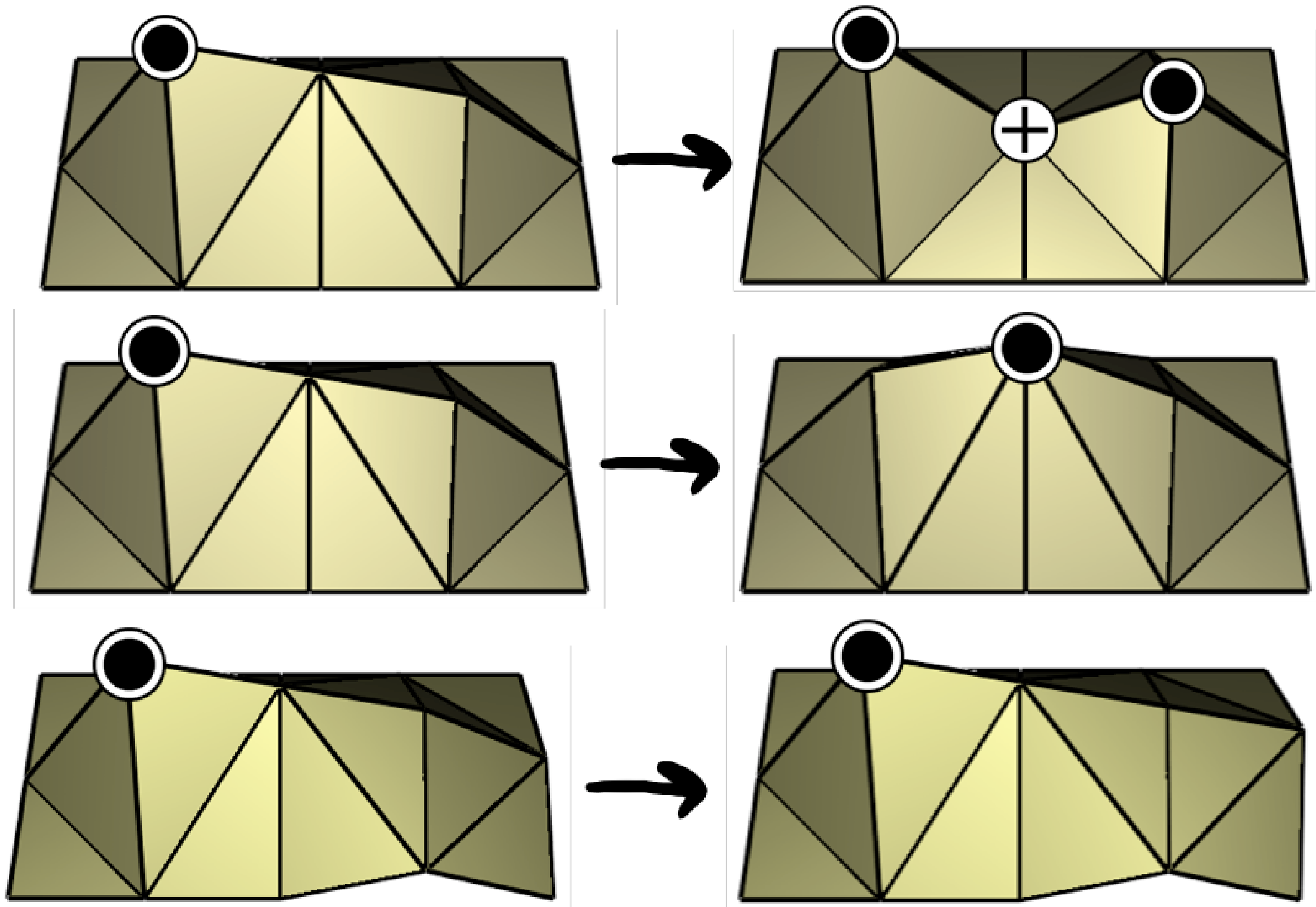


PL topological events

Merge/Split

Move

Nothing



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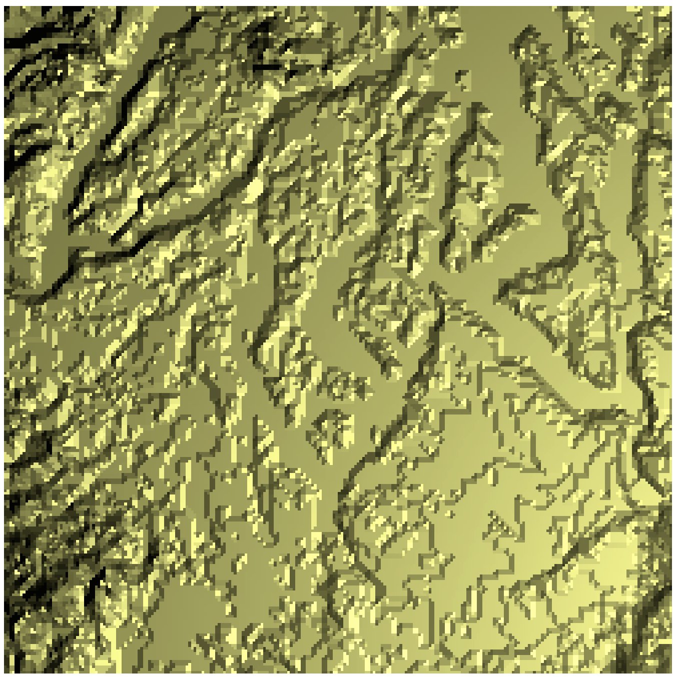
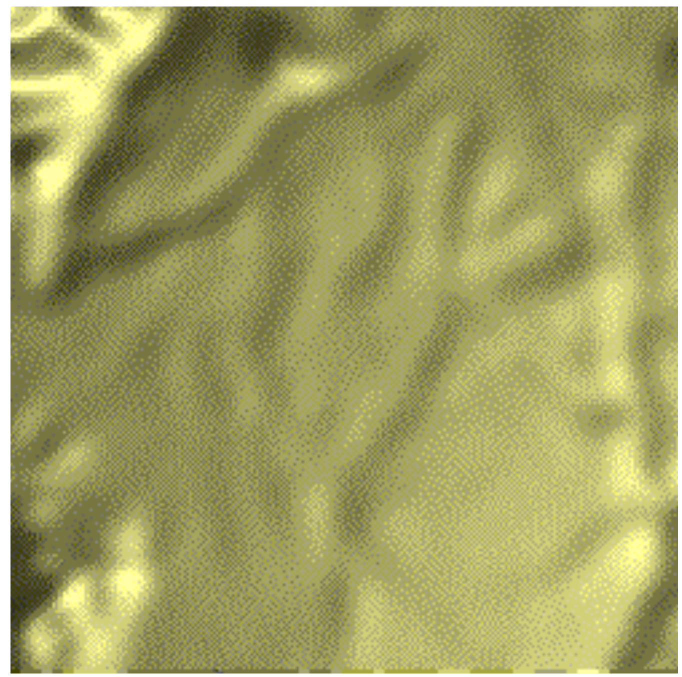
Laplacian Smoothing

$$I_t = \Delta I$$





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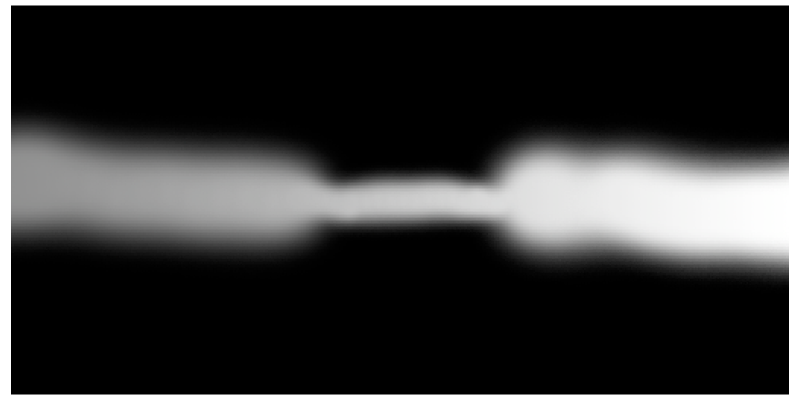
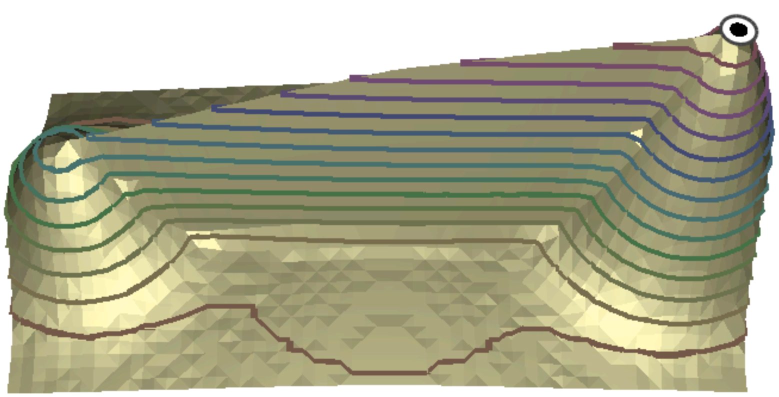
Laplacian Smoothing

Can create features

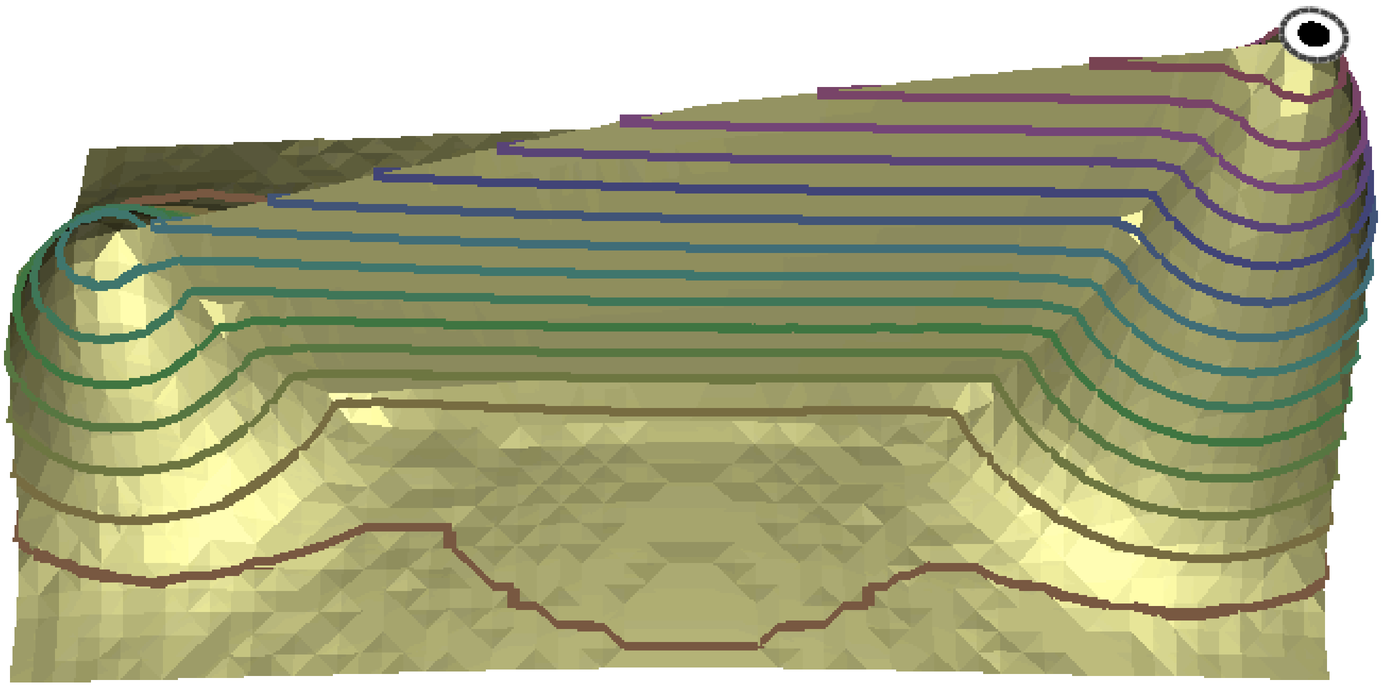
Ridge Bridge **blood vessels**

as in

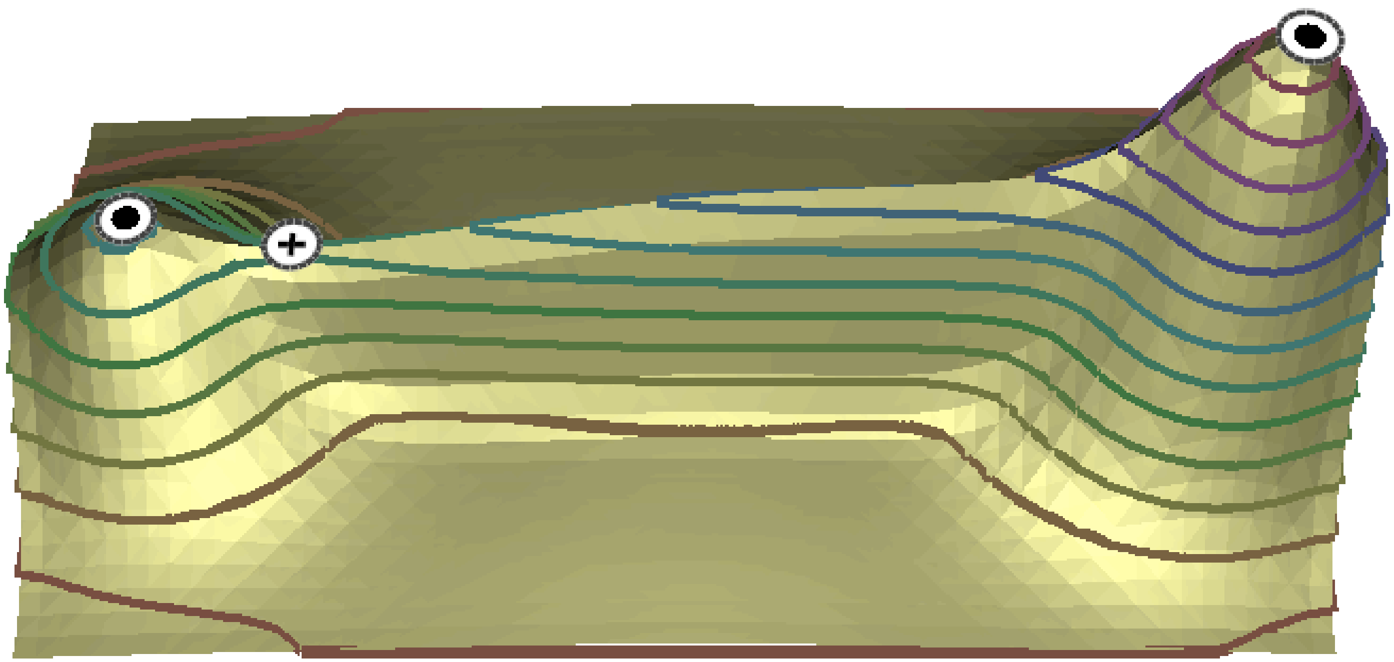




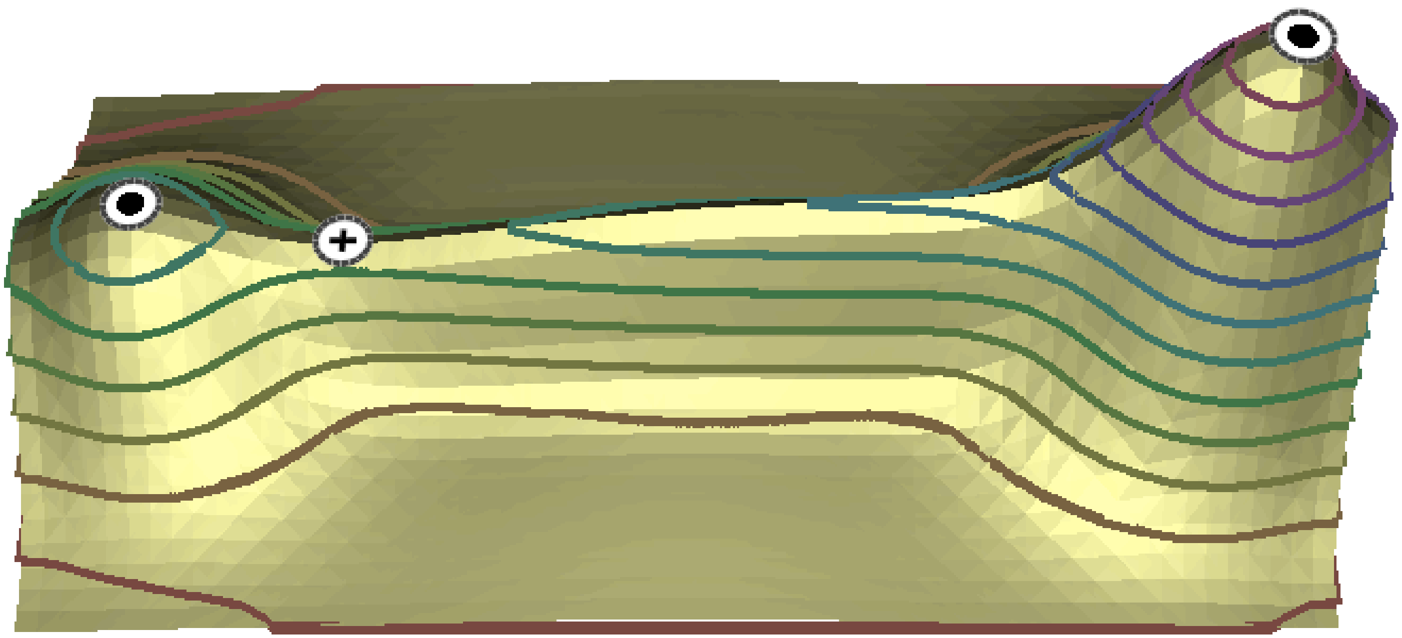
Ridge Bridge (t = 0.0)



