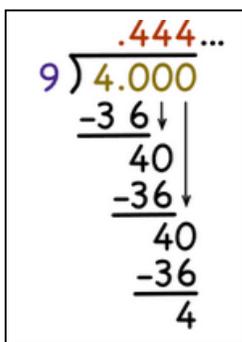


College Bound Math Solutions #13
week of January 26, 2015

Long division (using paper and pencil) used to be taught in 4th or 5th grade. Calculators have made it seem less important, but in fact it's a great introduction to other parts of math and computer science. There is plenty of help online, just googling long division algorithm.

Please read the intro on the Problems page, where it's claimed that the patterns that one sees emerging from long division are "interesting." I hope that you and your student will end up agreeing with me about that. Stanley Ogilvy found the relationships among $1/7$, $2/7$, $3/7$,... so fascinating that he began his book *Excursions in Mathematics* with them.



The image shows a handwritten long division problem: 9 divided into 4.000. The quotient is .444... The steps are: 9 goes into 40 (from 4.0) 4 times, leaving a remainder of 4. Bring down the next 0 to get 40. 9 goes into 40 4 times, leaving a remainder of 4. Bring down the next 0 to get 40. 9 goes into 40 4 times, leaving a remainder of 4. The pattern repeats.

1. (a) The example at the left shows how long division works on $\frac{4}{9}$. It is taken from the web page mentioned in the problem. Follow the same steps to divide 3 into 1.
 - (b) In the example, 9 is repeatedly divided into 40, to get 4 with a remainder of 4 which produces 40 again when the next zero is brought down for the next decimal place, thereby setting the stage for yet another 4, and so on.
 - (c) $\frac{2}{3} = 0.6666... = 0.\overline{6}$

2. The answers to (a) and (b) are given in the statement of the problem.
 - (c) Yes, the pattern does hold up. That pattern is (i) a repeating decimal (ii) with a period of 2 (iii) where the two digits form a multiple of 9. In fact, (iv) the repeating number always equals 9 times the numerator of the fraction!

3. In these answers, the zero to the left of the decimal point is deliberately left out.
 - (a) In case neither your student's calculator nor yours provides 12 decimal places, here is the answer: .142857142857
 - (b) .285714 The six digits appear in the third through eighth positions of (a).
 - (c) The repeating decimal of $\frac{3}{7}$, for example, appears starting at the second digit of (a). That is, $\frac{3}{7} = \overline{.428571}$

Note to math majors: You'll probably notice that for $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$, the decimal to six places rounds up and you get the approximation $\frac{\%}{7} \approx .857143$, whereas $\frac{\%}{7} = \overline{.857142}$ is an (exact) equality "in the limit," since as more and more decimal places are computed, the value gets as close as you like to that of the fraction.