Style Learning: Inductive Generalisation of Architectural Shape Patterns

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Abstract. Art historians and critics have defined the style as common features appeared in a class of objects. Abstract common features from a set of objects have been used as a bench mark for date and location of original works. Common features in shapes are identified by relationships as well as physical properties from shape descriptions. This paper will focus on how the computer recognises common shape properties from a class of shape objects to learn style. Shape representation using schema theory has been explored and possible inductive generalisation from shape descriptions has been investigated.

Keywords : Style, Inductive Generalisation, Knowledge Representation, Shape

1 Introduction

In this paper, design is viewed as an application of previous design knowledge to new design. According to Peirce (1923), humans have three modes of inference: deduction, induction and abduction. Within this classification of inference, the design process is hypothesised as the abduction that produces a particular instance from a general notion and specific data. Specific data are in the form of a preliminary statement of required characteristics, and general notions are working rules of some generalities induced from a set of particular events or objects (March, 1984). General notions are stored in memory and invoked to be used for understanding events or things (Marshall, 1995) and evaluating new design (March, 1984), as well as to be applied to production processes.

New designs are produced or created by instantiation, extending the space of known or possible solution, or creating a new state space from existing vocabulary elements and general concepts. Elements and concepts may be provided at the beginning of a design process and will be sought from memory to improve the design result. Those general notions, elements and concepts are design knowledge that is formulated in the designer's mind as a mental image by the reasoning and imaging process, and applied for producing new design. They affect the design processes on which design results depend. Design knowledge can be deduced from a set of design objects.

To support computer-based design procedures as well as human design activity, design knowledge should be learned from previous design objects. Design knowledge is acquired and stored in human (or computer) memory using several different learning methods, such as rote learning, learning from instruction, learning from teacher-provided examples and learning from observation and discovery (Michalski, 1983). The process of design knowledge learning in this research is considered as a form of "learning from teacher-provided examples", in that a set of shape objects is given with background knowledge[1], and then an inductive hypothesis is generated. It can be detected when something constantly appears peculiar, or is giving functional efficiency and visual goodness. One important design knowledge that characterises an artefact is the style that is embodied by common features appearing in a class of objects designed by an individual, a group or a period (Gombrich, 1960; Shapiro, 1961; Chan, 1992). Here, when we refer to style we mean shapes and shape patterns related style.

One objective here is to provide the computer with the ability to learn a concept from a class of objects, particularly style from shape objects, so called inductive style learning. To support inductive style learning in architectural design, many procedures need to be elaborated. In the style learning process, a class of drawings is provided as input data (examples), and then generalisation processes identify a style (inductive assertions). Input shape data are encoded with mathematics or symbols (Gero and Yan, 1994; Cha and Gero, 1999). The computer reads the shape data and recognises spatial relationships and patterns in terms of the law of visual organisation (Kohler, 1930; Wertheimer, 1945; Arnheim, 1954) and the recognition factors of typicality, similarity, frequency, dominance and multiplicity. Then conceptual shape descriptions are constructed in a hierarchical tree structure using predefined shape knowledge. Shape pattern schemas are generalised from a set of multiple representations for a single object or a set of representations for a class of shape objects using inductive generalisation. Generalised shape pattern schemas are the learned style that explains common properties for a class of objects. In this paper, generalisation theory and its possible applications for style learning using shape pattern representation are elaborated, and the style of Mario Botta will be explored.

2 Shape Representation

Shape representations and their manipulation have been studied in many areas of design computation, such as, shape grammars, morphology in spaces and logic representation (Stiny, 1975; Steadman, 1983; Coyne, 1988; Mitchell, 1990). This section primarily concerns with representations of implicit shape knowledge such as relationships of relationships, shape patterns and shapes complexity rather than physical shapes. Shape pattern representation was introduced and developed in Cha and Gero (1999) for design computation. Here we explain only important ideas of those shape representations for inductive generalisation. As a method schema theory has been explored to represent and explain shape characteristics.

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[1] According to Michalski (1983), "background knowledge is the assumptions and constraints imposed on the observational statements and general candidate inductive assertion, and any relevant problem domain knowledge".

