College Bound Math Problems #14
week of February 2, 2015

The table below is used in all the problems. The first row is filled like this:

In the first column put **1**.
Double it (to get 2) and add 1 (to get **3**). Put the result in column **a**.
Square that (to get 9), subtract 1 (to get 8) and divide by 2 (to get **4**).
Put that in column **b**.
Finally, add 1 (to get **5**) and put the result in **c**.

The next row starts with 2. The same operations then give 5, 12 and 13.

#	a	b	c
1	3	4	5
2	5	12	13

1. Check to see that the row starting with 2 has been done correctly, like this:

   Double it, add 1 and put the result in column **a** of the same row.
   Square that result, subtract 1 and divide by 2. Put this result in column **b**.
   Add 1. Put the result in column **c**.

   Now pick *any* other positive integer, put it at the start of the first empty row and do the same operations. (That is, double it and add 1 to get **a**, etc.)

2. In each row, let a be the number in column **a**. Similarly b is the number in column **b** and c is the number in column **c**. For each row show that $a^2 + b^2 = c^2$. For example, in the row that begins with 1, the values are $a = 3$, $b = 4$ and $c = 5$, so each side is 25. $a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$ and $c^2 = 25$. Show that this works for the row that begins with 2 and also for the row that you filled in starting with a different number in Problem #1.

3. Using a different row in the table choose another number as the starting number and show that you get results for which $a^2 + b^2 = c^2$. Do this again with another number. In each case you will find that $a^2 + b^2 = c^2$. If you are doing this with large numbers, you may wish to use a calculator.

   Please read #4, even if you don't do it.

4. [**Optional:**] What's going on? Why do those steps (doubling, adding, squaring, ...) always turn out to make $a^2 + b^2 = c^2$?

   Hint: $b = \dfrac{a^2 - 1}{2}$ **Offer:** If you solve it, come to MathLab for a prize.