

Homework 4

Students are welcome to work together, but *every student must write up their own solutions, independently!* I strongly encourage students to use LaTeX for writing up their solutions. Please see the course web-page for a template file.

Question 1 (25 points). Consider the language

$$\text{HAMPATH} = \left\{ G = (V, E), s, t \mid \begin{array}{l} G \text{ is an undirected graph, } s \text{ and } t \text{ are vertices of } G, \text{ and there exists a path} \\ \text{from } s \text{ to } t \text{ that visits every vertex of } G \text{ exactly once} \end{array} \right\}.$$

Show that this language is self-reducible.

Question 2 (25 points). Consider the language

$$\text{3-COLOR} = \left\{ G \mid \begin{array}{l} \text{The vertices of graph } G \text{ can each be labeled Red, Green or Blue} \\ \text{such that no adjacent vertices have the same label} \end{array} \right\}.$$

Show that this language is self-reducible.

Question 3 (25 points). Prove that language $L' = \{a^i b^j c^k d^l \mid i, j, k, l \geq 0, i + k = j + l\}$ is a member of class L.

Question 4 (25 points). Consider the language

$$\text{ODD-CYCLES} = \{G \mid G \text{ is an undirected graph that has an odd cycle} \}.$$

Give an algorithm that shows that this language is in NL.