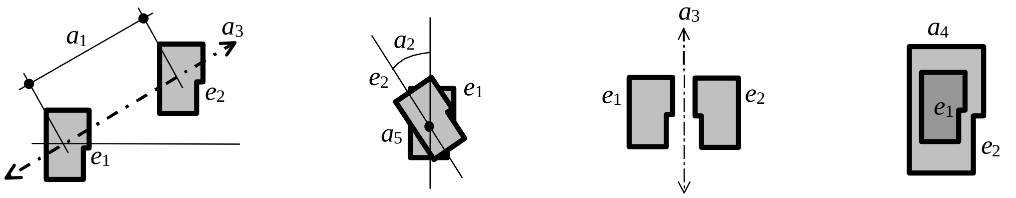
2.1. Spatial relationship and representation

A shape is composed of subshapes, and shapes may have relationships between each other. Shapes are recognised explicitly and implicitly. A shape that is initially represented explicitly is a *primary shape* and a shape that exists only implicitly in a primary shape is an *emergent shape* (Gero and Yan, 1994; Gero and Jun, 1998). Among those shapes, *bounded polyline shapes*[2] are considered as shapes in this paper. In a group of shapes, each shape can be described with respect to another shape using spatial relationships, especially isometric transformation. The initial or primary shape can be represented as a referent, and a relationship is represented as a predicate in a propositional shape description with arguments.

$$S = R\{E, A\} \qquad (1)$$

Where A: arguments for relationship
E: referent shape
R: relationship between shapes
S: shape

Isometric transformations are closed transformations that transform one shape into another shape without losing any properties. Isometric transformation relationships are the most fundamental spatial relationships upon which all shape representations, such as topology, shape semantics and patterns, can be founded. These are relationships between congruent shapes. There are four kinds of isometric transformations: translation, reflection, rotation and scaling. Examples and representations are given in Figure 1. Even though these relationships are well known, they are presented here for consistency and completeness.



(a)
$e_2 = t_1\{e_1, (a_1, a_3)\}$
t_1 = translation
a_1 = distance

(b)
$e_2 = t_2\{e_1, (a_2, a_5)\}$
t_2 = rotation
a_2 = angle
a_5 = rotation centre

(c)
$e_2 = t_3\{e_1, a_3\}$
t_3 = reflection
a_3 = axis

(d)
$e_2 = t_4\{e_1, a_4\}$
t_4 = scale
a_4 = scale

Figure 1 Basic isometric transformation relationships and their representations: (a) translation, (b) rotation, (c) reflection and (d) scale.

2.2 Group shape representation

A group of shapes or patterns that are congruent or similar may be arranged in a pattern, and a shape or pattern can be explained with respect to another shape or pattern

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[2] A bounded polyline shape is an enclosed polyline shape, for any point on the boundary of which there exists at least one circuit composed of line segments which start from and end at the point without covering any line segment more than once.

recursively, then this group shape can be described using an isometric transformation relationship representation and a nesting operator ($\prod_{i=1}^{n}$). The nesting operator applies a transformation factor to elements recursively until all shapes or patterns in a pattern are described by another shape or pattern. It provides a description of how the given pattern is constructed from the primitives. The general representation of a relationship between two shapes or patterns is $e_i = \{e_{i-1}, a_k\}$, and a shape pattern is an arrangement of a congruent shape group or pattern group $\{e_1, e_2, \ldots, e_n\}$. Thus the pattern in this shape group can be described with a nesting operator as:

$$S = \prod_{i=1}^{n} t_k\{e_i, a_k\} \tag{2}$$

The nesting operator denotes n recursive applications of isometric transformation t_k to shape elements e_i with transformation arguments a_k.

2.3 Similarity in Shape

Similarities in shapes are identified by attributes, physical structure (Gero and Jun, 1995), continuous transformation (March and Steadman, 1971; Steadman, 1983; Mitchell, 1990), or organising structure (Falkenhainer *et al*, 1989/90). Shapes are recognised and categorised in terms of their attributes, such as, colour, line type, thickness, and so on. Shapes that have the same physical structure in terms of topology and geometry are regarded as similar shapes: congruent shapes. They are transformed in various ways, for example, stretch, shear, perspective, rubber sheet. Group shapes that are composed of different subshapes but have the same compositional relationships are called analog shapes.