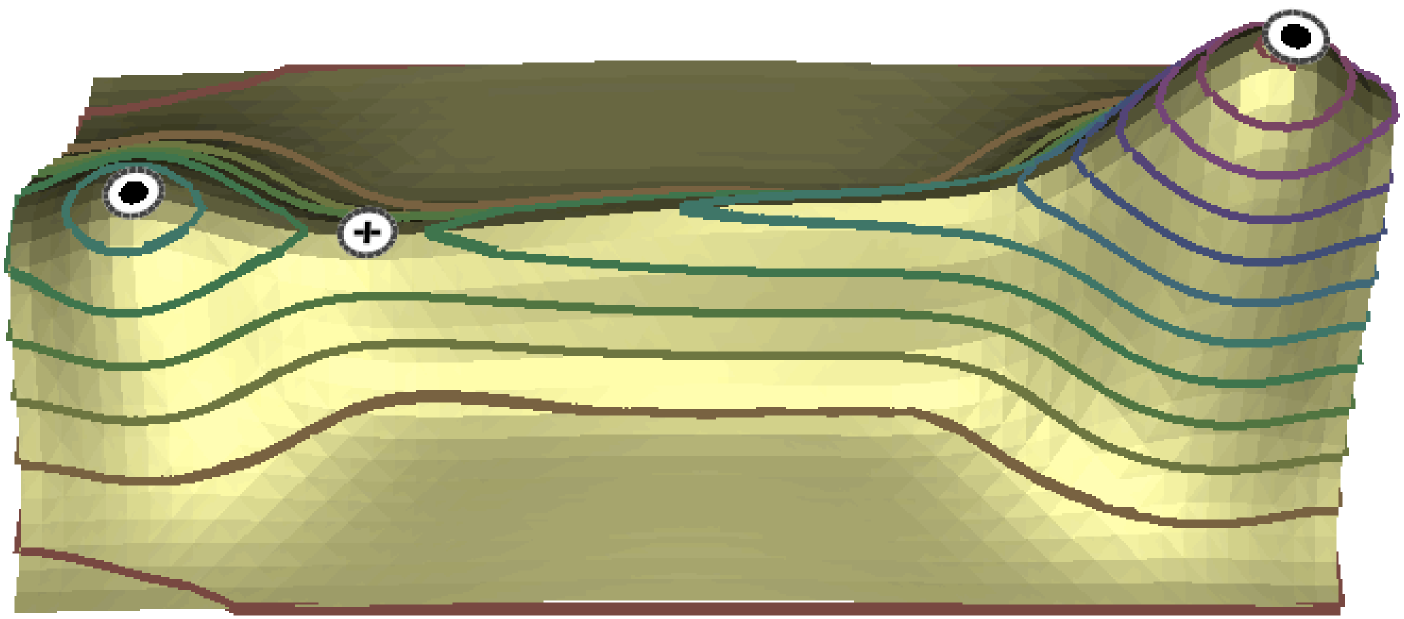
Ridge Bridge (3.5)



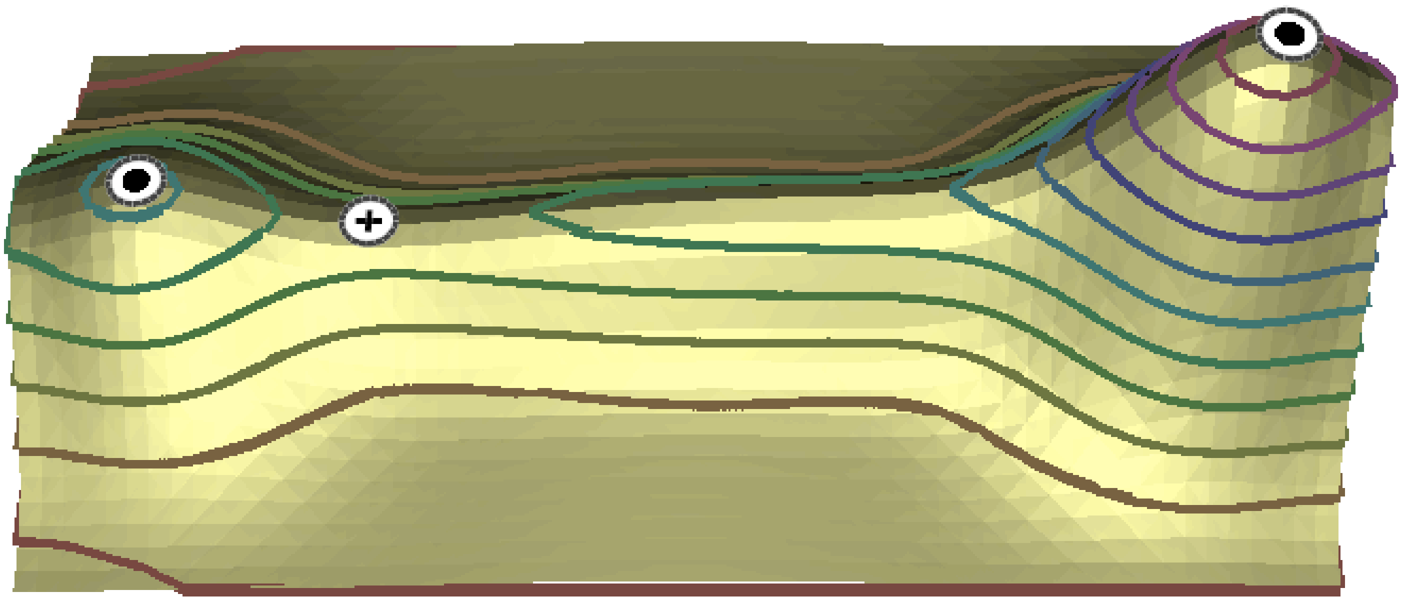
Ridge Bridge (7.0)



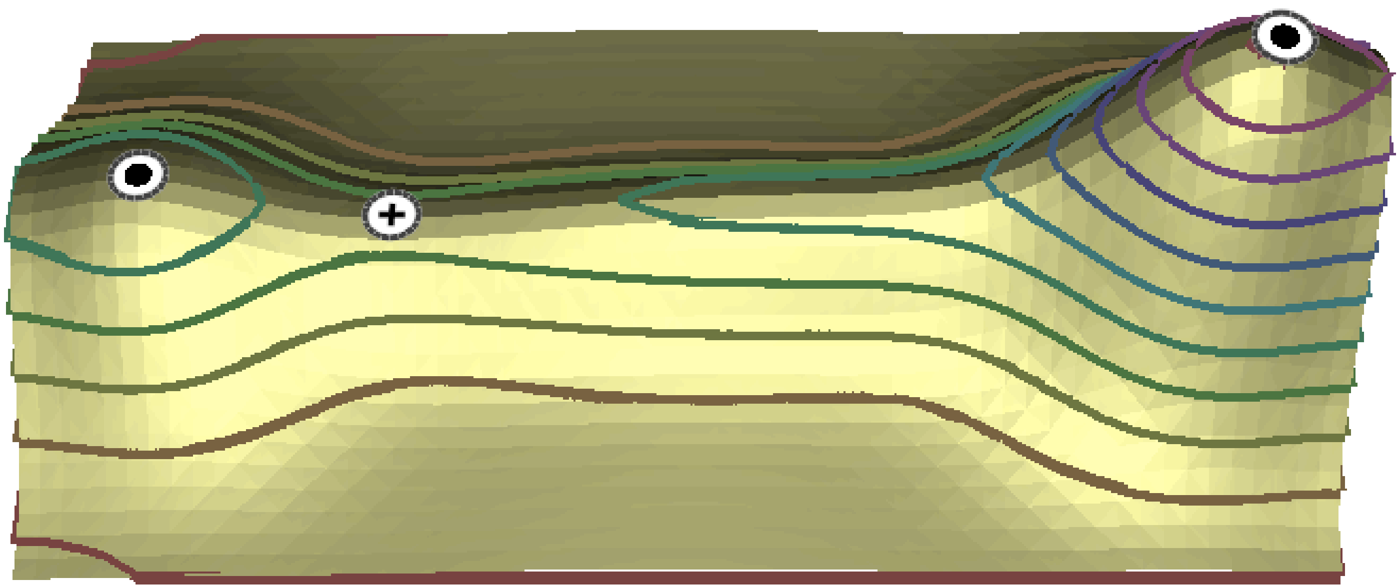
Ridge Bridge (10.5)



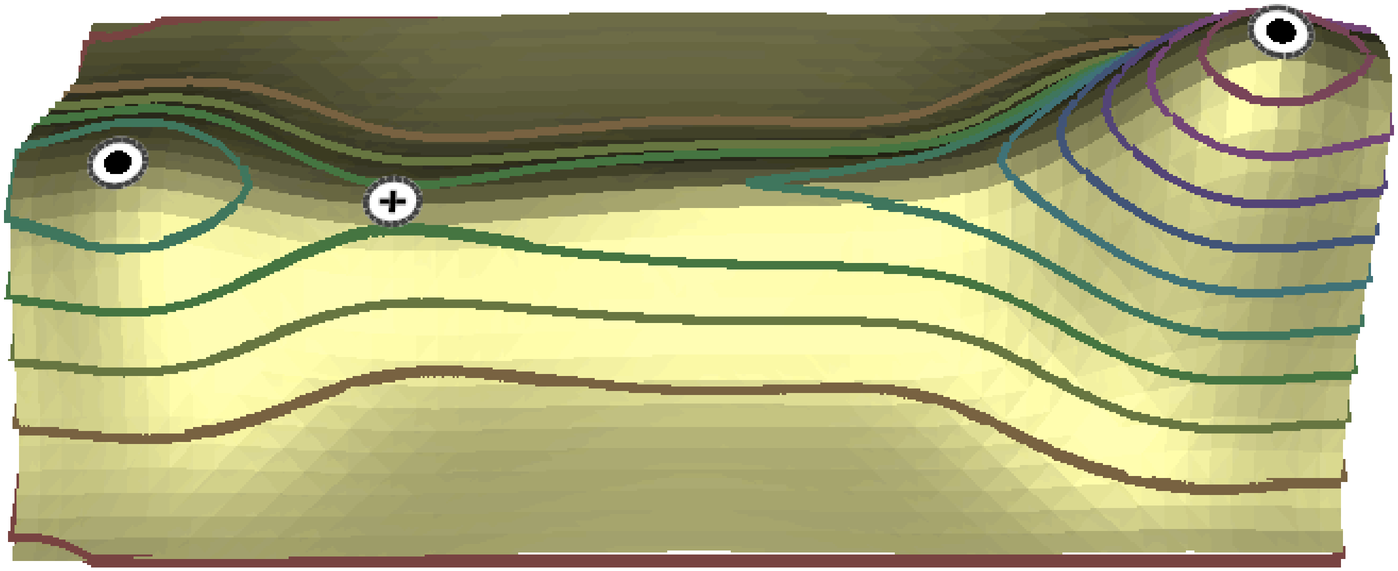
Ridge Bridge (14.0)



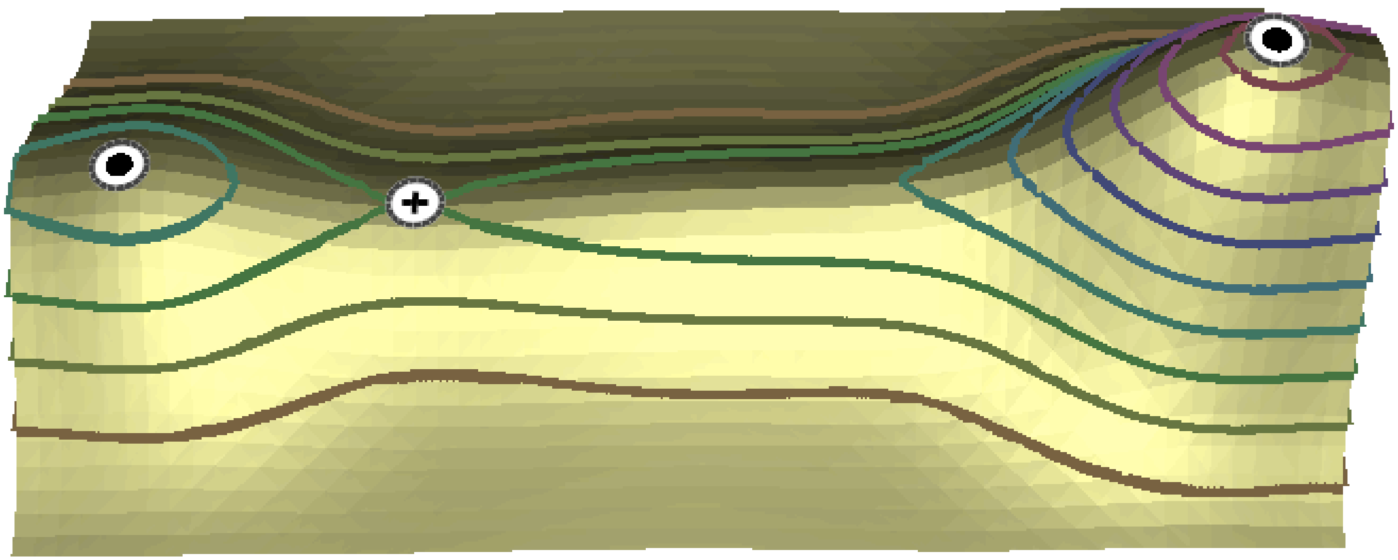
Ridge Bridge (17.5)



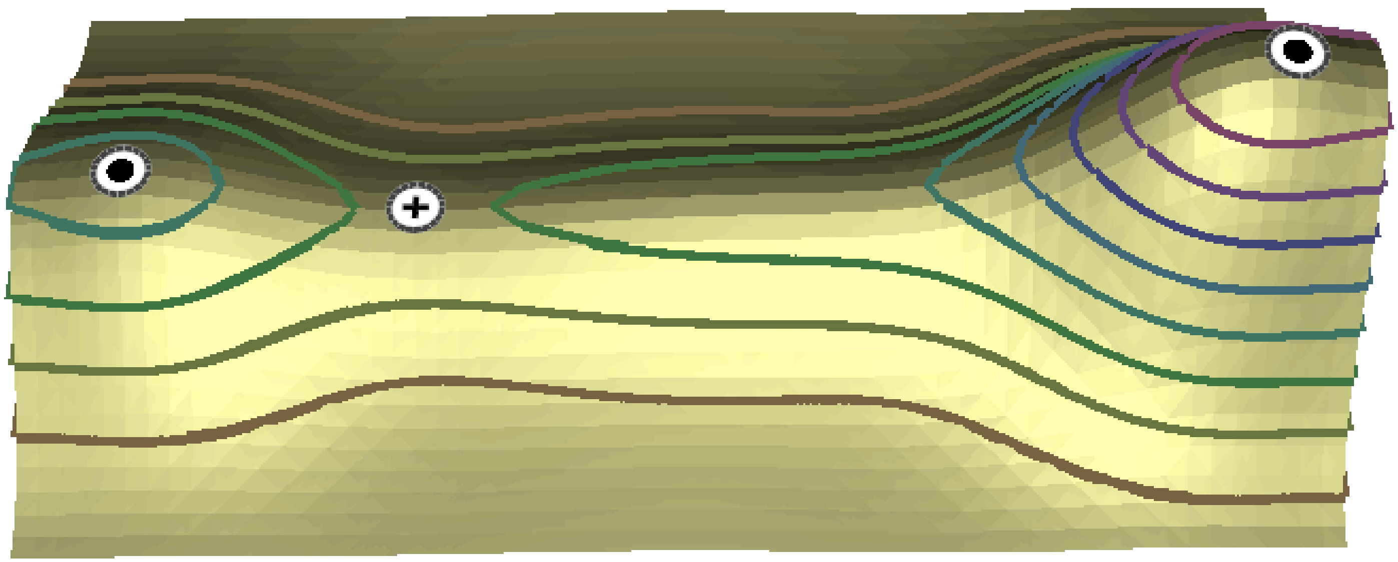
Ridge Bridge (21.0)



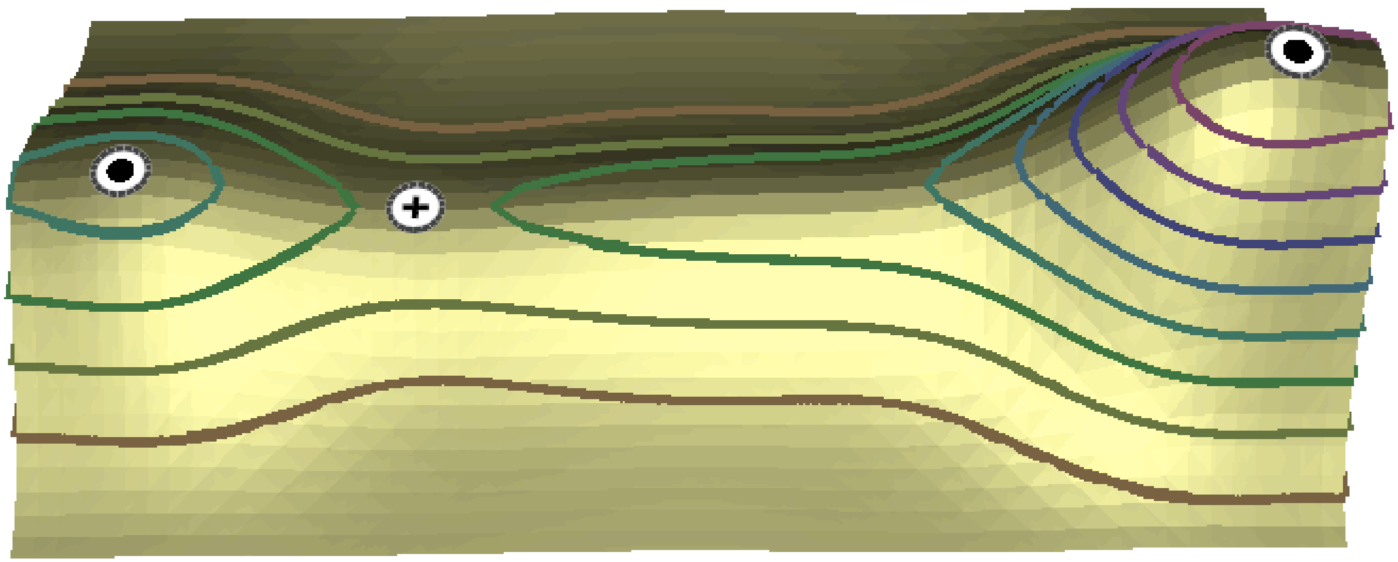
Ridge Bridge (24.5)



Ridge Bridge (28.0)



Ridge Bridge (28.0)

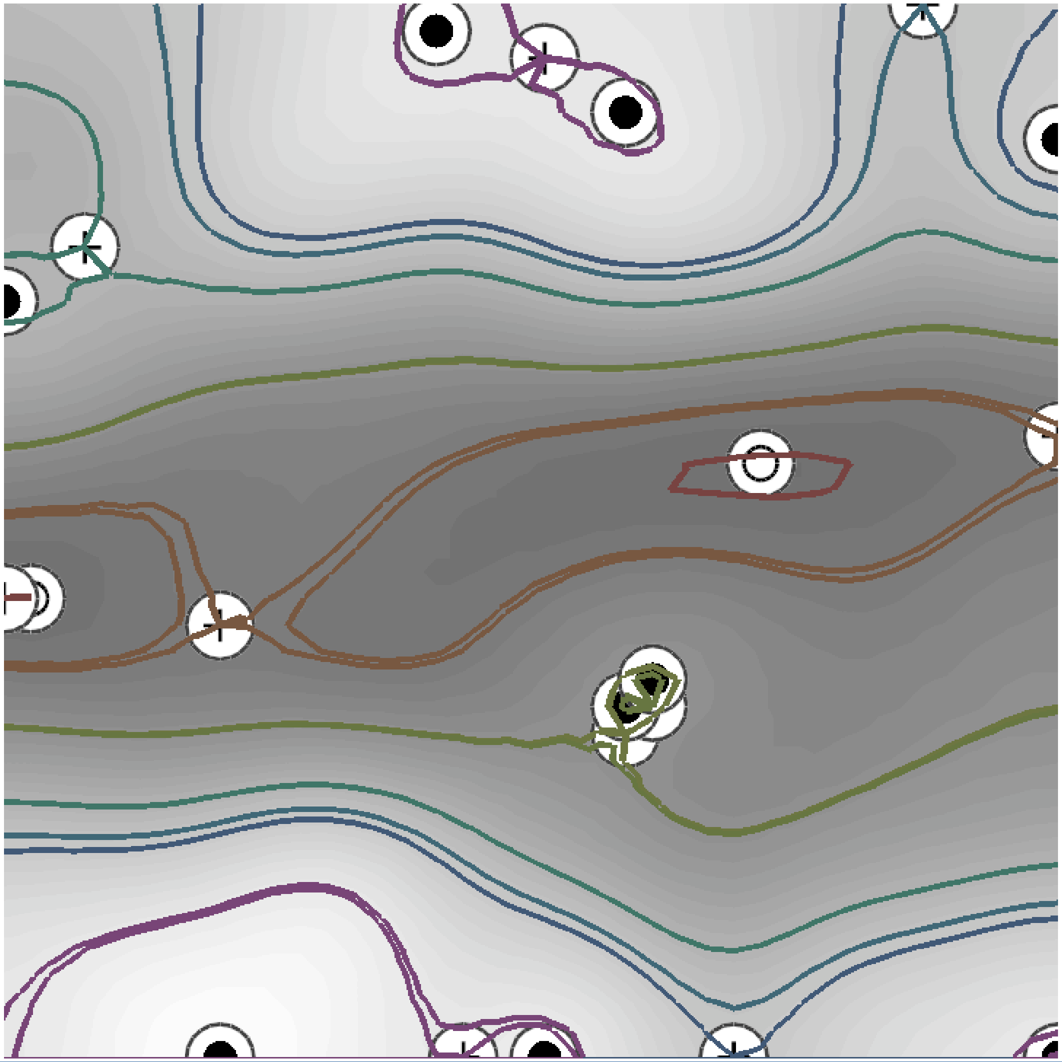


2/3

Ridge Bridge (28.0)

