Solutions & Comments for Mentors, #5
for the week of November 3, 2014

Note to mentors: Each problem here follows up on the one numbered the same in Set #4, so you may find some things in the solutions to #4 that are relevant here. These problems go a bit further. A student who hasn't done any of Set #4 yet should do some of them first.

1. (a) \[1 \frac{4}{5} \times 4 \frac{1}{2} = (1 + \frac{4}{5})(4 + \frac{1}{2})\] by the definition of mixed numbers.

   (b) "No decimals until part (c),” so use F.O.I.L. as in Set #4 to get:
   \[(1 + \frac{4}{5})(4 + \frac{1}{2}) = (1 \cdot 4) + (1 \cdot \frac{1}{2}) + (\frac{4}{5} \cdot 4) + (\frac{4}{5} \cdot \frac{1}{2}) = \frac{5 + 32 + 4}{10} = 4 + \frac{31}{10} = 4 + 3 \frac{1}{10} = 8 \frac{1}{10}\]

   (c) Start as at right, but by ignoring decimal points we have a result that's 100 times too big and we need to move the decimal point 2 places to the left from its implicit position at the right end of "810." to get the correct answer, 8.10 or 8.1, which is consistent with the answer in (b).

2. \[3^0 = 1, \quad 3^1 = 3, \quad 3^2 = 9, \quad 3^3 = 27, \quad 3^4 = 81.\]

   (a) \[3^2 = 9 = 3^2, \quad \text{so } x \text{ must be } 2.\]

   (b) \[3^{x-1} = 27 = 3^3, \quad \text{so } x - 1 \text{ must be } 3, \quad \text{which requires} \quad x \text{ to be } 4.\]

   (c) \[3^{2x} = 81 = 3^4, \quad \text{so } 2x = 4 \quad \text{and } x \text{ is } 2.\]

3. Since 3 circles span the 3 inches each circle must have a width of \(\frac{\sqrt{3}}{2} = 1,\) measured along its diameter. So that diameter must be 1 and the height of the rectangle must be 2, as given. It also follows that each radius is \(\frac{1}{2} \).

   (a) Since each radius is \(\frac{1}{2} \), the area of (any) one circle is \(\pi r^2 = \pi \left(\frac{1}{2}\right)^2 = \frac{1}{4} \pi .\)

   (b) The total area of 6 circles is \(6 \times \frac{\pi}{4} = \frac{6}{4} \pi = \frac{3}{2} \pi\)

   (c) The gray portion is a 3 by 2 rectangle with the 6 circles removed. That's \(6 - \frac{3}{2} \pi\).