Structural similarity between shapes can be decided from these shape descriptions. Properties of shape elements are disregarded in structural matching, they are generalised as element variables. Predicates and arguments are considered to determine similarity. Consider the two shapes, S_a and S_b in Figure 2 to determine their similarity. In terms of surface similarity, there are no common properties, shape S_a is composed of curves or ovals, shape S_b is composed of straight lines, triangles or squares. But if we decompose them and describe them using sub-shapes and their relationships, then their similarity can be identified. The shape S_a is composed of four ovals that have a 90^o rotation relationship, and can be described as $S_a = \prod_{i=1}^{4} t_2\{\text{Oval}_i, (90^o, a_5)\}$. The shape S_b is composed of four triangles that have a 90^o rotation relationship and can be described as $S_b = \prod_{i=1}^{4} t_2\{\text{Triangle}_i, (90^o, a_5)\}$. Comparing these two shape descriptions, we can identify that they share the same predicate (t_2) and arguments (90^o, a_5), thus they can be regarded as similar shape patterns in terms of the relationship, even though the physical properties in the element shapes are different.

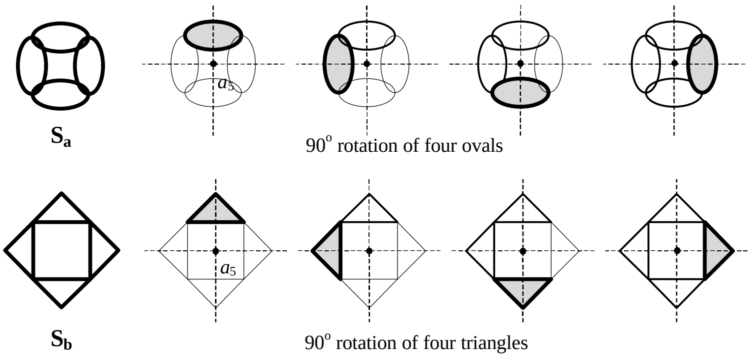


Figure 2. Similarity between shape S_a and S_b by 90^o rotation relationship.

2.4 Shape pattern schema representation

A shape pattern schema is an organised body of knowledge about spatial relationships between shapes that describes the patterns, syntactic structure, and the characteristics of shape patterns. In perceptual categories of shapes, objects are perceived not only by their properties, such as, colour, material, but also by their organising structures specified by spatial relationships. Perceptual categories exist by virtue of similar structural descriptions. According to Rumelhart (1980), a schema is "a higher order relational structure for representing the generic concepts upon which all information processings depend". It is a network of inter-relationships that represents essential characteristics of things or concepts rather than a list of features. The network may be in a hierarchical tree structure with nodes and paths. It is generalised from multiple repetitions (Piaget, 1952) and describes a prototype (or a generic concept) for a group of things or situations (Minsky, 1975).

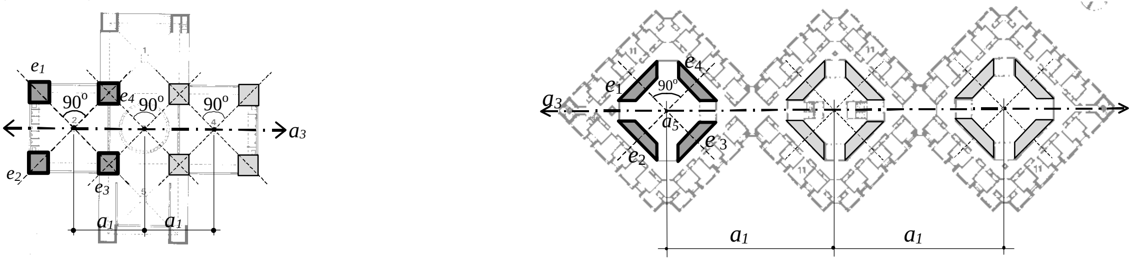
The shape pattern schema is represented with a set of sub-elements and their relationships. It is generalised from multiple shape pattern representations or a class of shape pattern descriptions. It is in a form of hierarchical tree structure and its low-level nodes in a hierarchical tree structure can be turned into variables and embedded in each other.