3/3

Puget Sound (3.5)



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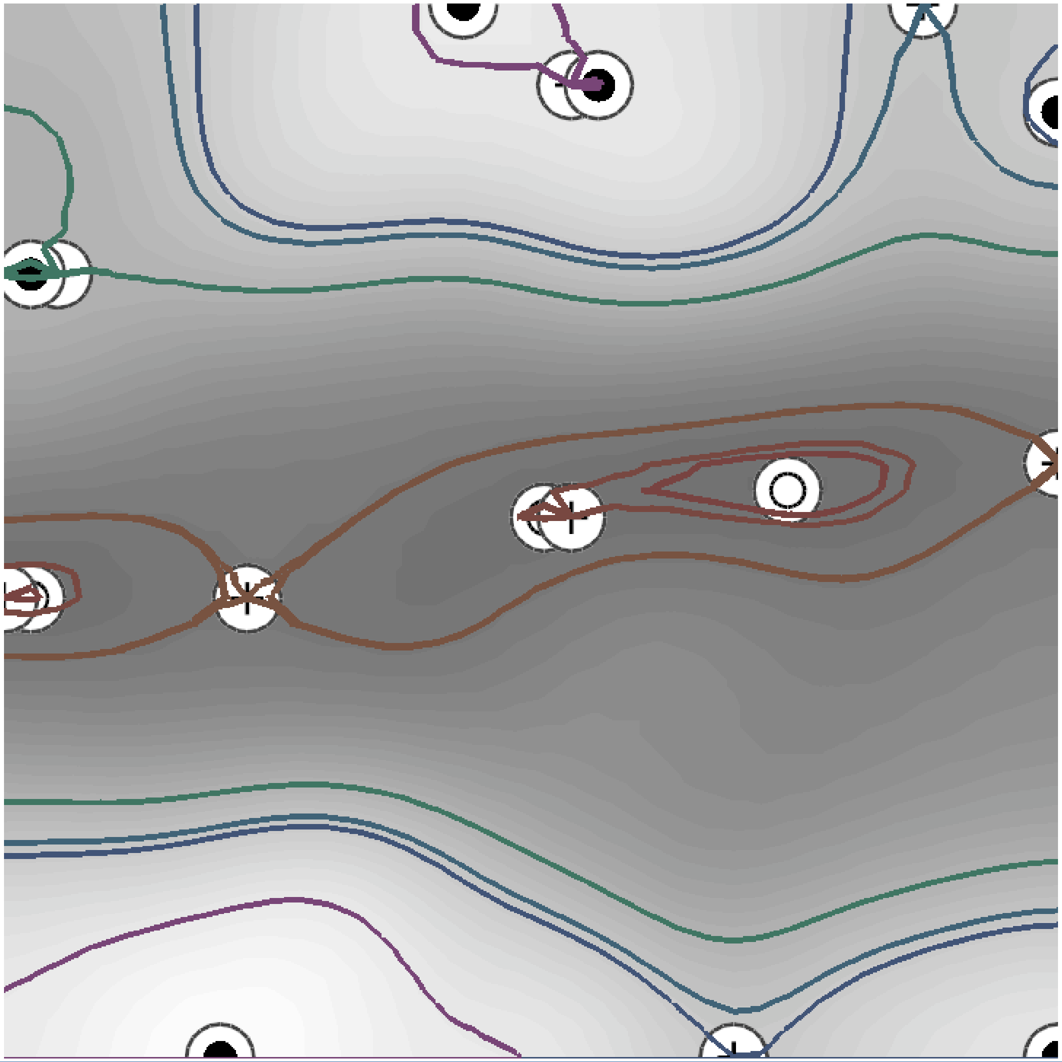
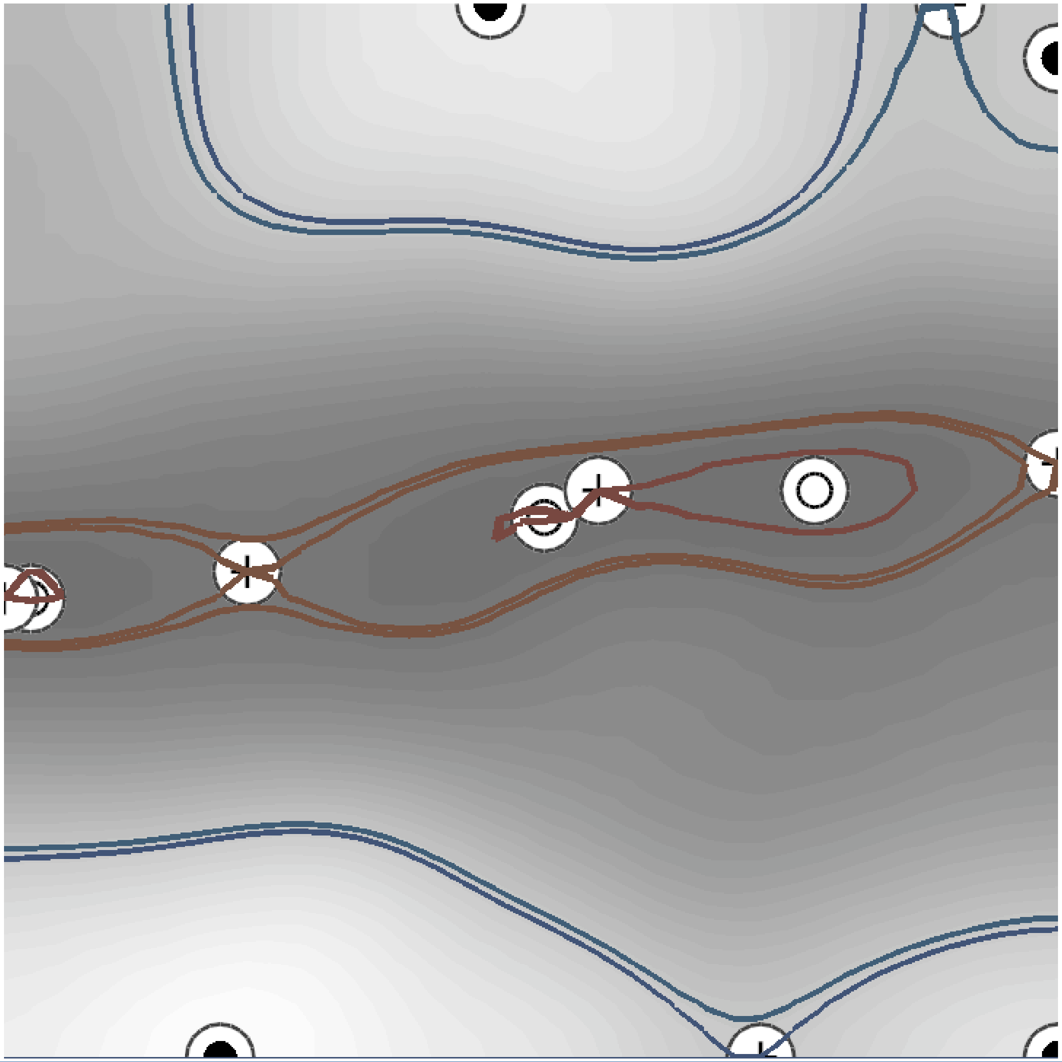
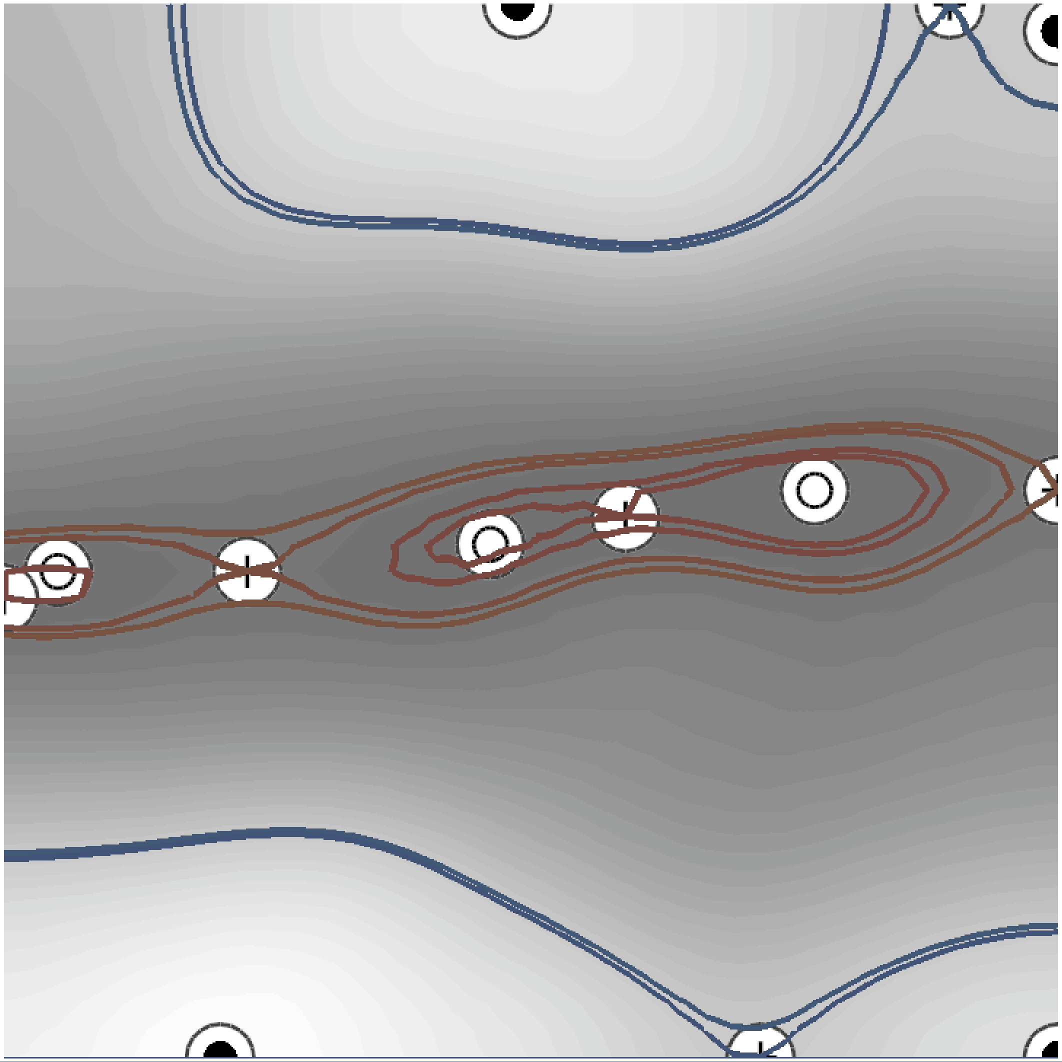
Puget Sound (8.4)



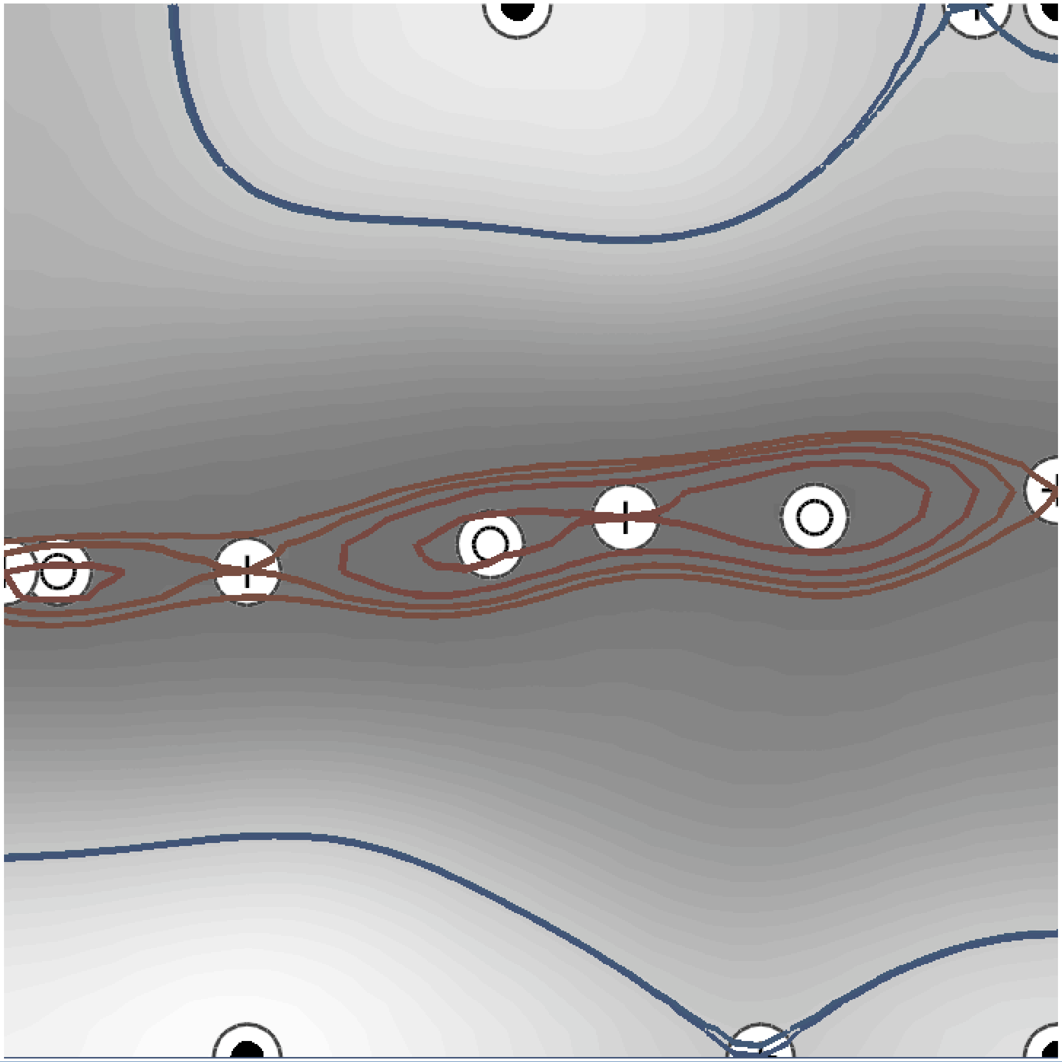
Puget Sound (13.3)



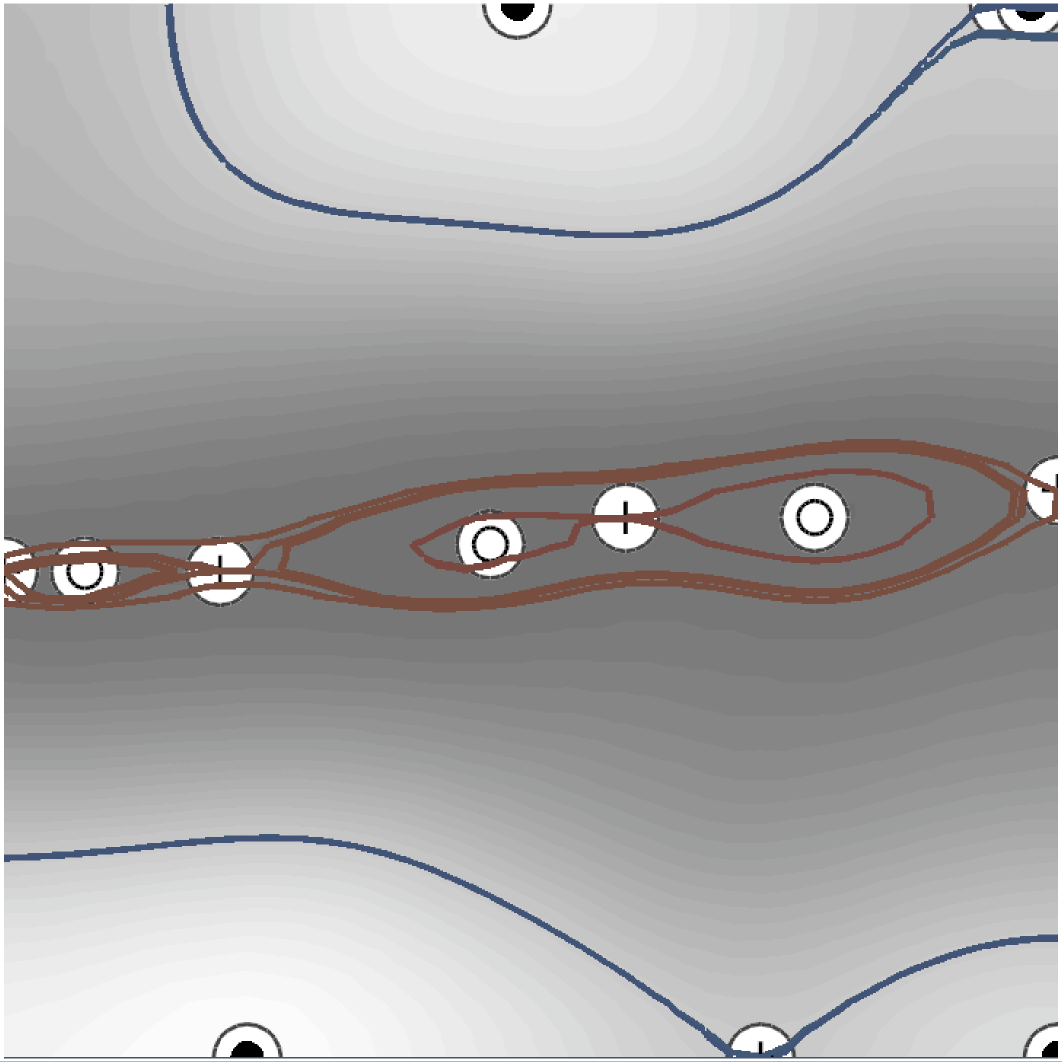
Puget Sound (18.2)



