

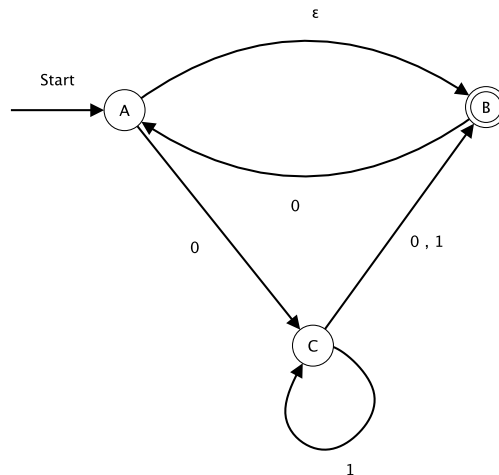
Homework 1

Students are welcome to work together, but *every student must write up their own solutions, independently!* I strongly encourage students to use LaTeX for writing up their solutions. Please see the course web page for a template file. When problems require you to draw a state machine, feel free to include a hand-drawn picture with your typed-up solutions.

Question 1: Give the state diagram of the DFA that recognizes the language $L = \{w \mid w \text{ contains at least two 1s and at most one 0}\}$. For the automaton, you can use either a graph editor, e.g., yEd, or simply a picture/scan of a handwritten automaton.

Question 2: Give the state diagram of the NFA that recognizes the language $L = L_1 \circ L_2$, where $L_1 = (001)^*$ and $L_2 = (0 \cup 010)^*$.

Question 3: Convert the following NFA to an equivalent deterministic finite automaton:



Provide both the state diagram as well as the definition of each component of the proposed $M = (\Sigma, Q, S, \mathcal{A}, \delta)$.

Question 4: Prove that the language $L = \{a^n b^m a^n \mid m, n \geq 0\}$ is not regular. *Hint:* Use the pumping lemma and read carefully the case analysis from the notes. Remember that your proof should show that regardless of which pumping length p one chooses, you can come up with a string w such that all possible partitions of w to xyz contradict one of the properties of the pumping lemma.

Question 5: Let w be a string over alphabet $\{a, b\}$ and let $w^{\mathcal{R}}$ be string w written backwards, e.g., $w = aab$ and $w^{\mathcal{R}} = baa$. Construct a state diagram from the PDA (push-down automaton) that recognizes language $L = \{ww^{\mathcal{R}} \mid w \in \{a, b\}^*\}$.