**Solutions & Comments for Mentors, #3**
week of October 20, 2014

**Mentors:** Each problem builds on the one before it, so everyone should start with #1, which provides practice with decimals, squares and square roots. #2 is about approximation, the basis of gut checks for answers that make sense; it also hints at interpolation. Finally, #3 is an opportunity to plot something that's familiar (a circle) but not the usual line or parabola.

1. *Square* and *square root* undo each other.

   (a)  $1.7 \times 1.7 = 2.89$            (c)  $1.8 \times 1.8 = 3.24$

   (b)  $\sqrt{2.89} = 1.7$ (by undoing)        (d)  $\sqrt{3.24} = 1.8$ (by undoing)

2. Finding $\sqrt{3}$ by a series of guesses.

   The square of 1.5 is 2.25 (which is less than 3).
   The square of 1.7 is 2.89 (closer to 3 though still less than it).
   The square of 1.8 is 3.24 (more than 3).

   1.7 has a square (2.89) that differs from 3.00 by 0.11, which is closer than 1.8, which has a square (3.24) that differs from 3.00 by 0.24.

   Since the ratio of the two errors is about 1 to 2 (0.11 to 0.24), it is reasonable to expect the answer to be about twice as far from 1.8 as from 1.7, which would be approximately 1.73. That's the best we can do with two decimal places.

   Students with calculators always seem eager to use them. Here they should square 1.73 and 1.74 to see which has a square closer to 3.

3. Using Pythagoras' Theorem to find the ends of radii.

   The theorem tells us that the legs, $x$ and $y$, satisfy the equation $x^2 + y^2 = 2^2$ (= 4), since the hypotenuse is 2.

   In one solution $x = 1$ and $y = \sqrt{3}$, which gets plotted as the point $(1, \sqrt{3})$. It is a solution because $1^2 + (\sqrt{3})^2 = 1 + 3 = 4 = 2^2$.

   Similarly, $(2,0)$ is a solution because $2^2 + 0^2 = 4 + 0 = 4 = 2^2$.

   