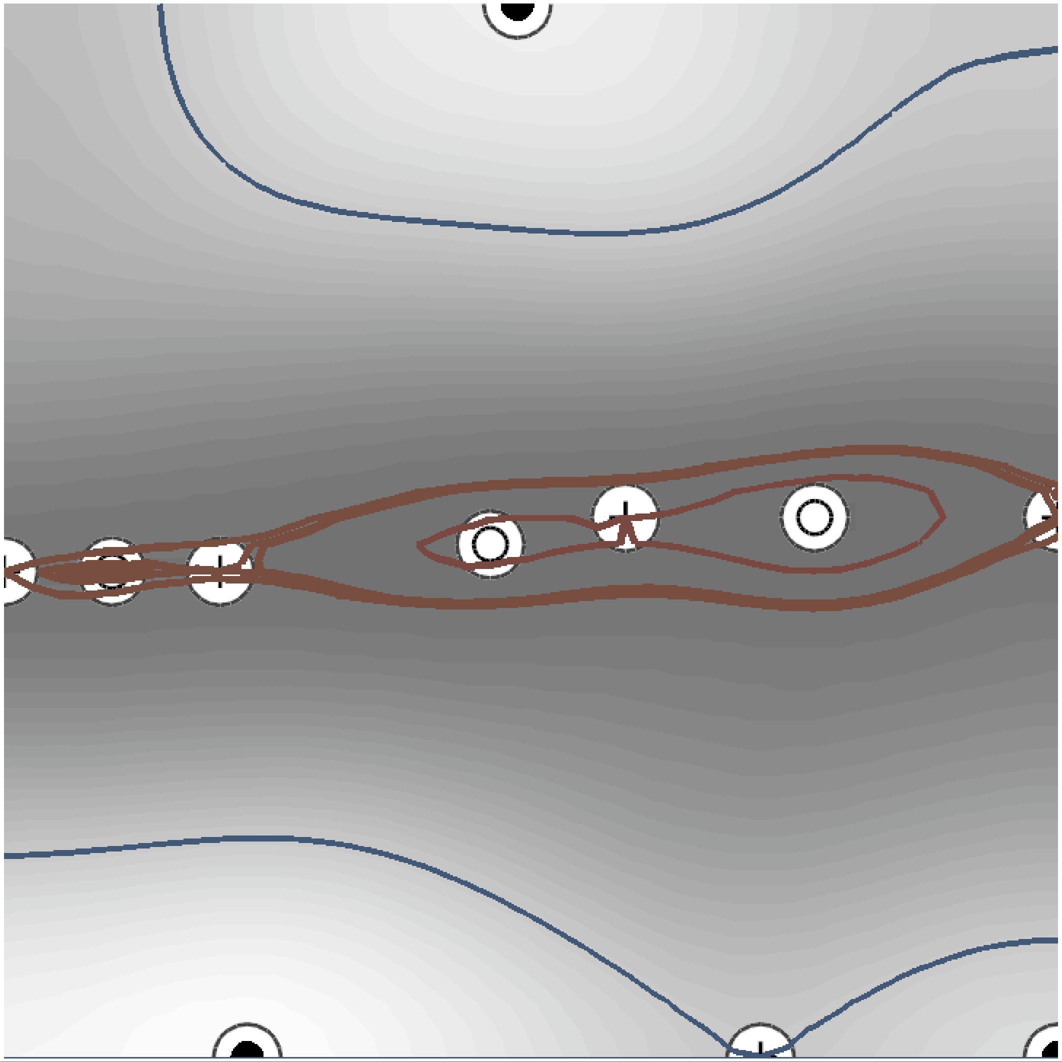
Puget Sound (23.1)



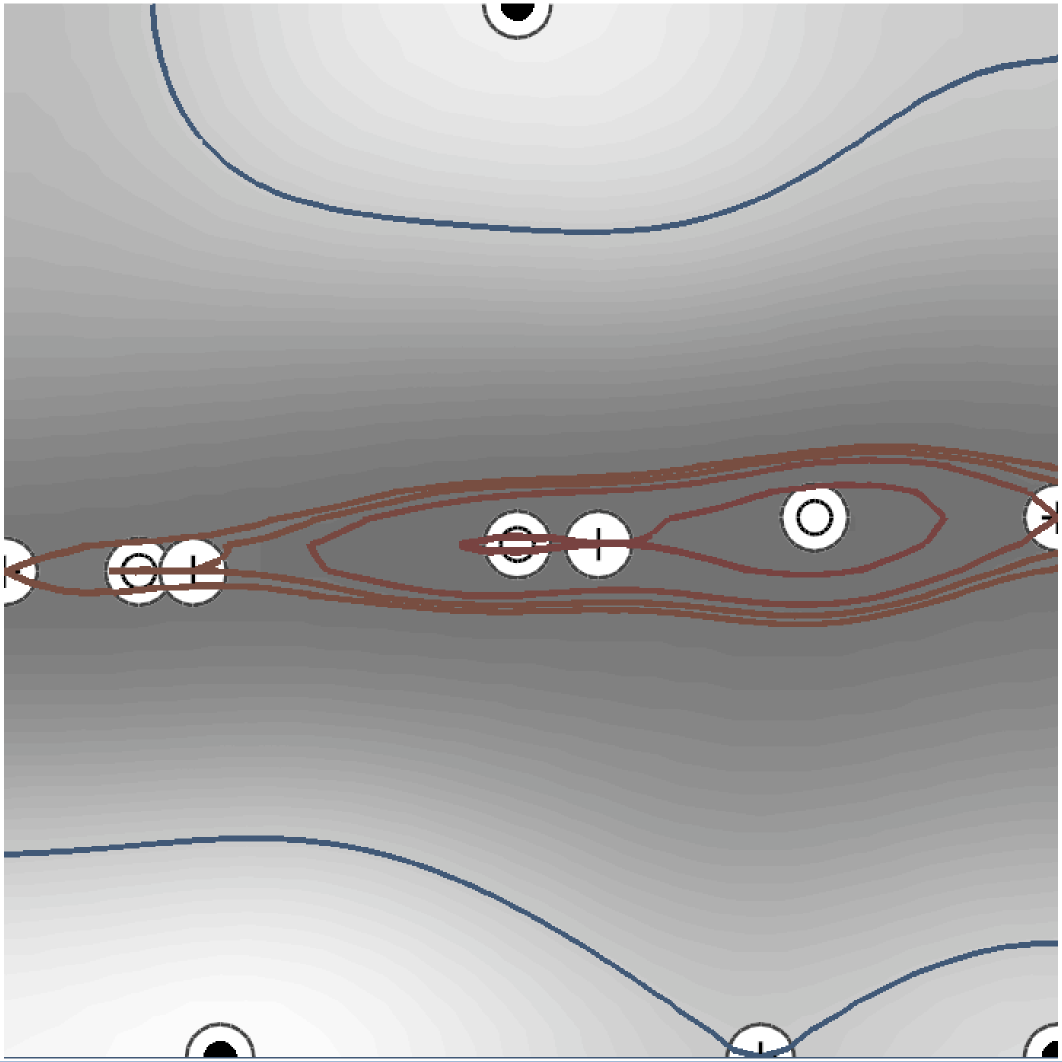
Puget Sound (28.0)



Puget Sound (32.9)



Puget Sound (37.8)



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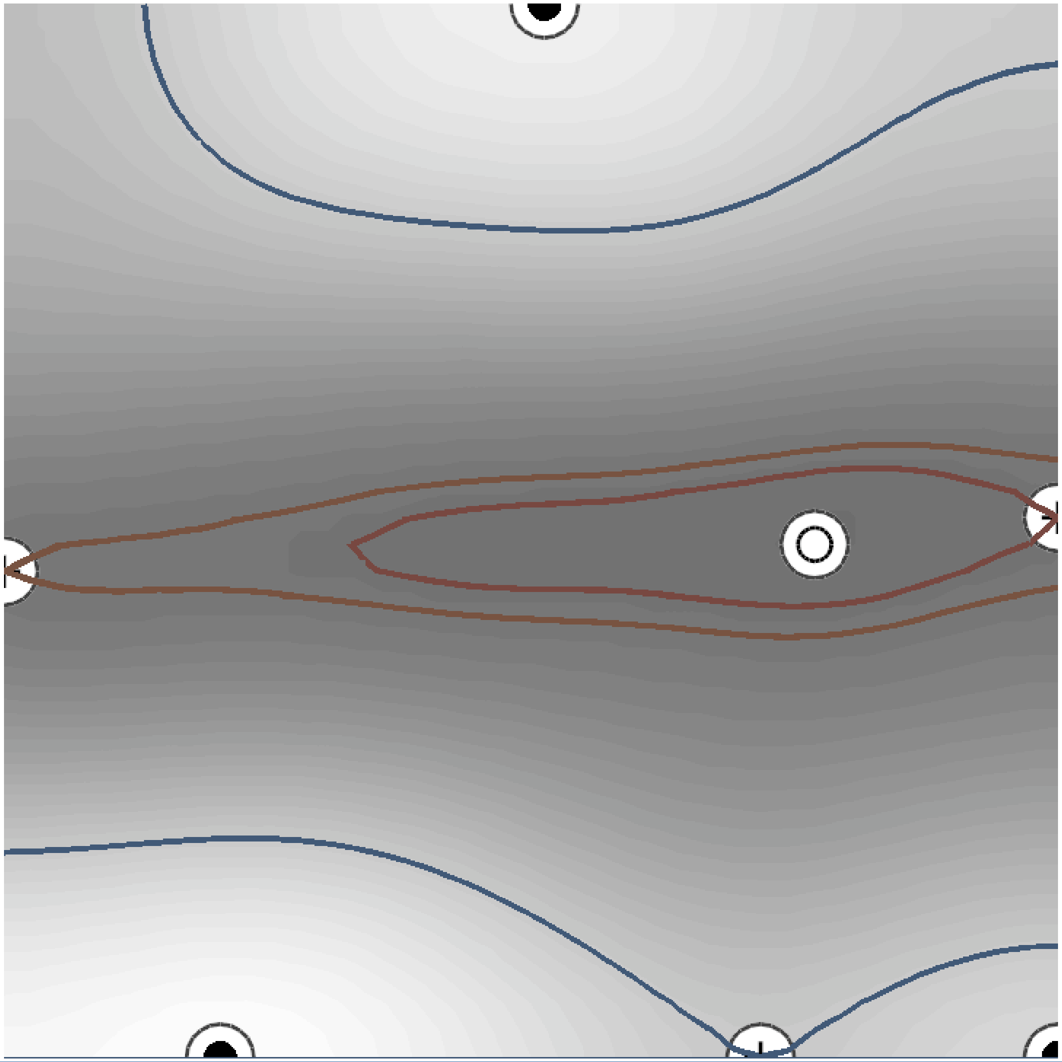
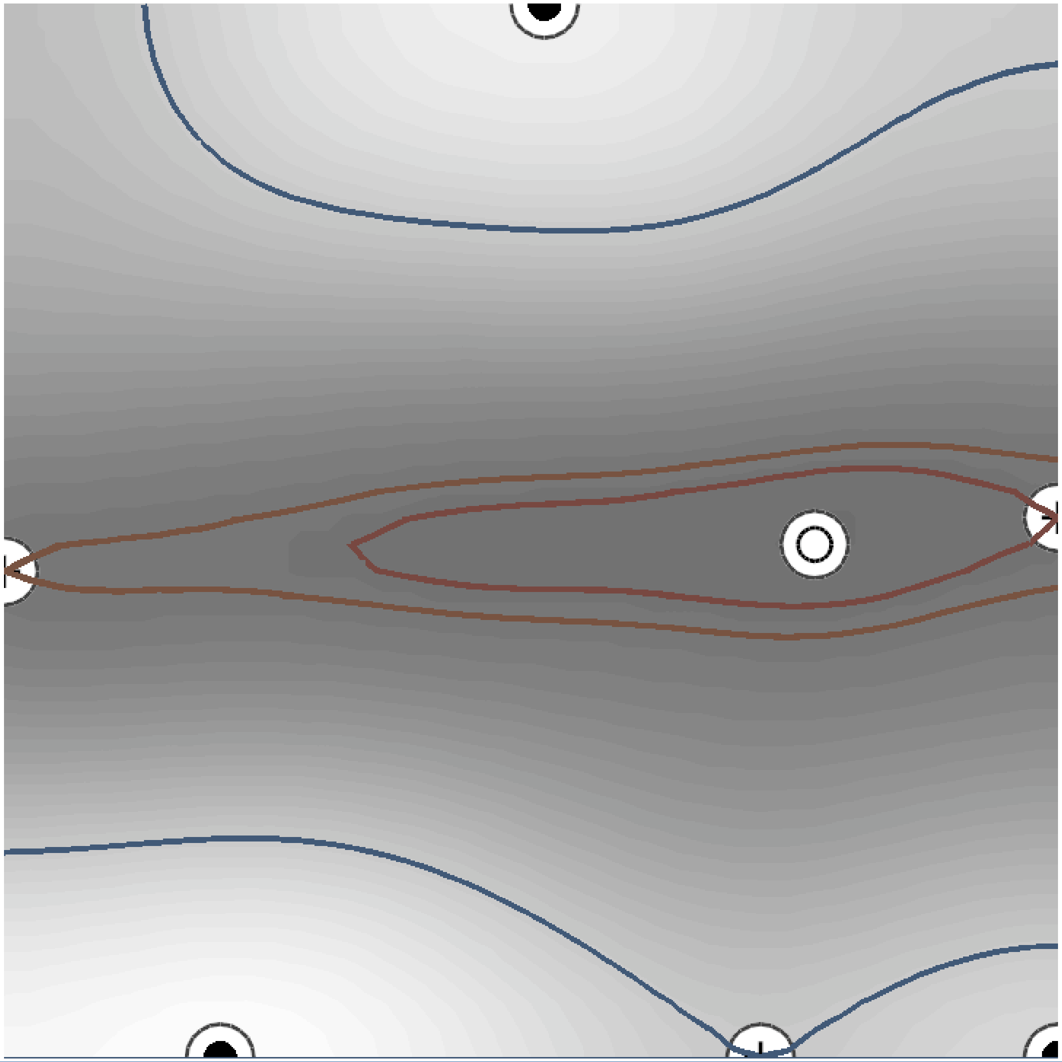
Puget Sound (42.7)



Puget Sound (42.7)



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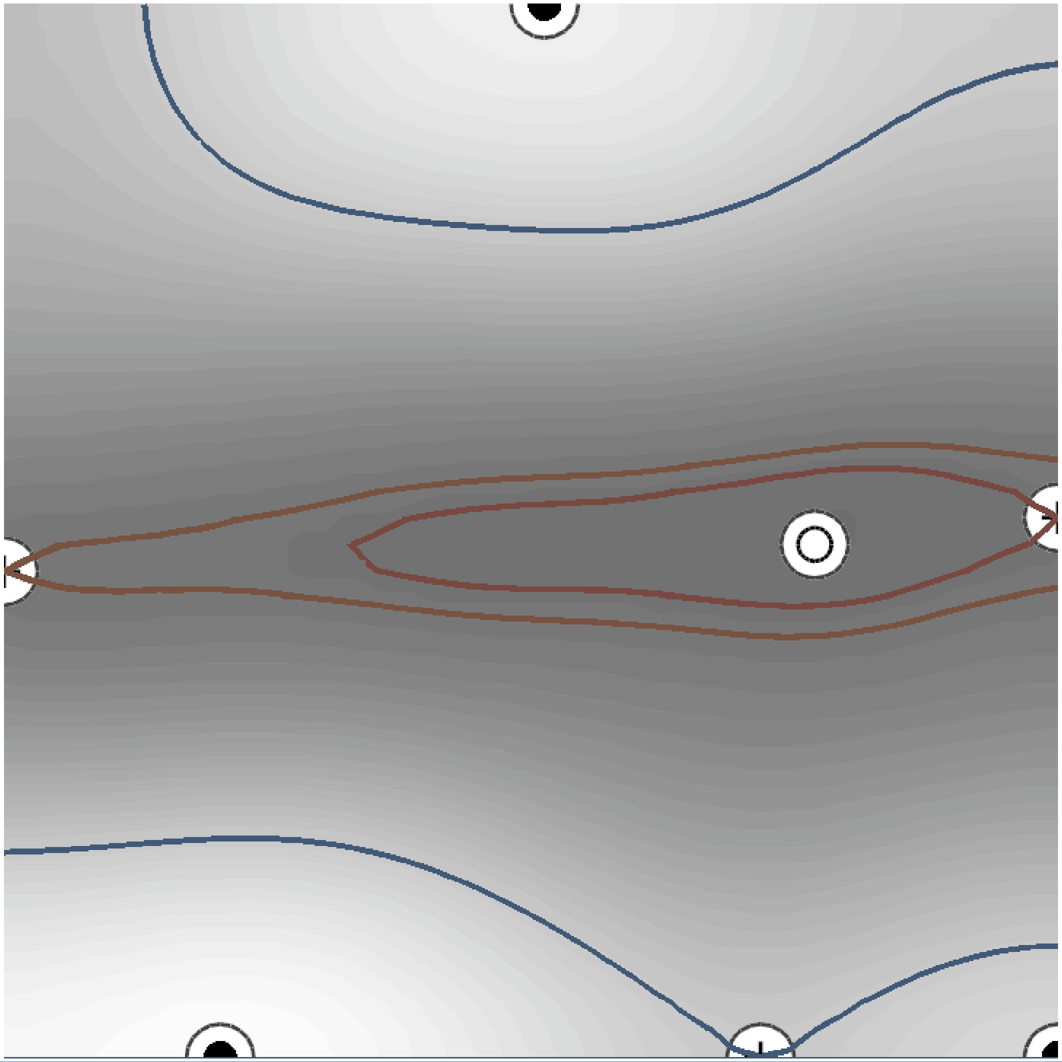
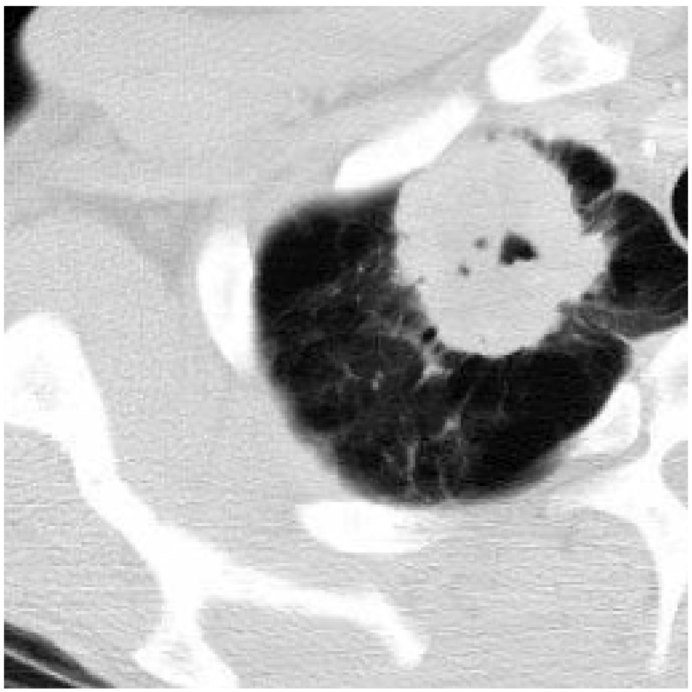
Puget Sound (42.7)