2.4.1 Variability in shape pattern schema

A shape pattern schema is not a representation of a specific shape pattern, rather it represents spatial characteristics for a set of shape patterns. Each shape pattern is a specific instantiation of the shape pattern schema. From the notion of schema as a network of elements and relationships, lower level schemas as well as lower level elements and relationships can be regarded as variables that can be instantiated. In shape pattern schemas, shape elements and lower level relationship elements or schemas are considered as variables.

Suppose we have two shape patterns that are described as N_1 and N_2, Figure 3. These two shape patterns can be generalised into a shape pattern schema S_1 that is a translation of three 90^o rotation relationships. High-level relationships, such as, the translation and

90^o rotation are fixed, but low level elements and arguments can be turned into variables (x_{ei}, x_{a1}).



$N_1 = \prod_{j=1}^{3} t_1 \{ \prod_{i=1}^{4} t_2 [e_{i,j}, (90^o, a_5)], (a_1, a_3)\}$ $N_2 = \prod_{j=1}^{3} t_1 \{ \prod_{i=1}^{4} t_2 [e_{i,j}, (90^o, a_5)], (a_1, a_3)\}$

$$S_1 = \prod_{j=1}^{3} t_1 \{ \prod_{i=1}^{4} t_2 [x_{ei,j}, (a_2, a_5)], (x_{a1}, a_3)\}$$

Figure 3. Shape patterns and shape pattern descriptions.

2. 4. 2 Embeddedness in shape pattern schema

One schema may be embedded in one or many others in various ways. A schema may be a part of other schema, many schemas may share a subschema, or a schema can be embedded within itself.

Part: A subschema or a set of subschemas is embedded in a shape pattern schema if that subschema is a part of this shape pattern or schema.

Sharing: Many shape pattern schemas may share a single shape pattern schema or a set of shape pattern schemas. If two complex shapes overlap each other, the conjunction part of two shapes is represented with a shape pattern schema or a set of shape pattern schemas. These shared shape pattern schemas are embedded in shape pattern schemas of those complex shapes.

Recursion: A schema is a recursive schema when it embeds within itself. If a shape pattern schema share the same patterns with every levels of a shape pattern, then the first shape pattern is embedded into the second shape pattern schema recursively.

3 Style Learning

3.1 Definition of style

Style is the constant forms and qualities that can be identified implicitly and explicitly in a group of objects. It characterises and discriminates one object group from other groups. Forms and qualities shared by designs of an individual, a group or a period characterise a style. A style arises in various ways, and many factors are involved to characterise it. Environment (culture, technology, physical environment) influences styles for objects, designs or manufacturing processes produce styles, human cognition creates and affects styles. Thus a process of emerging style can be induced from the known situation and style, or a style can be induced from the situation and the process.

Style has been explained in several ways. Gombrich (1960) and Simon (1975) defined style as a way of doing things, and asserted that a particular selection from the alternatives or a particular production process produces a style. A group of people has particular ways of thinking and doing things, and of choosing one out of a number of satisfactory solutions that has specific characteristics, and has specific ways of producing solutions. Style expresses characteristics which are created or selected by the cognitive structure of a period, a group or an individual. It appears in the final objects from which we mostly recognise and classify.

Schapiro (1961) defined style as "the constant form and sometimes constant elements, qualities, and expression in the art of an individual or a group". Schapiro emphasises not only the elements but also a system of forms with a quality and meaningful expression. Thus style is recognised from the forms and qualities of artefacts identified by repetitious elements, qualities or expressions. Elements are physical parts of objects that can be recognised explicitly, such as the pointed arch for Gothic and Islamic architecture, the round arch for Roman and Byzantine, floral shapes for Art Nouveau. Qualities emerge from relationships between elements, or relationships between these relationships. Relationships include interpretation of physical relationships as well as a kind of syntax or compositional patterns, such as cool and hot, hard and soft, proportion, rhythm, symmetry, balance, gradation, etc. Form relationships are a kind of syntax or compositional patterns that can be analysed and expressed formally.

This thesis follows the definition of Schapiro that style is constant forms and qualities, particularly with regard to replication of shape qualities.

