TCP Performance

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TCP Responsibilities in Congestion Control

- A TCP source sees two types of loss indications:
  - Triple duplicate ACKs (TD)
  - Time-outs (TO)
- A TD event cuts \( cwnd \) by half.
- A TO event sets \( cwnd \) to 1.
- With smaller window sizes, the source must “stop and wait” frequently and thus reduce traffic rate.
Assumptions/Simplifications

- The time needed to send all packets in a window is smaller than the round trip time.
- When a packet loss to the $k$-th packet in a round, the rest of the packets in that round are lost too.
  - This is largely due to the FIFO queueing of routers.
- These behaviors are generally, but not always, observed in the real world.

Analysis of TD-Only Scenarios

- TD Period (TDP): a period between two triple-duplicates indications.
- $A_i$: the duration of the $i$-th TD period, $\text{TDP}_i$
- $Y_i$: the no. of packets sent in $\text{TDP}_i$
- $W_i$: the window size (cwnd) at the end of $\text{TDP}_i$
- $b$: the no. of packets ack-ed per ACK.
  - In many TCP implementations, $b=2$.
- Our goal: throughput $B = E[Y] / E[A]$
Evolution of Window Size

Packets Sent during a TD Period
A TD period starts immediately after a TD loss indication and thus $cwnd$ is $W_{i-1}/2$.

At each round, the window is incremented by $1/b$ and the no. of packets sent per round is incremented by 1 every $b$ rounds.

Let $\alpha_i$ be the first packet lost in TDP$_i$ and $X_i$ the round where this loss occurs.

After packet $\alpha_i$, $W_i - 1$ more packets are sent.

We have $Y_i = \alpha_i + W_i - 1$


Let $p$ be the probability of packet loss.

$$P[\alpha = k] = (1 - p)^k \cdot p \Rightarrow E[\alpha] = \frac{1}{p}$$

It follows that

$$E[Y] = \frac{1 - p}{p} + E[W]$$

Next, we must figure out $E[W]$ and $E[A]$. 
\( r_{i,j} \): the duration (round trip time) of the \( j \)-th round in TDP\( _i \).

The duration of TDP\( _i \) is \( A_i = \sum_{j=1}^{X_i+1} r_{i,j} \)

We consider \( r_{i,j} \) be a random variable independent of the size of congestion window, and thus independent of \( i \) and \( j \).

It follows that \( E[A] = (E[X] + 1)RTT \) where \( RTT = E[r] \) is the average round trip time.

During TDP\( _i \), the window size increases between \( W_{i-1}/2 \) and \( W_i \) linearly with slope \( 1/b \), that is,

\[
W_i = \frac{W_{i-1}}{2} + \frac{X_i}{b} \quad \Rightarrow \quad E[W] = \frac{2}{b} E[X]
\]

The fact that \( Y_i \) packets are transmitted in TDP\( _i \) is expressed by

\[
Y_i = \sum_{k=0}^{X_i/b - 1} \left( \frac{W_{i-1}}{2} + k \right) b + \beta_i \\
= \frac{X_i W_{i-1}}{2} + \frac{X_i}{2} \left( \frac{X_i}{b} - 1 \right) + \beta_i \\
= \frac{X_i}{2} \left( \frac{W_{i-1}}{2} + W_i - 1 \right) + \beta_i
\]
**Average Window Size**

- Thus \( \frac{1-p}{p} + E[W] = \frac{E[X]}{2} \left( \frac{E[W]}{2} + E[W] - 1 \right) + E[\beta] \)
- Assuming \( \beta_i \) be uniformly distributed 1 and \( W_i \), and thus \( E[\beta] = E[W]/2 \), we have
  \[
  E[W] = \frac{2 + b}{3b} + \sqrt{\frac{8(1-p)}{3bp}} + \left( \frac{2 + b}{3b} \right)^2
  \]
- For small values of \( p \),
  \[
  E[W] \approx \sqrt{\frac{8}{3bp}}
  \]

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**Average TCP Throughput**

- It follows that
  \[
  E[X] = \frac{2 + b}{6} + \sqrt{\frac{2b(1-p)}{3p} + \left( \frac{2 + b}{6} \right)^2}
  \]
- \( B(p) = \frac{1-p}{p} + E[W] \)
  \[
  B(p) = \frac{1-p}{p} + \frac{2 + b}{3b} + \sqrt{\frac{8(1-p)}{3bp} + \left( \frac{2 + b}{3b} \right)^2}
  \]
- RTT \[
  B(p) \approx \frac{1}{RTT} \sqrt{\frac{3}{2bp}}
  \]
Discussion

- TCP favors flows with short RTT.
  - Find a server near you for downloading
- The relationship between packet loss rate $p$ and throughput is not linear.
  - When $p$ is increased 4 times, throughput drops to half.
- Large $b$ values is bad for throughput.
  - Hence $b=2$ in common implementations
  - Why not $b=1$?