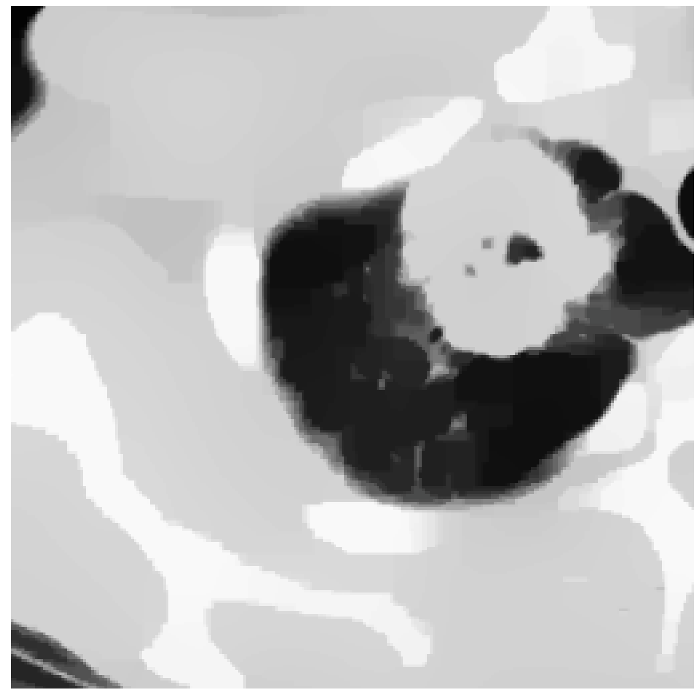


3/3

Anisotropic Smoothing

$$I_t = c(x,y,t)\Delta I + \nabla c \cdot \nabla I$$





- Can also create features

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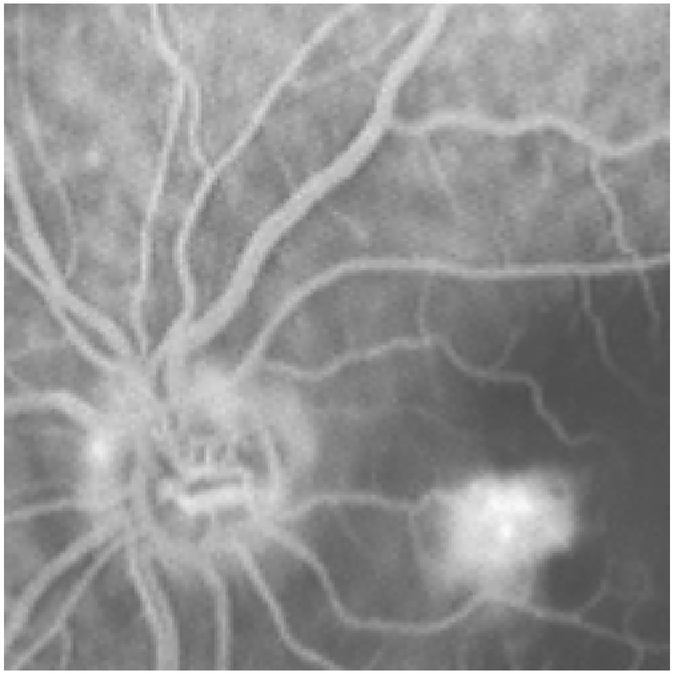
Sharpening

$$I_t = I_0 - smoothed(I)$$



- Don't want to create or destroy features

Algorithm

1. Obtain proposed values from the filter function
2. Identify edge flips and sort in time
3. Detect and prevent disallowed events
4. Goto step 1

1. Obtain proposed values

$$\bar{p}_{l+1} = F(\Delta t, p_l)$$

Proposed values at step l+1

Filter function

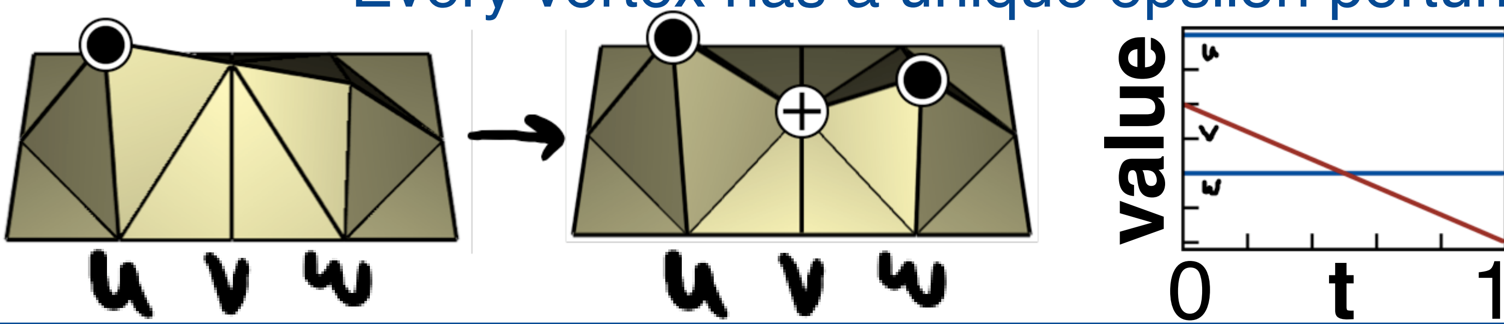
Step size

Current values at step l

2. Identify flips and sort

Function values change linearly from p_l to $\bar{p}_{l+1}$

- An edge between adjacent vertices flips at most once
- Must resolve flip time exactly
  - Otherwise vertex ordering becomes cyclic
  - Every vertex has a unique epsilon perturbation



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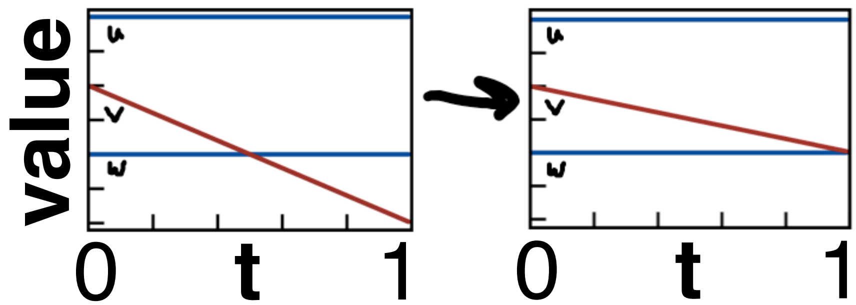
3. Detect & prevent disallowed events

Process events in order

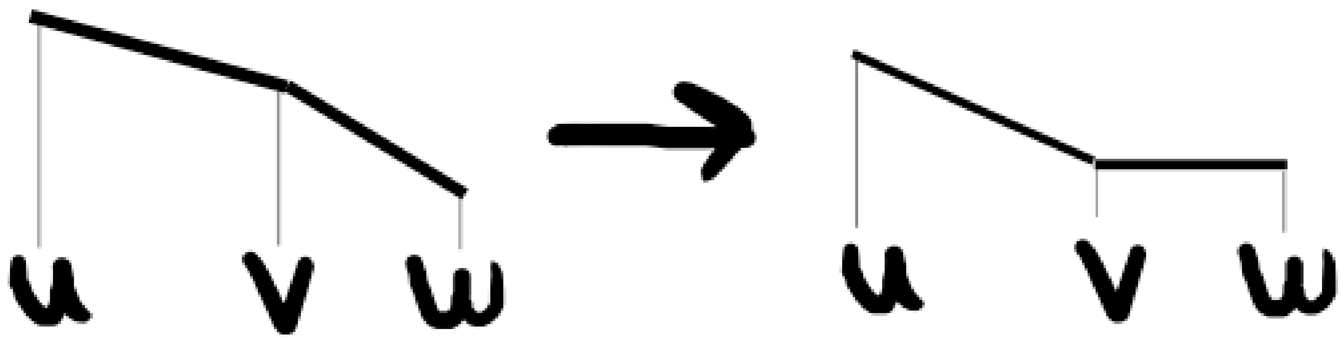
- If we reach a disallowed event between vertices (v, w), set $\bar{p}(v), \bar{p}(w)$ to values infinitesimally before the event



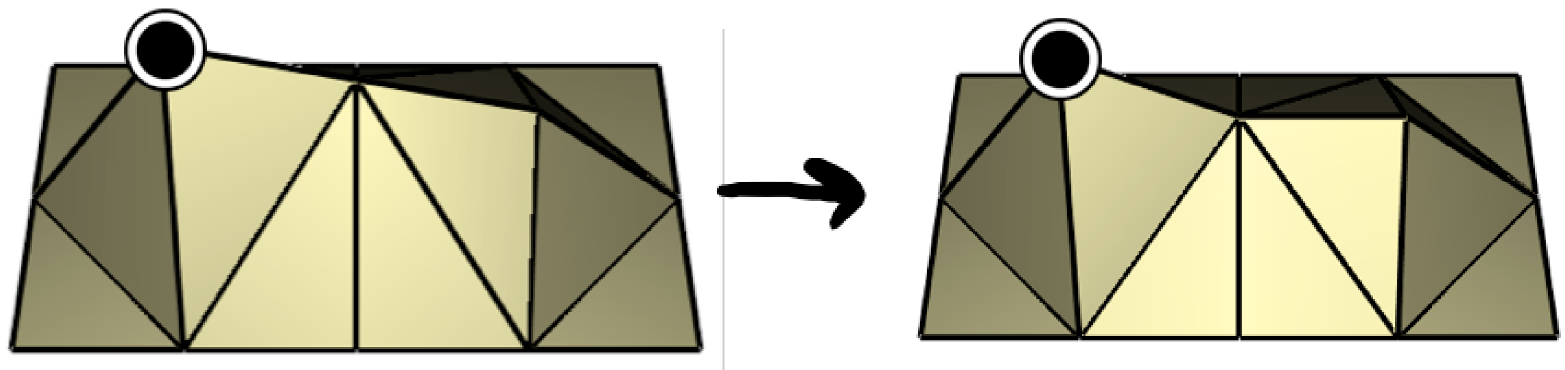
- Must re-identify events involving *v,w*
- May need to rewind the event queue

3. Detect & prevent disallowed events

1D:



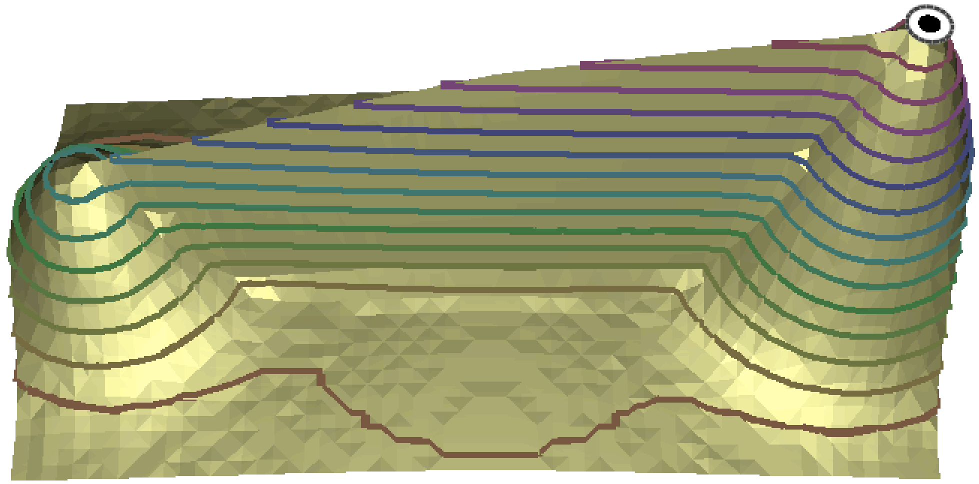
2D:

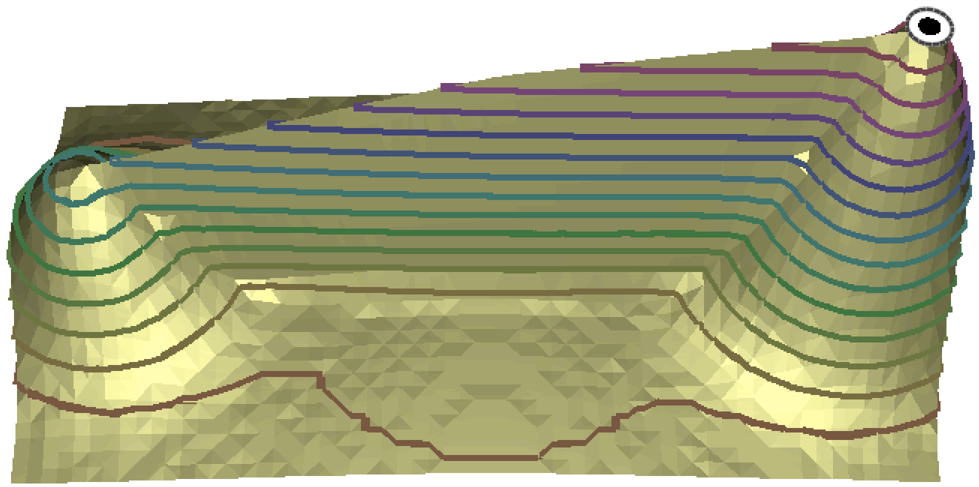


Algorithm

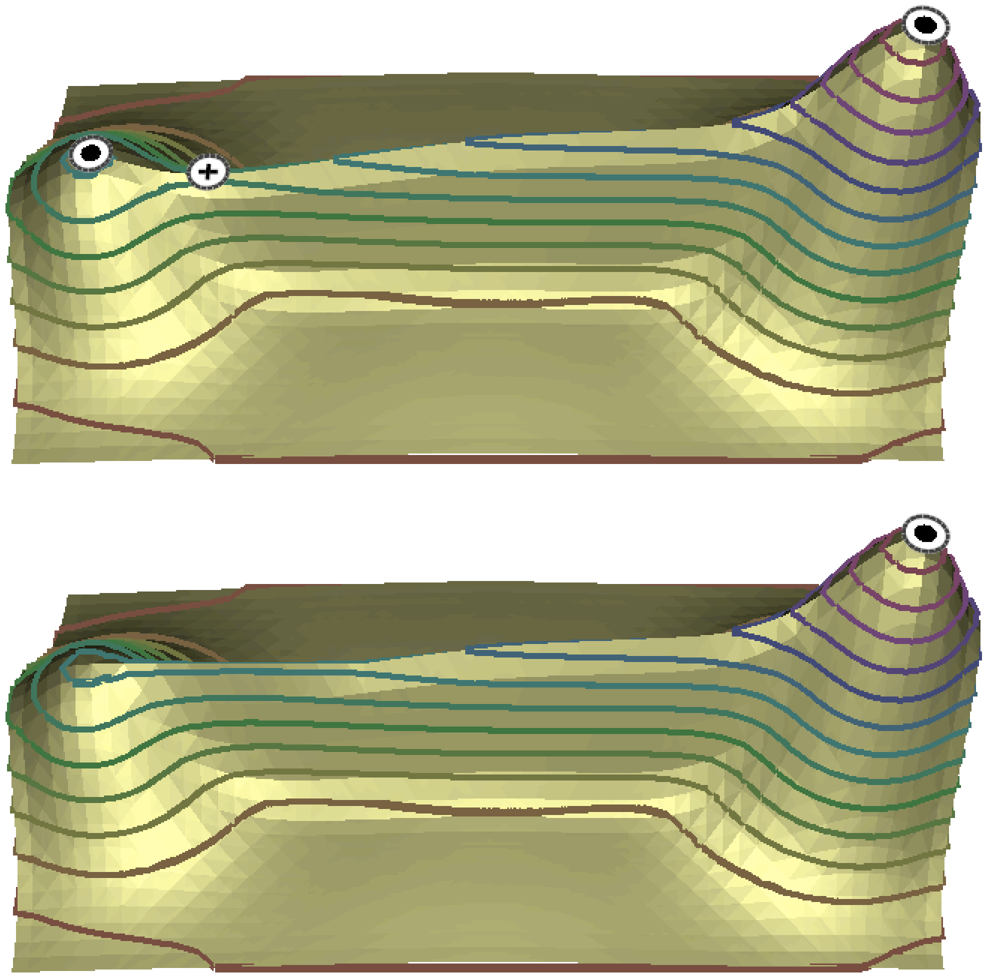
1. Obtain proposed values from the filter function
2. Identify edge flips and sort in time
3. Detect and prevent disallowed events
4. Goto step 1

Results: Ridge Bridge (0.0)





Results: Ridge Bridge (3.5)