3.2 Style as a form element

Form elements identifying style are replicate physical parts in a group of objects that categorise and characterise this group. Form elements appearing constantly in works of an individual or a group contribute to style. If an object has enough form elements characterising a certain style, this object may be considered to belong to this style. Style, recognised from the same form elements, is regarded as a part of the knowledge structure about objects, therefore it is represented by a set of elements composed of sub-elements and their relationships which are embedded into a knowledge structure. It is represented as follows:

$$\text{Style}(N) = \{(UM), (UF)\} \tag{3}$$

Where F: form elements
N: name of style
M: members of style

The form element here is a physical structure that is visually perceived and can be represented in a Cartesian coordinate system. In architecture, form elements are architectural vocabularies that are simply some small, usually carefully chosen subset of the structures handled by some basic design system (Mitchell, 1990). Traditionally, architectural elements are walls, doors, columns, floors, rooms, windows, arches, ornaments, etc. (Atkinson and Bagenal, 1926; Vitruvius, 1973; Krier, 1983. Columns, beams and wall panels are elements of construction, and rooms are elements of composition. Architectural buildings are composed of those elements. Logical representation of those elements for design has been well studied by Mitchell (1990) and Coyne (1988). Repetition of the same types of those elements in a group of objects characterizes a style. For example, ribbed vaults repeatedly appear in most Gothic and Islamic architecture. So it might be assume that a building belongs to the Gothic style in architecture if it has the ribbed vaults under certain circumstances,

Style(Gothic) = {(Paris Cathedral, Laon Cathedral, Nayon Cathedral, Reims Cathedral),
(Pointed Arches, Flying buttresses, Pyramidal roofs, Cruciforms,
Pinnacles, **Ribbed vaults**, Stained-glass windows, etc)}

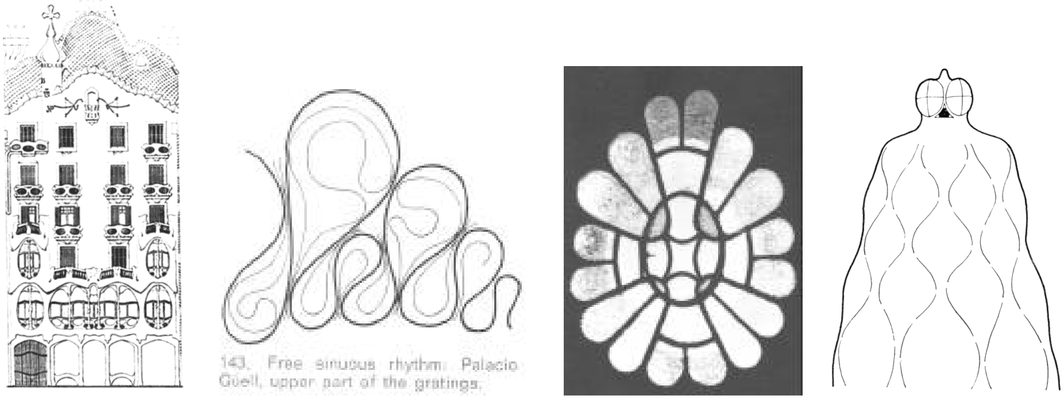
Architectural buildings are composed of not only functional elements but also visual form elements that are represented with points, lines, planes, or, volumes. The form has properties such as shape, size, color, texture, position, orientation (Ching, 1979). For instance, floral shapes characterise Art Nouveau style, keyhole shapes characterise Kahn’s windows, L-shapes characterise Botta’s architectural facades.

3.3 Style as shape pattern

Distinct and replicate syntax or compositional patterns in a set of objects, which can be analysed formally, characterise style (Schapiro, 1961). The structural view of style is that style is about relationships, rather than simple features. Relationships are the way elements are organised. Knowledge of an object can be represented with elements and their relationships in hierarchical structures.

With shapes, the schema is a hierarchy structure that is composed of spatial relationships between subshapes or semantics of shapes, and explains the characteristics of a shape. Sets of shapes or semantics of shapes in patterns identify relationships, and sets of congruent relationships specify higher relationships. Primitives, relationships and relationships of relationships are constructed into hierarchy structures. A set of such hierarchy structures appearing in a set of shape objects characterises the style of the object group.

