Homework #3

- (10pt) Prove that \( \sum_{i=1}^{L} ix^i = \frac{x + (Lx - L - 1)x^{L+1}}{(1-x)^2} \)

- (10pt) Find the \( E[X^2] \) of the exponential distribution with parameter \( \lambda \).

- (5pt) Using the result of the previous question to find the average queuing time \( W \) of a M/G/1 with exponentially distributed service times. (Hint: your result should be identical to the \( W \) of M/M/1.)

- (10pt) Verify that the solution of \( \beta \) in G/M/1 systems is \( \sigma = \frac{\lambda}{\mu} \) when G is an exponential distribution with rate \( \mu \).

TCP uses the equation \( RTT = RTT^* w + New^*(1-w) \) to compute the moving exponential average of round trip times, where New is a new sample of round trip time and \( w \) is a constant between 0 and 1. Suppose initially \( RTT = R \) and all the subsequent samples are 2R (probably due to newly developed heavy traffic).

- (8pt) Show that \( RTT_k \), the value of RTT after receiving \( k \) samples, is \( 2R - Rw^k \).

- (7pt) Show that it takes \( k = -1/\ln(w) \) packets for \( RTT_k \) to be \( \geq (1-1/e)R + R \), that is, the gap between the real round trip time and the average round trip time to be less than around one third of \( R \).
(10pt) Create an algorithm that generates a random variable having density function

\[ f(x) = 20x(1-x)^3, \quad 0 < x < 1 \]

Hint: Use the rejection method. An easy auxiliary random variable (the \(Y\)) is the uniform random variable from 0 to 1. Write \(g(x)\), pdf of \(Y\), and find an upper bound of \(f(x)/g(x)\). You must show the steps to derive the upper bound \(c\).