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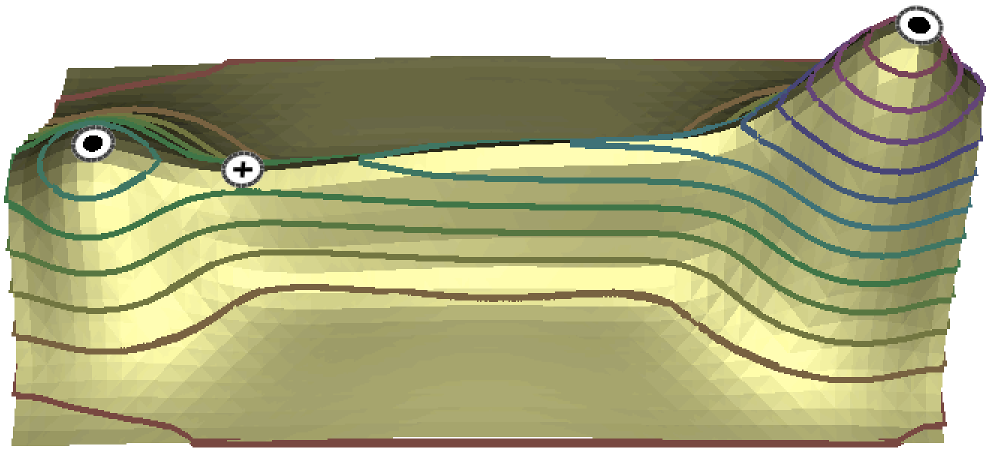
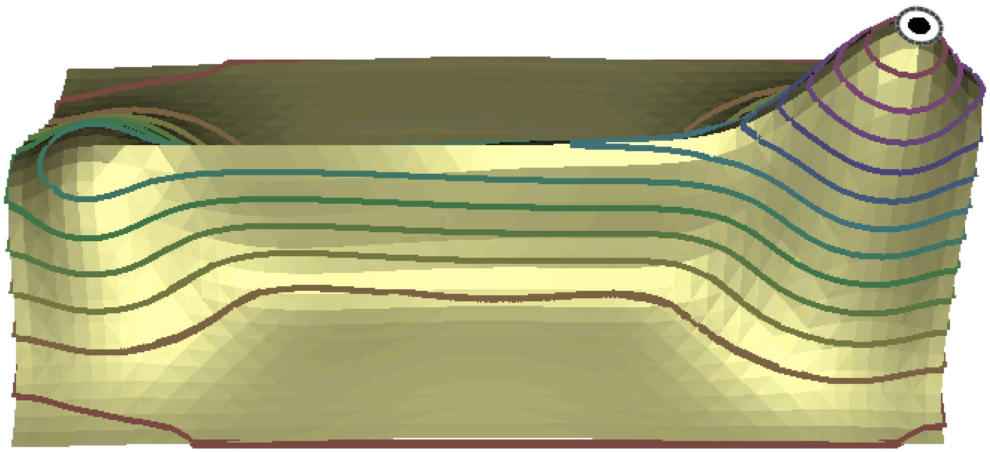
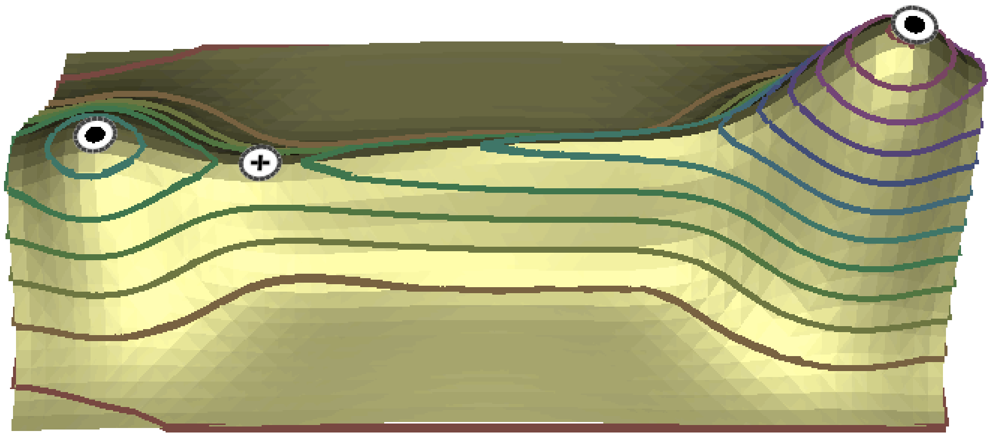
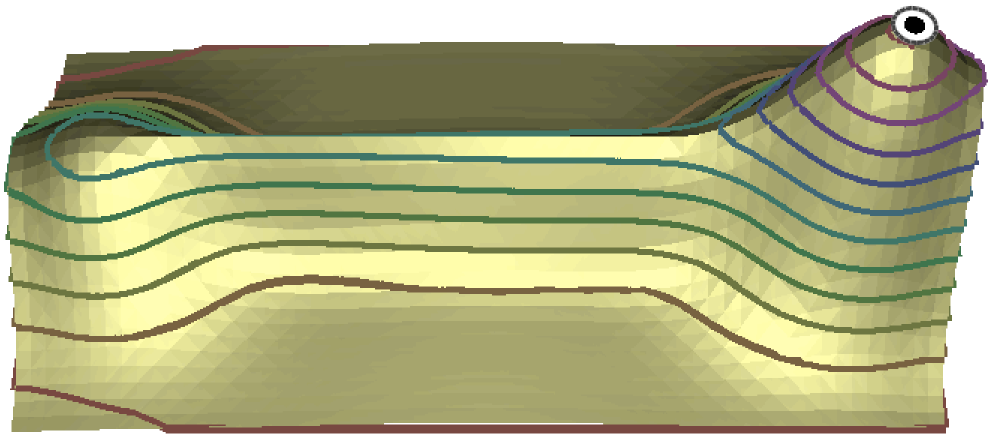
Results: Ridge Bridge (7.0)



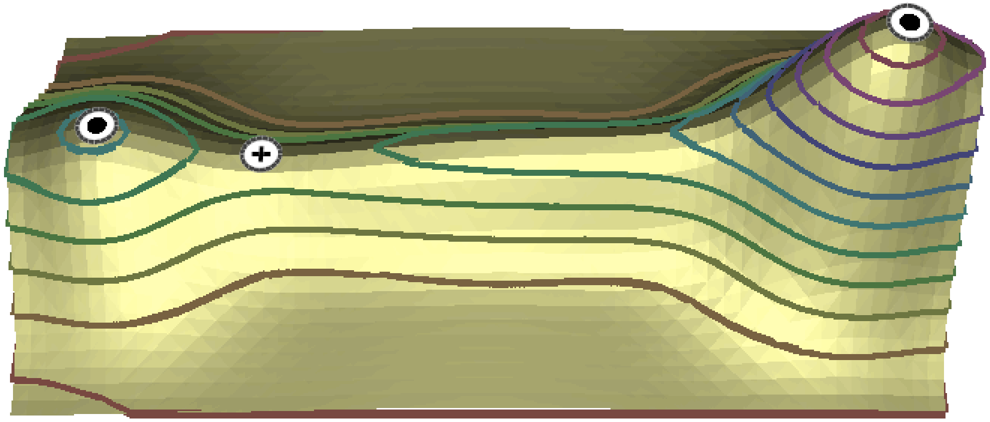


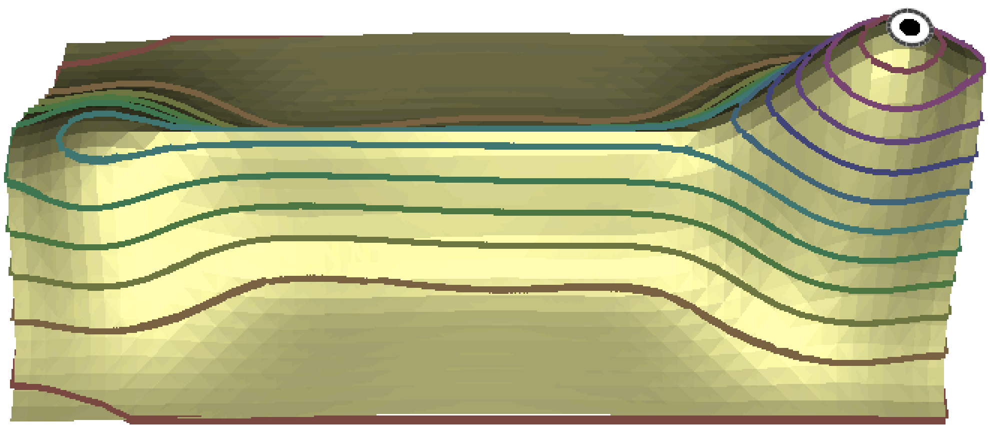
Results: Ridge Bridge (10.5)



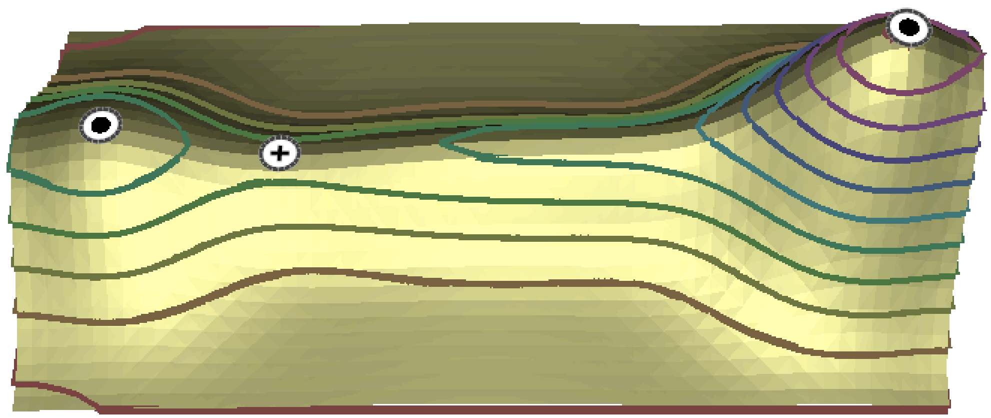


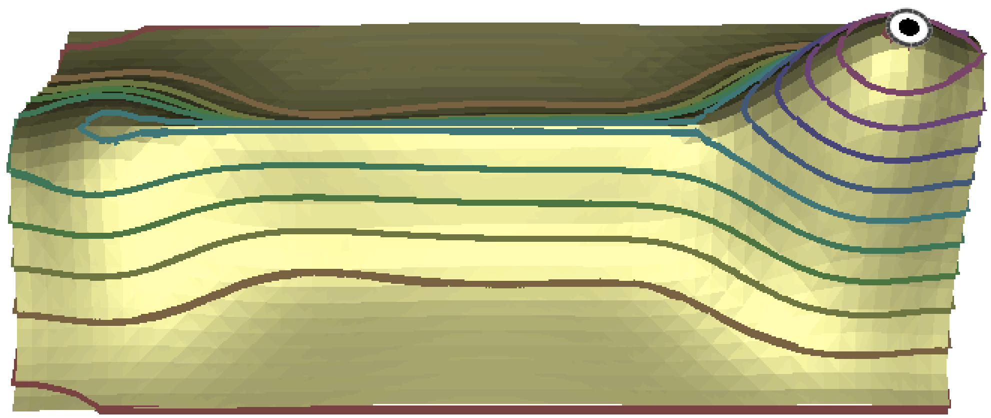
Results: Ridge Bridge (14.0)



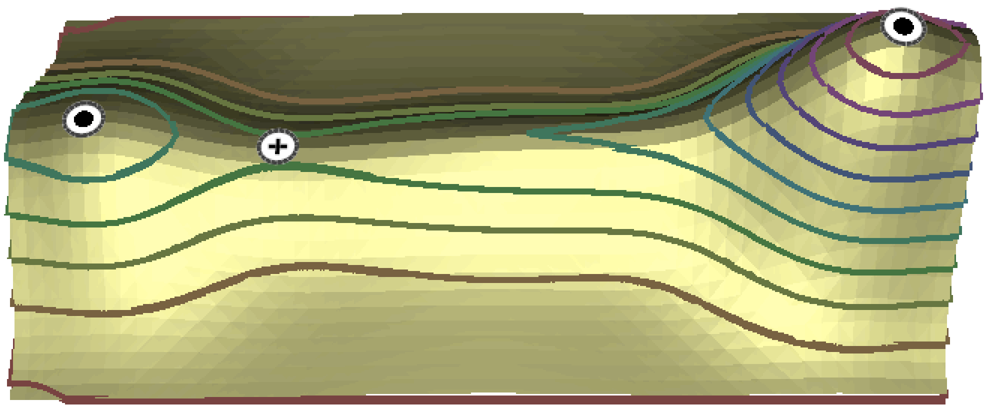


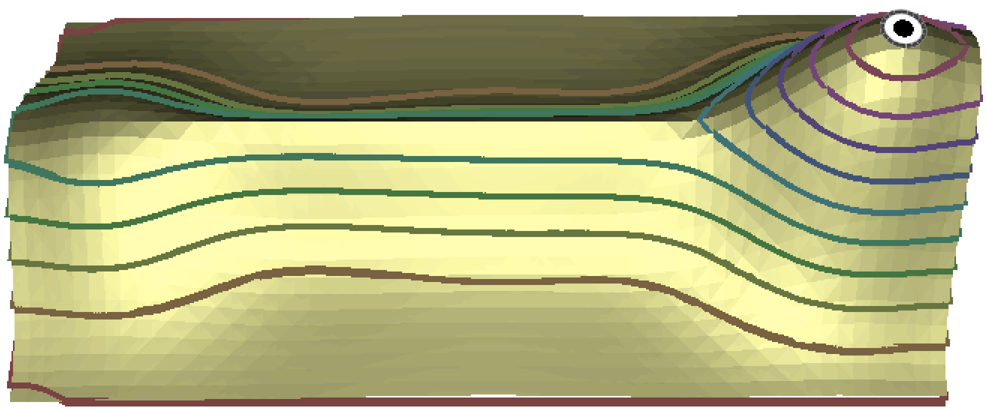
Results: Ridge Bridge (17.5)





Results: Ridge Bridge (21.0)





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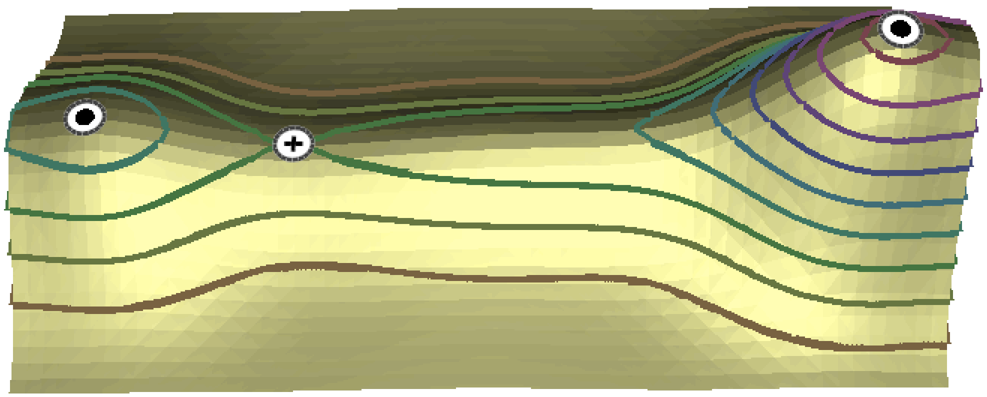
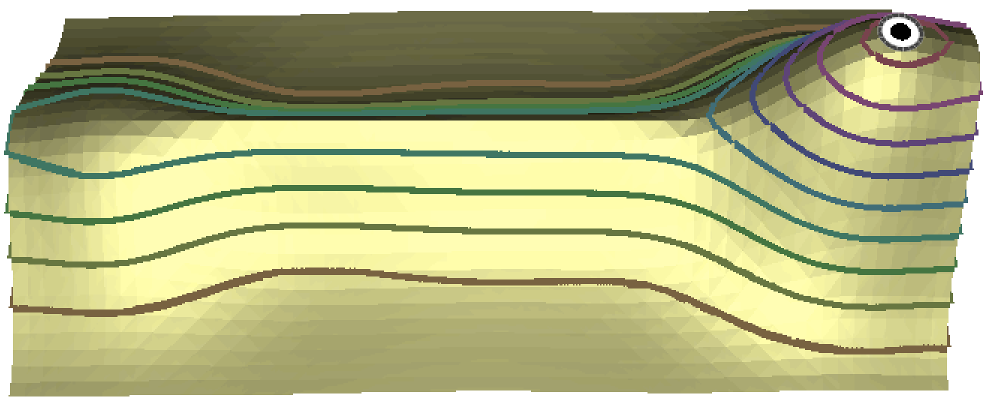
Results: Ridge Bridge (24.5)





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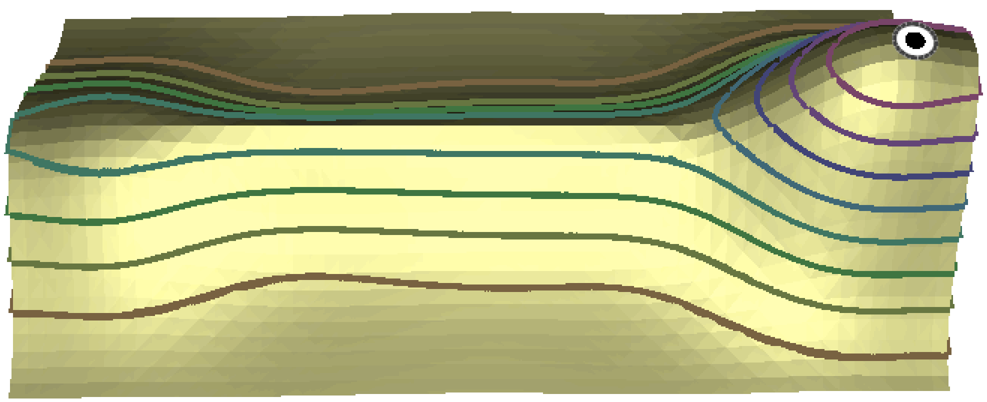
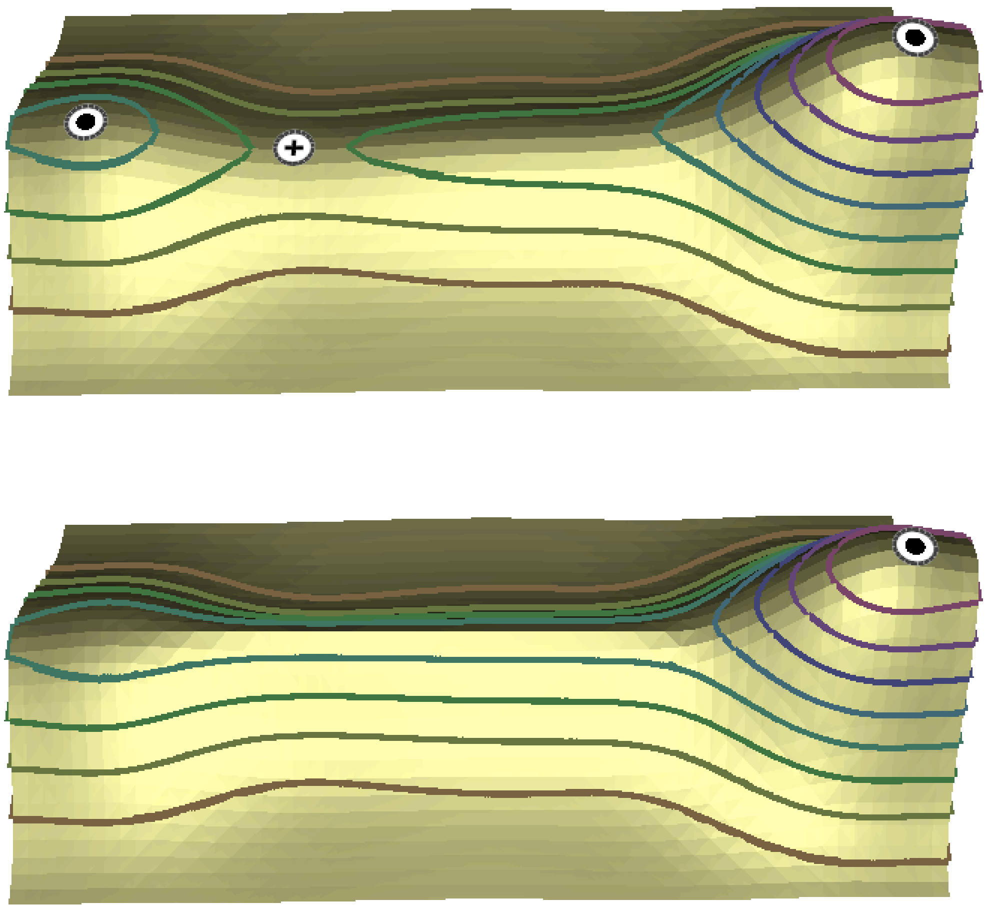
Results: Ridge Bridge (28.0)

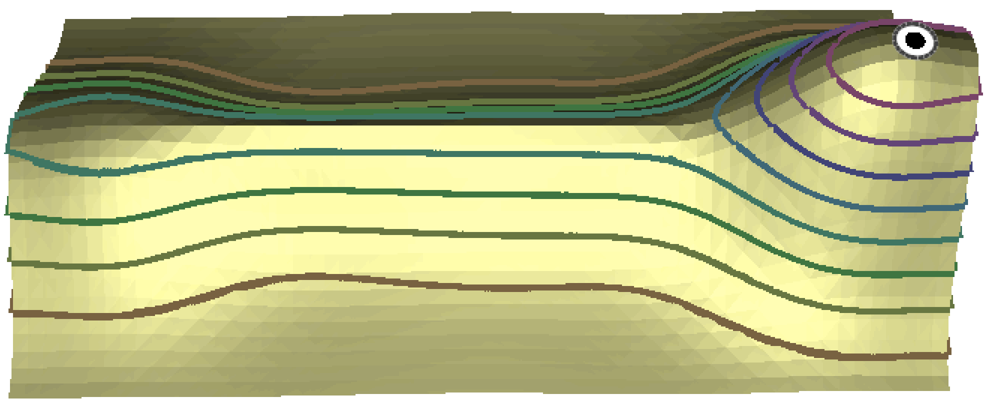


Results: Ridge Bridge (28.0)



