College Bound Math Problems #13
week of January 26, 2015

Each of these problems asks you to divide one positive integer into another, in order to see the interesting patterns of results.

In order to get a feel for how those patterns emerge, you will need to lay aside your calculator for a moment, pick up your pencil, and carry out long division (google <u>long division algorithm</u> if you need to).

1. (a) Use <u>long division</u> to divide 3 into 1 to get 0.3333... where the threes keep repeating forever, so this is an example of a <u>repeating decimal</u>. This result is also written $0.\overline{3}$ with a bar over the number that keeps repeating. There is a similar example at http://www.virtualnerd.com/pre-algebra/rational-numbers/fraction-to-repeating-decimal-conversion.php.

   (b) How does the long division procedures show you that the three in (a) will repeat forever?

   (c) The result in (a) is for $\frac{1}{3}$. Now find the repeating decimal for $\frac{2}{3}$.

2. (a) Use long division to divide 11 into 3 to show that 3/11 is 0.27272727... = $0.\overline{27}$ Here the number under that bar is 27, which has two digits, so this repeating decimal is said to have a period of two.

   (b) Divide 11 into 4, to get the repeating decimal $0.\overline{36} = 4/11$

   (c) Notice that 27 and 36, the repeating numbers in (a) and (b), are each multiples of 9. Do you expect that this pattern will also hold for 1/11 and other multiples of 1/11 like 2/11, 5/11, 6/11, etc.?

3. This is the one with the most remarkable patterns. You may use a calculator here.

   (a) Find 1/7 to 12 decimal places. Notice that the seventh through twelfth digits are the same ones in the same order as the first through sixth. This suggests that 1/7 equals a repeating decimal of period 6, and that is true.

   (b) Find 2/7 to 6 decimal places. Check to see that these numbers show up in the same order somewhere in the answer to part (a).

   (c) Same as part (b), but for at least one of: 3/7, 4/7, 5/7, or 6/7.