From four of Gaudi’s buildings and ornaments in Figure 4, replicate shape elements and shape patterns can be induced that characterise Gaudi style. Reflection patterns, gradation patterns and translation patterns can be identified as shape patterns of some aspects of Gaudi’s style.

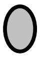


(a) (b) (c) (d)

Figure 4 Architectural buildings of Antonio Gaudi: (a) Casa Batllo, (b) upper part of the gratings in Palacio Guell, (c) window in the Chapel of the Colonia Guell, (d) Casa Mila Roof.

Style(Gaudi) = {(Casa Batllo, Gratings, Window, Casa Mila Roof),
(reflection, gradation, translation)}

Some shape patterns are constructed from low-level shapes or patterns. The gradation pattern may be generalised from different shapes and patterns, Figure 5. Among these gradation patterns, sub-patterns of gradations are readily decomposable. The reflection pattern as a sub-pattern of gradation is composed of two reflections of shapes. Elements in the gradation patterns in Gratings and Casa Mila are also translated on the translation axis.

Gradation = { gradation of reflections ($\prod_{i=1}^{2}\mathrm{t}_4\{\prod_{j=1}^{2}\mathrm{t}_2[\prod_{k=1}^{2}\mathrm{t}_2(e_{k,j,i}, a_3), a_3], a_4\}$ $e =$ ),
gradation of shapes ($\prod_{i=1}^{4}\mathrm{t}_4\{\mathrm{t}_1[e_i, (a_1, a_3)], a_4\}$ $e =$ ),
gradation of shapes ($\prod_{i=1}^{4}\mathrm{t}_4\{e_i, a_4\}$ $e =$ ),
gradation of translations (($\prod_{i=1}^{3}\mathrm{t}_4\{\mathrm{t}_1[\prod_{j=1}^{3}\mathrm{t}_1(e_{j,i}, (a_1, a_3)), (a_1, a_3)], a_4\}$
$e =$ )}

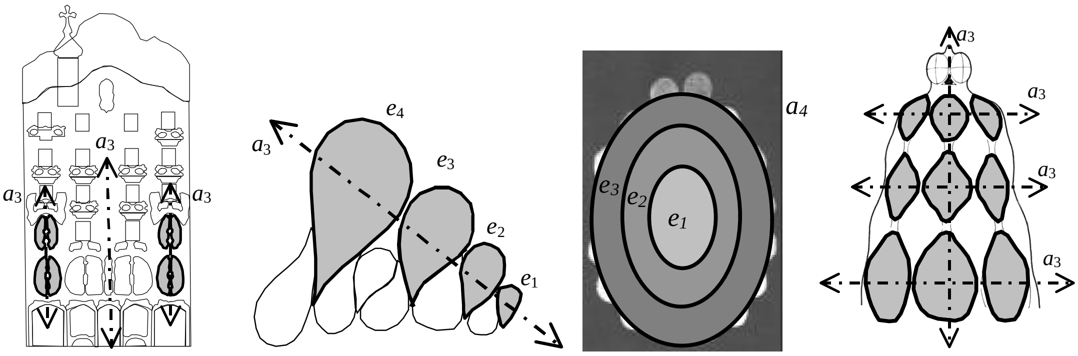


Figure 5 Gradation patterns constructed from various sub-elements.

4 Inductive Generalisation

4. 1 Theory of Generalisation

The generalisation process starts from specific facts or phenomena, or parts of them with background knowledge (Dietterich and Michalski, 1983), then derives a complete and correct hypothesis. Specific facts or phenomena are provided as observational statements that represent them, as well as background knowledge for constraints and preference criterion. This process is expressed as follows:

F **|<** **H** (F generalises to H) (4)

Where F: facts
H: hypotheses
|<: generalisation

In shape pattern generalisation, sets of shape pattern descriptions or less generalised shape pattern schemas are provided as facts, then shape pattern schemas are derived as inductive assertions. Derived shape pattern schemas from a class of shape patterns represent the style of shapes. This is expressed as follows:

D **|<** **S** (D generalises to S) (5)

Where D: descriptions of shape patterns or less generalised shape pattern schemas
S: generalised shape schemas
|<: generalisation

The dropping condition rule: A concept description can be generalised by simply removing one or more conjunctively linked expressions. With the condition of the law of simplification, the generalisation rule can be derived.