2/3

Results: Ridge Bridge (28.0)



3/3

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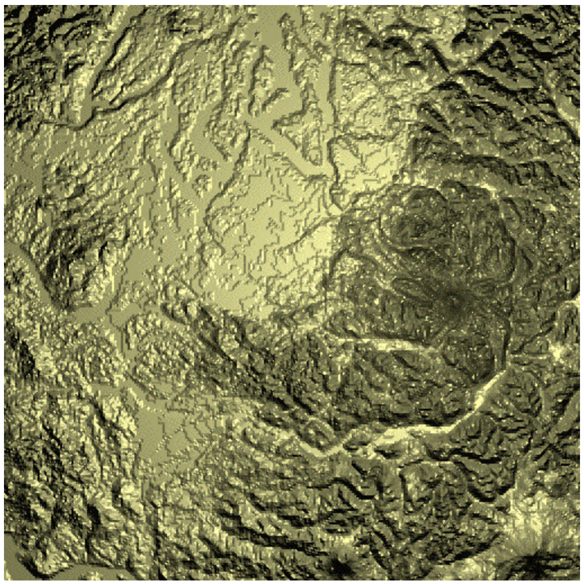
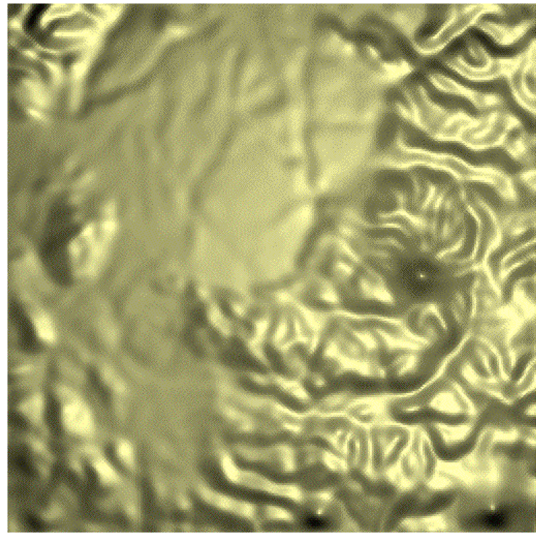
Results: Laplacian Smoothing

original



uncontrolled



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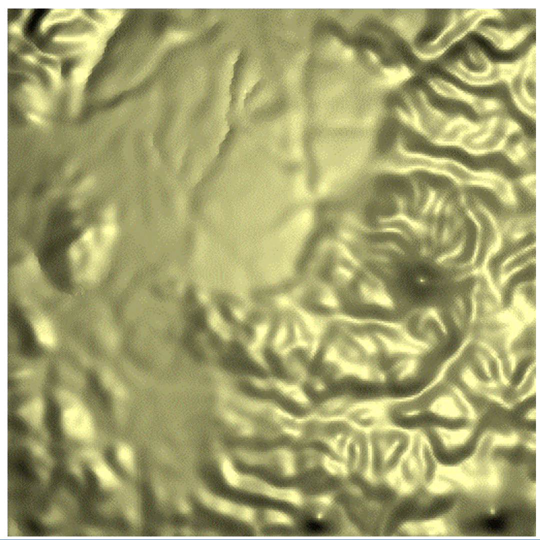

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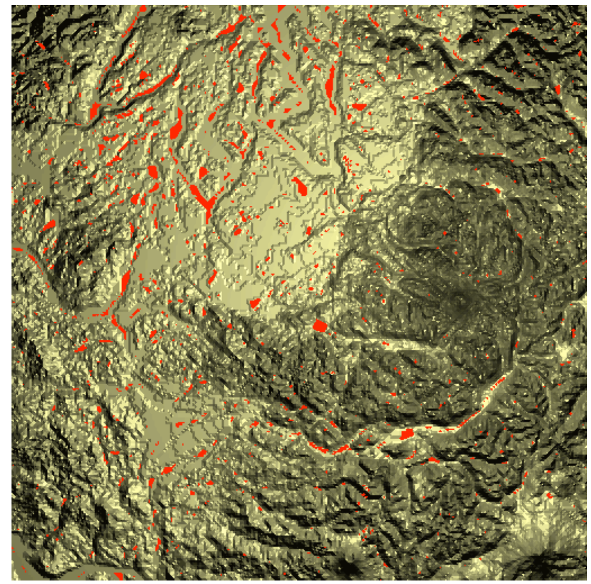

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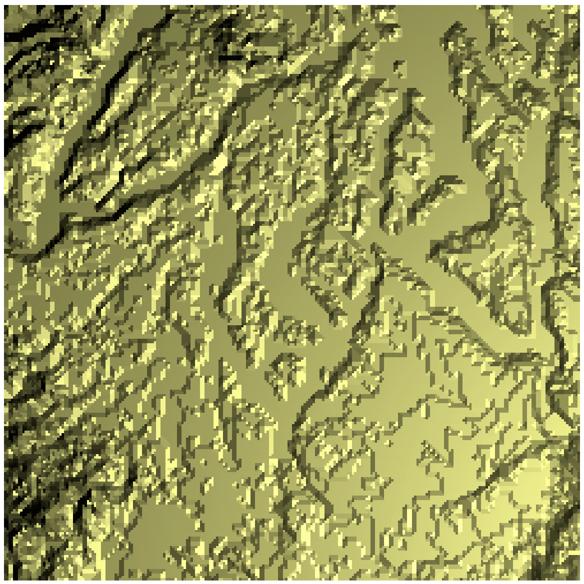
Results: Laplacian Smoothing

original



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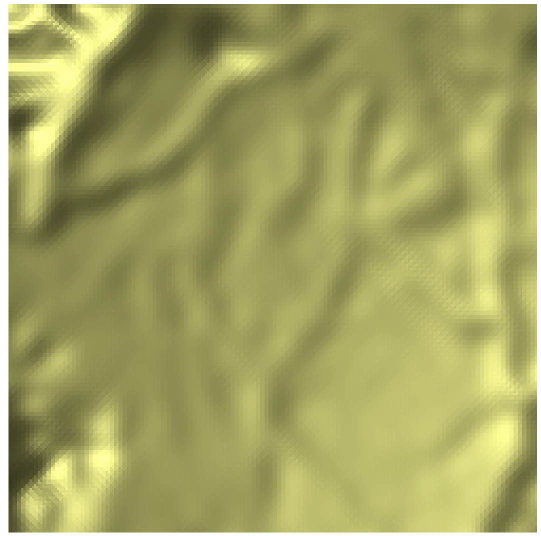

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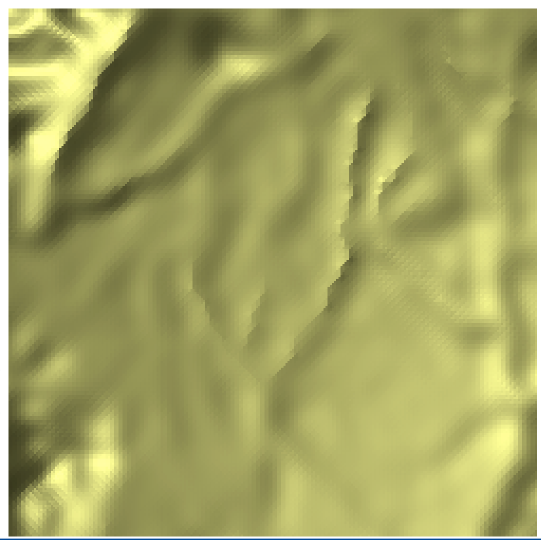

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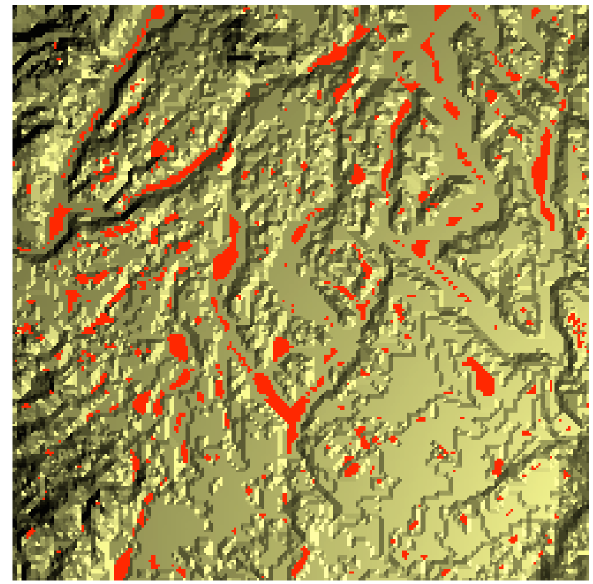
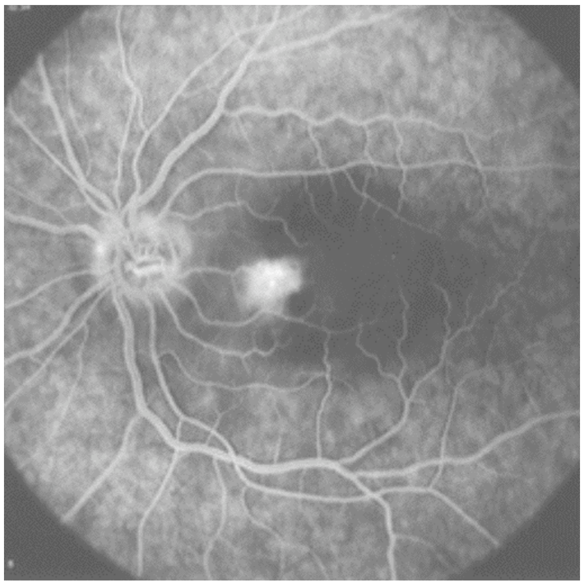


Results: Sharpening

original



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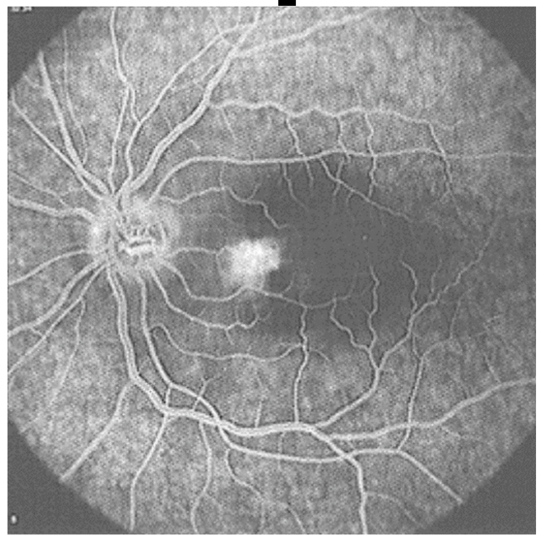

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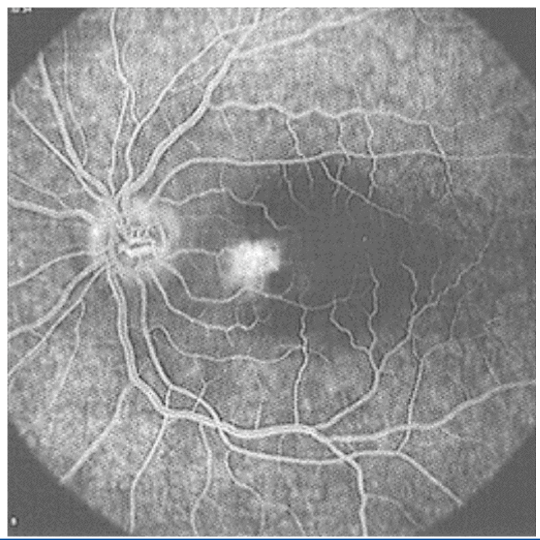

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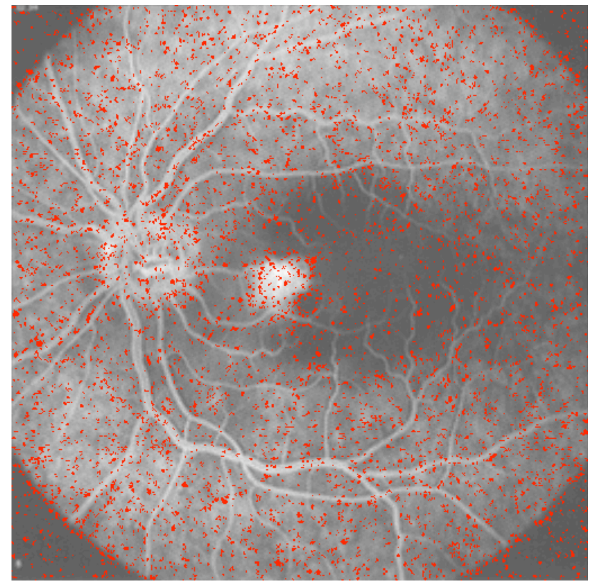

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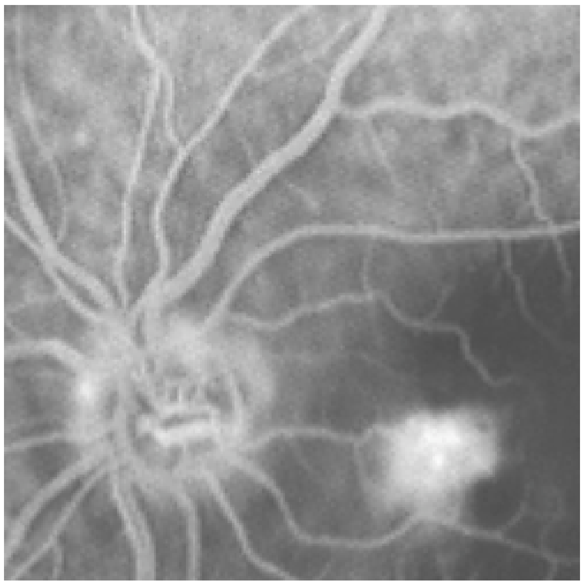
Results: Sharpening

original



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controlled

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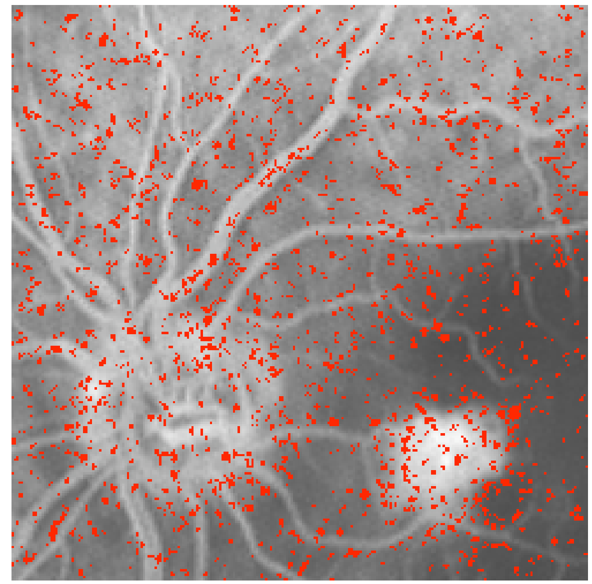

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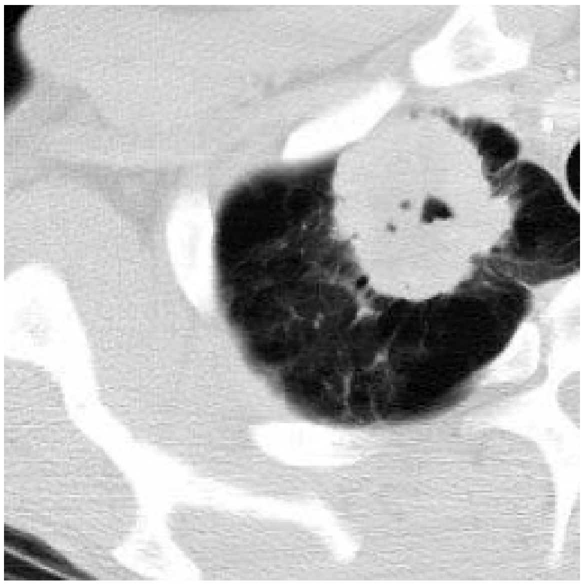
Results: Anisotropic Smoothing

original



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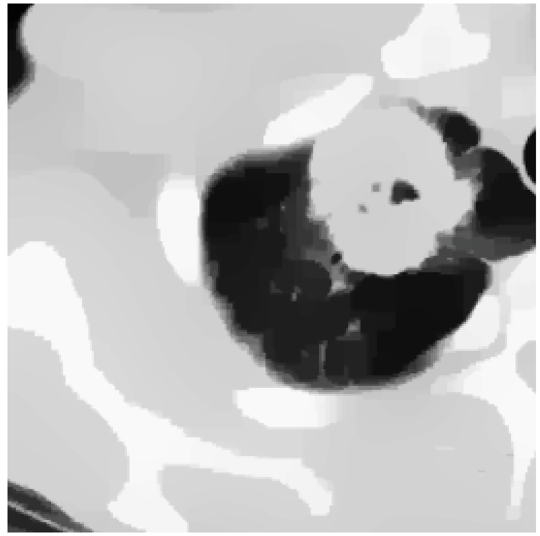

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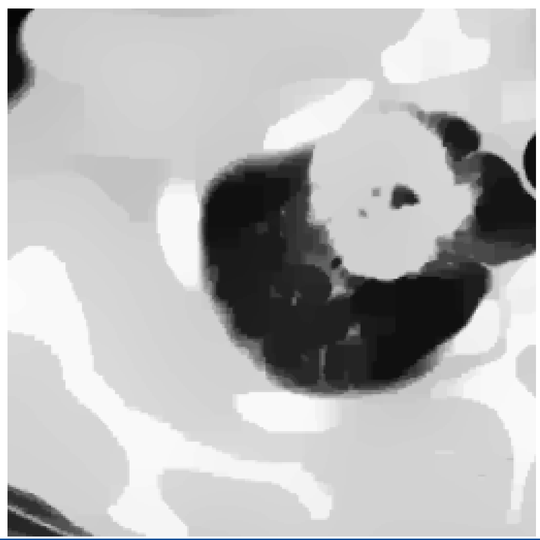