Taking Timeouts into Account

- Timeout (TO): the timer of a missing ACK fires before three duplicate ACKs are received.
- The initial TO period is denoted as $T_0$.
- After a TO, $cwnd$ is reduced to 1, allowing for the retransmission only for the lost packet.
- If the retransmission fails (another TO), the TO period is set to $2T_0$.
- If the retransmission fails a second time, the TO period is set to $4T_0$.
- The maximum TO period is 64 $T_0$. 
Evolution of Window Size

Notations

- $Z_{i}^{TO}$: the duration of a sequence of TO.
- $Z_{i}^{TD}$: the time interval between two consecutive TO sequences.
- Define $S_{i} = Z_{i}^{TD} + Z_{i}^{TO}$
- $M_{i}$: the no. of packets sent during $S_{i}$.
- $n_{i}$: the no. of TD periods in interval $Z_{i}^{TD}$
- $Y_{ij}$: the no. of packets sent in the $j$-th TDP in $Z_{i}^{TD}$
- $A_{ij}$: the duration of TDP$_{ij}$
- $X_{ij}$: the no. of rounds in TDP$_{ij}$
- $W_{ij}$: the window size at the end of TDP$_{ij}$
- $R_{i}$: the no. of packets sent in $Z_{i}^{TO}$
Notice that the “packet counts” \((Y_{ij}, M_i, R_i)\) includes retransmissions and our results are for **throughput**, not **goodput**. We have

\[
M_i = \sum_{j=1}^{n_i} Y_{ij} + R_i \implies E[M_i] = E\left[ \sum_{j=1}^{n_i} Y_{ij} \right] + E[R]
\]

\[
S_i = \sum_{j=1}^{n_i} A_{ij} + Z_i^{TO} \implies E[S_i] = E\left[ \sum_{j=1}^{n_i} A_{ij} \right] + E[Z_i^{TO}]
\]

Assuming that \(n_i\) is an independent sequence of random variable and independent of \(Y_{ij}\) and \(A_{ij}\):

\[
E\left[ \sum_{j=1}^{n_i} Y_{ij} \right] = E[n] \times E[Y] \quad \text{and} \quad E\left[ \sum_{j=1}^{n_i} A_{ij} \right] = E[n] \times E[A]
\]

Let \(Q\) be the probability that a TDP ends with a TO. We have \(Q = 1/E[n]\).

The throughput can be expressed as

\[
B = \frac{E[Y] + Q \times E[R]}{E[A] + Q \times E[Z^{TO}]}
\]

Since \(Y_{ij}\) and \(A_{ij}\) do not depend on TO, we can use previous results of \(E[Y]\) and \(E[A]\).

We still need to figure out \(Q, E[R]\), and \(E[Z^{TO}]\).
In the penultimate round, packet $k+1$ is lost.

The probability that the first $k$ packets are ACKed in a round of $w$ packets, given there is one or more losses in the round is

$$A(w,k) = \frac{(1 - p)^k p}{1 - (1 - p)^w}$$

The probability that $n$ packets are sent and $m$ of them are acknowledged in the last round is

$$C(n,m) = \begin{cases} (1 - p)^m p, & m < n \\ (1 - p)^n, & m = n \end{cases}$$

In the last round, the probability a loss in a window of size $w$ causes TO is given by

$$Q'(w) = \begin{cases} \sum_{k=0}^2 A(w,k) + \sum_{k=3}^w A(w,k) \sum_{m=0}^2 C(k,m) & w \leq 3 \\ w > 3 \end{cases}$$

< 3 packets successfully sent in the penultimate round, and thus < 3 sent in the last round. No way to produce 3 duplicate ACKs

≥ 3 packets successfully sent in the penultimate round, allowing ≥ 3 sent in the last round

But < 3 last round packets get thru to cause duplicate ACKs. No way to produce 3 duplicate ACKs
Solving $Q$

- After some fun with algebra, we have

$$Q'(w) = \min\left(1, \frac{(1-(1-p)^3)(1+(1-p)^3(1-(1-p)^{w-3}))}{(1-(1-p)^w}\right)$$

- $Q$, the probability a TO occurs at the end of a TDP is $Q(E[W])$, where $E[W]$ has been solved previously.

Solving $E[R]$

- Next, we find $E[R]$, the average no. of packets sent during an $Z^{TO}$.
- In a $Z^{TO}$ there are $k$-1 consecutive losses followed by a successful transmission, that is,

$$P[R = k] = p^k(1-p)$$

- Thus, $E[R] = \frac{1}{1-p}$
Solving $E[Z^{TO}]$

- The first six TO have length $2^{i-1}T_0$, \( i = 1 \ldots 6 \)
- Following TO have length $64T_0$
- The duration of a sequence of $k$ TO is
  \[
  L_k = \begin{cases} 
    (2^k - 1)T_0 & k \leq 6 \\
    (63 + 64(k - 6))T_0 & k > 6 
  \end{cases}
  \]
- Thus,
  \[
  E[Z^{TO}] = \sum_{k=1}^{\infty} L_k P[R = k] 
  = T_0 \frac{1 + p + 2p^2 + 4p^3 + 8p^4 + 16p^5 + 32p^6}{1 - p}
  \]

Solving $B(p)$

\[
B(p) = \frac{E[Y] + Q \times E[R]}{E[A] + Q \times E[Z^{TO}]}
\]

\[
= \frac{1 - p}{p} + \frac{E[W] + Q'(E[W])}{1 - p}
\]

\[
\approx \frac{1}{RTT (E[X] + 1) + Q'(E[W])E[Z^{TO}]}
\]

\[
\approx \frac{1}{RTT \sqrt{\frac{2bp}{3}} + T_0 \min\left(1,3\sqrt{\frac{3bp}{8}}\right)p(1 + 32p^2)}
\]