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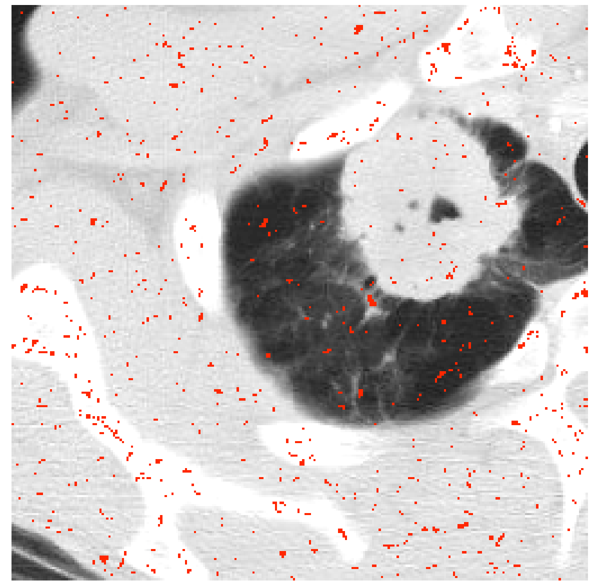
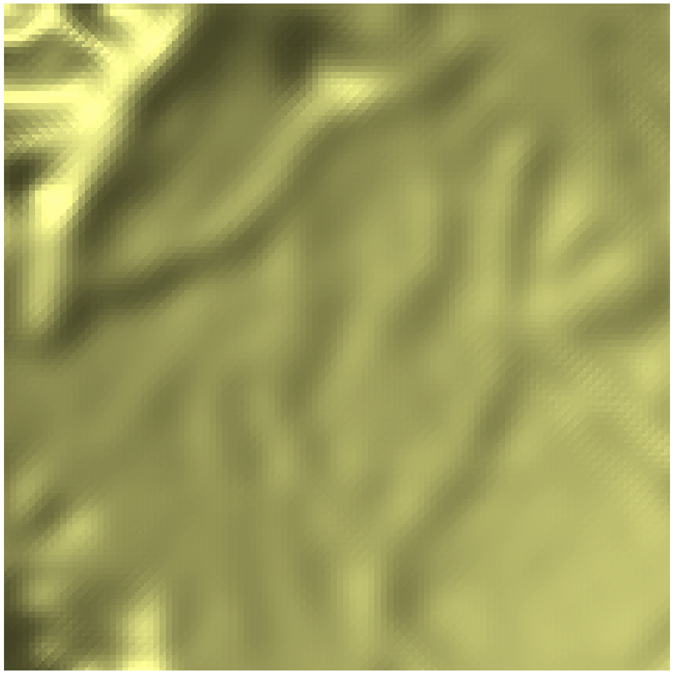


Local artifacts

Due to slowing time



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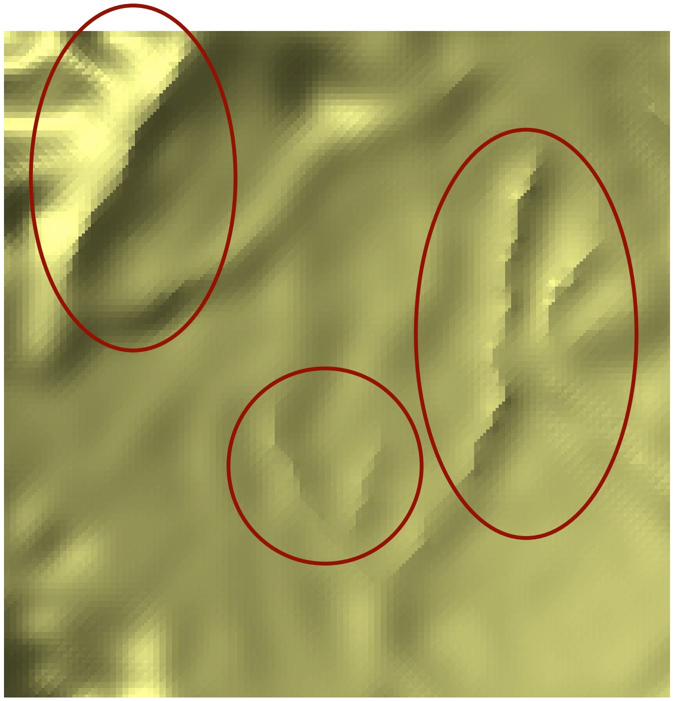

controlled

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No progress guarantee

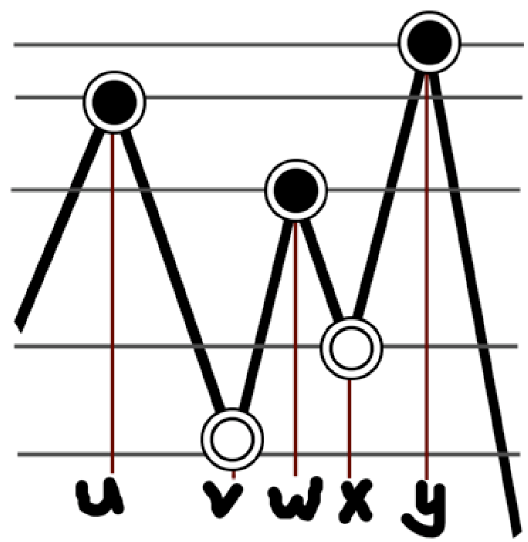
Terminate when proposed values same as current

Never saw topology control prevent progress everywhere

Results: Persistence

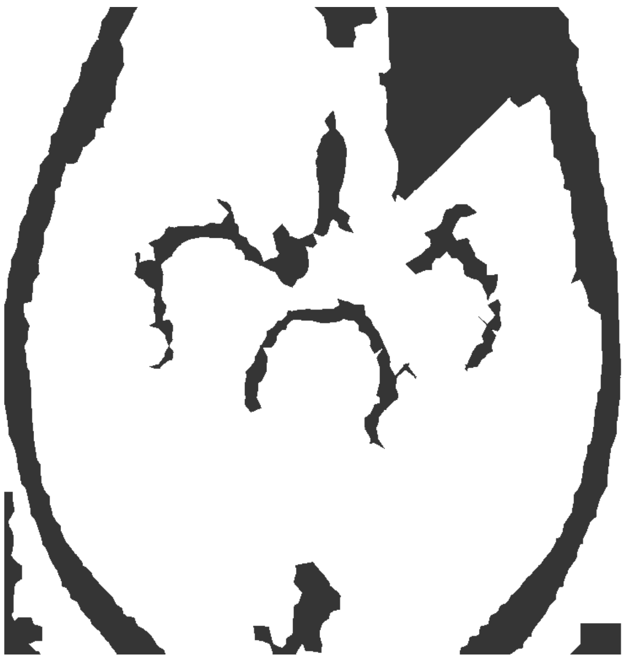
- Measures the importance of a feature
- Common measure is difference in value between an extremum and its paired saddle
- We can track the time it takes for an extremum (and its paired saddle) to be annihilated under smoothing

1D:



Results: Persistence

Difference
in value

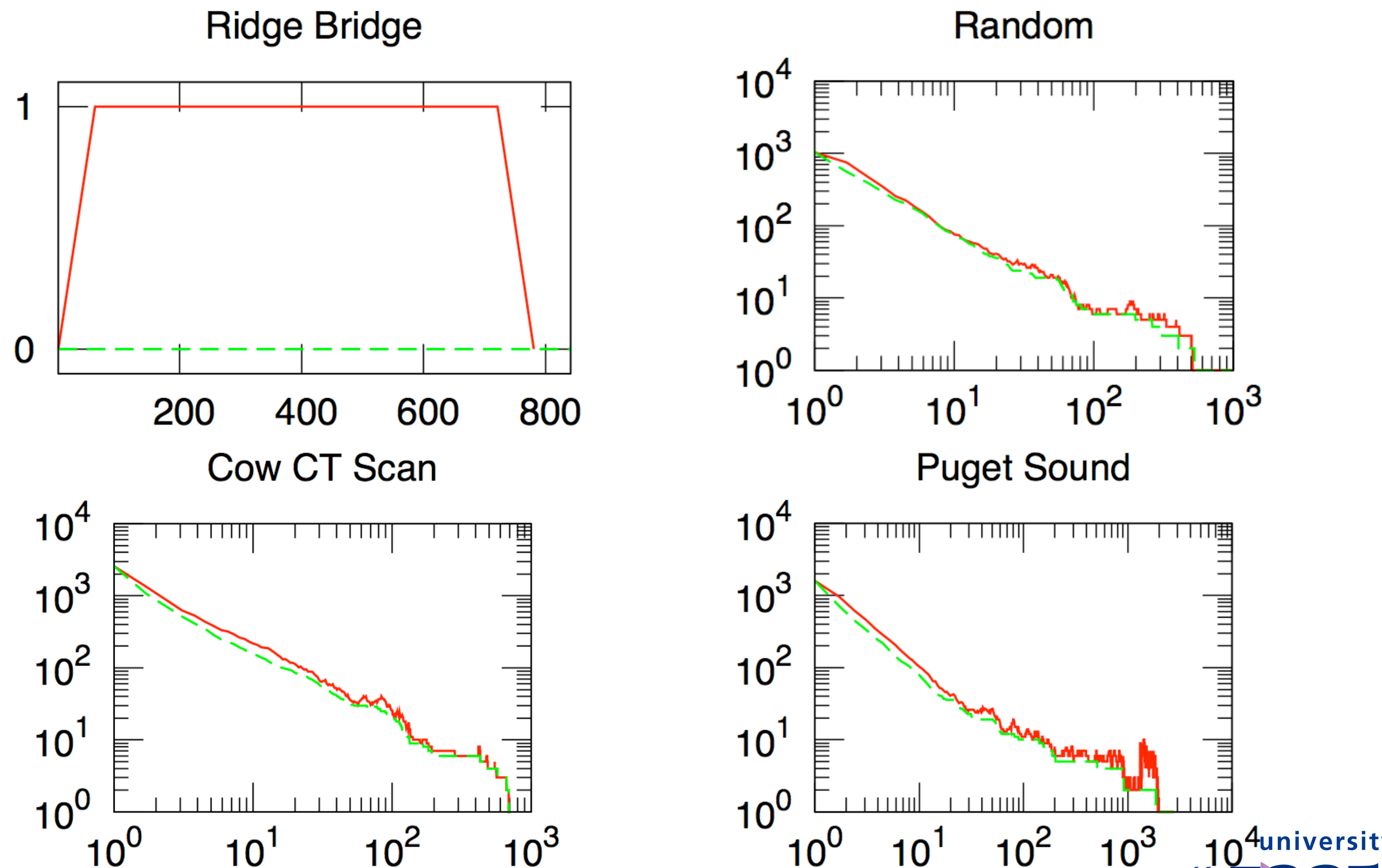


Cow CT Scan

Anisotropic
diffusion lifetime

Features shaded black

Critical Points Over Time



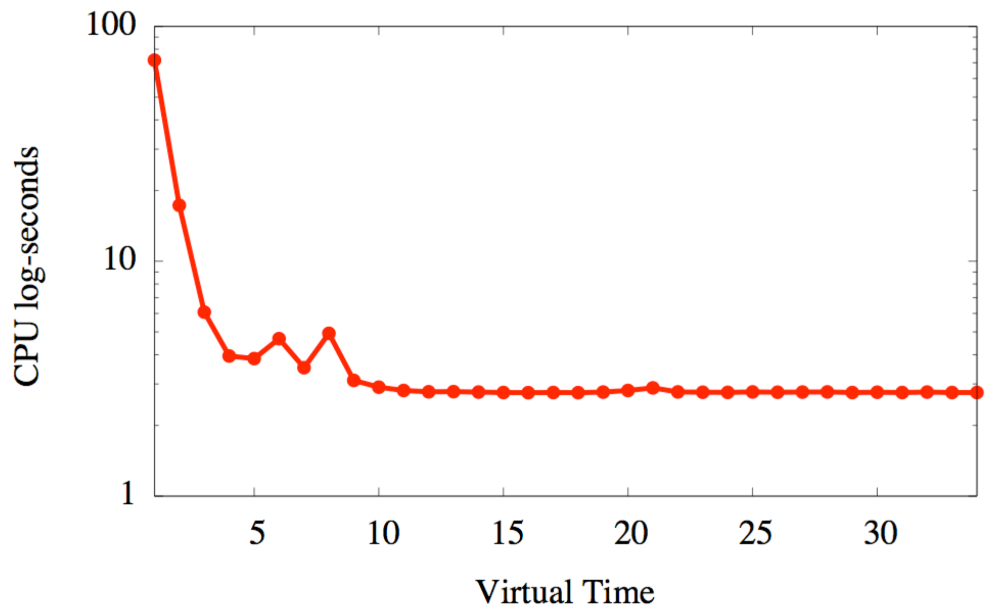
Performance

Performance related to number of edge flips and number of undesired topological events

Roughly corresponds to number of critical points

Performance

- Laplacian smoothing Puget Sound to one global maximum is 2.2x slower
- 44% of the time is spent in the first 3 steps



Future Work

Extend to 3D

Predict disallowed events to distribute undesired artifacts

Conclusion

A simple algorithm that controls topology changes when filtering

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Denis Zorin <dzorin@mrl.nyu.edu>

Thanks to: Chris Wu, Chee Yap, Adrian Secord,
NYU CS Colleagues, and the reviewers

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Fin

Filters

Discrete Laplacian smoothing (diffusion)

$$p_L^{l+1}(v) = p^l(v) + \Delta t \sum_{edges(v,w)} (p^l(w) - p^l(v))$$

Sharpening

$$p_S^{l+1}(v) = p^l(v) + \Delta t (p_L^n(v) - p^0(v))$$

Discrete Anisotropic smoothing ([Perona and Malik 1990])

$$p_{AD}^{l+1}(v) = p^l(v) + \Delta t \sum_{edges(v,w)} \frac{p^l(w) - p^l(v)}{1 + \|p^l(w) - p^l(v)\|^2 / k^2}$$

Feature

- Sweep a plane down. Notice how at a maximum a new contour is born! Notice how it merges with another contour at a saddle!
- Same with minimums and sweeping upwards!

Feature

Hills and valleys until it gets complicated -- max and mins (pics from original slide)

Feature

Sweep a plane down data set

- Maximum creates a contour
- Minimum creates a contour
- Saddle merges two contours
- Talk to denis to get terminology consistent

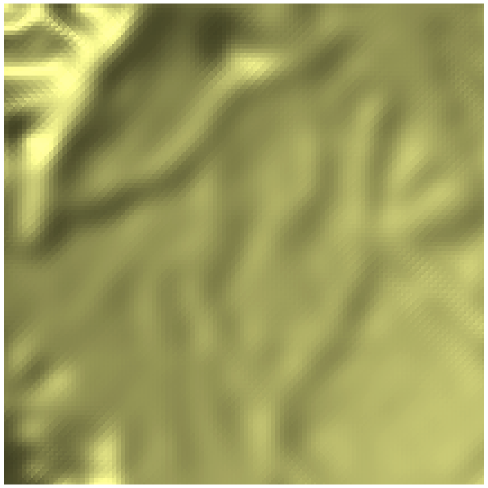
Feature

The set of contours from an extremum until the saddle which merges the contours with another extremum's contours, as we sweep a plane upwards/downwards

Single contour component from appearance at max/min until merger with another component at saddle as isovalue decreases/increases

No guaranteed progress

Terminate when proposed values same as current
Can topology control prevent progress everywhere?
Locally:



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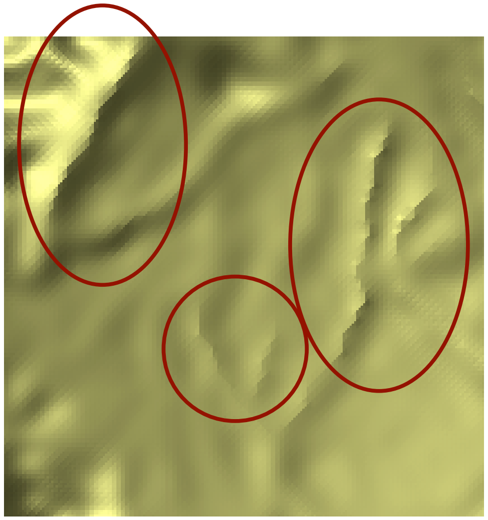

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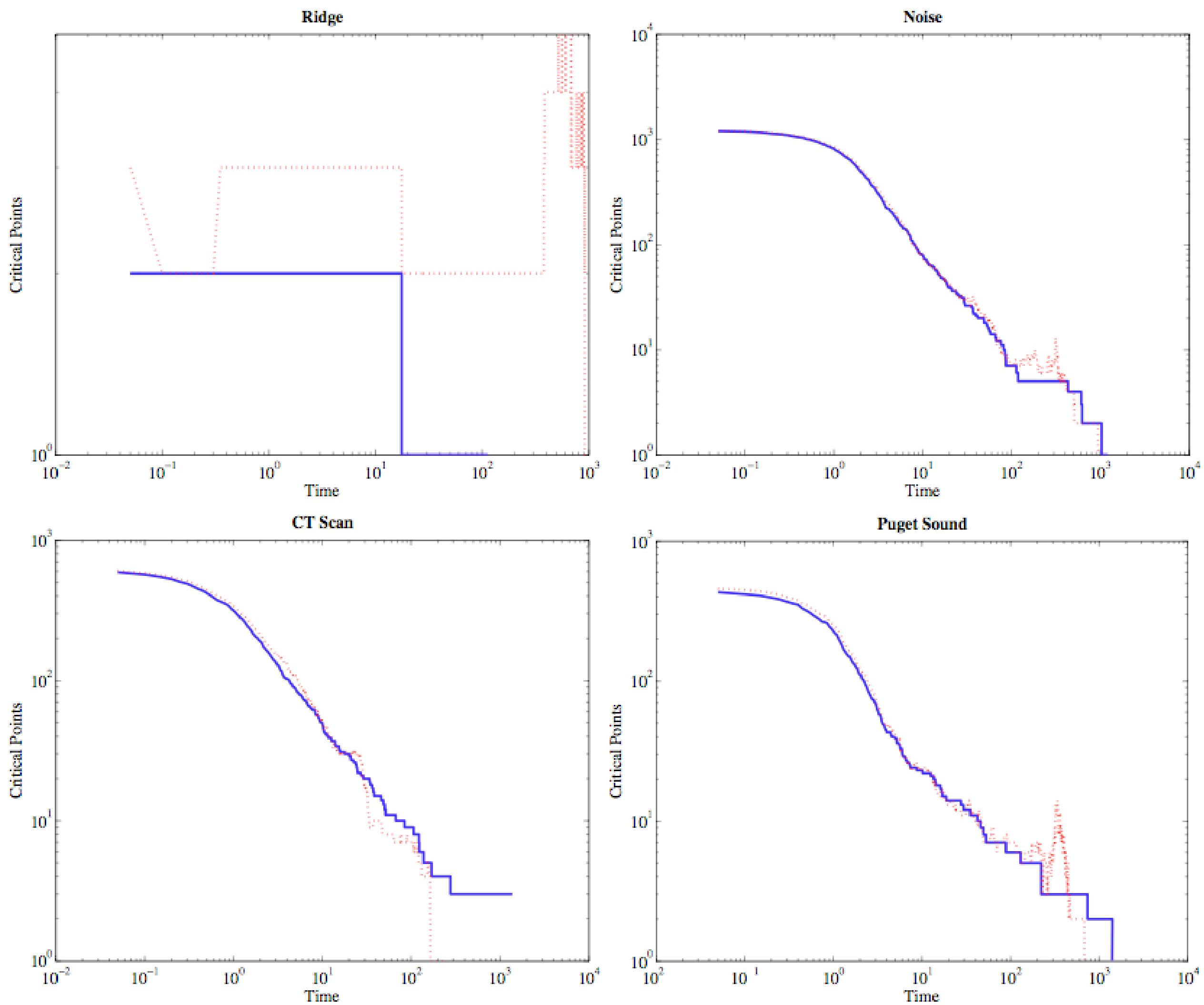
Critical Points Over Time



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