$P \,\&\, Q \Rightarrow P$ (Law of simplification) (6)

P & Q ::> K **|<** **P ::> K** (Dropping condition rule) (7)

Where P, Q: concept description
K: name of concept
::>: the implication linking a concept description with a concept name
|< : generalisation
⇒ : implication

The turning constants into variables rule: A concept description can be generalised by transforming constants into variables. If some description F(x) holds for x being a constant a or constant b, and so on, then the rule generalises these observations into a statement that F(x) holds for every value of x.

$$F(a) \;\&\; F(b) \;\&\; \ldots \quad |< \quad \forall\, x, F(x) \qquad (8)$$

where $a, b, \ldots$: constants
$F(x)$: descriptions dependent on variable x
x : variable

The climbing generalisation tree rule: In hierarchical structure descriptions, nodes are generalised to the lowest parent node.

$$[L = a] \;\&\; [L = b] \;\&\; \ldots [L = i] \quad ::> \quad K \quad |< \quad [L = s] \quad ::> \quad K \qquad (9)$$

where $a, b, \ldots$: constants
L : structured descriptor
s : the lowest parent node

4.2 Inductive Generalisation in Shape Patterns

The generalisation process is the process that transforms descriptions or less generalised schemas into more general descriptions using generalisation rules, such as the dropping condition rule, the climbing generalisation tree rule and the turning constants into variables rule. The shape pattern representation introduced in section 2 is used for inductive generalisation in shape and its examples are presented in this section.

4.2.1 The dropping condition rule

A concept description can be generalised by simply removing one or more conjunctively linked expressions. With the condition of the law of simplification, the generalisation rule can be derived. In shape objects, embedded shapes or shape patterns are implied as the conjunction of two shapes or shape patterns. If these two shapes or shape patterns are members of a class that are linked to a style, then the embedded shapes or shape patterns may characterise the style by the dropping condition rule.

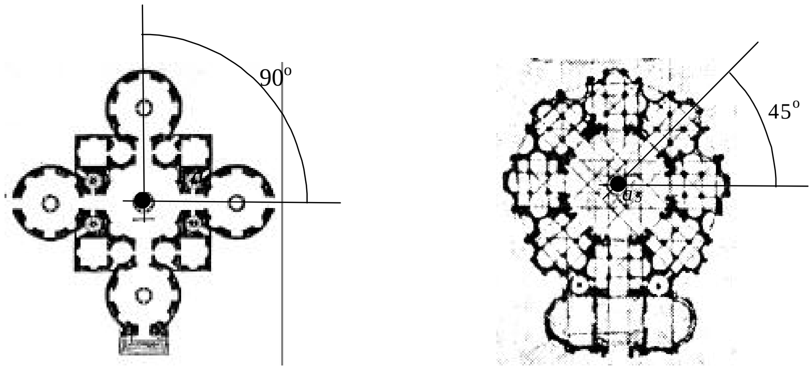
$$S_1 \;\&\; S_2 \quad \Rightarrow \quad S_1 \qquad (10)$$

$$S_1 \;\&\; S_2 \quad ::> \quad K \quad |< \quad S_1 ::> \quad K \qquad (11)$$

Where K : style
S : shape or shape pattern

Suppose there are two shapes, Figure 6, that characterise some aspects of Renaissance style. The first shape S_a has a 90^o rotation pattern in that four circular shapes are rotated by 90^o around a rotation centre a_5 and the second shape S_b has a 45^o rotation pattern in that eight shapes are rotated by 45^o around a rotation centre a_5. These are represented as follows:

$$S_a = \prod_{i=1}^{4} t_2\{e_i, (90^o, a_5)\}$$
$$S_b = \prod_{i=1}^{8} t_2\{e_i, (45^o, a_5)\}$$



(a) (b)

Figure 6. Renaissance architecture: (a) 90^o rotation pattern (Serlio's church plan), (b) 45^o rotation pattern (Leonardo's church).

The first shape pattern S_a is embedded in the second shape pattern S_b because the predicate t_2 of the shape pattern S_a is the same as the predicate t_2 of the shape pattern S_b, the rotation angle of the shape pattern S_a is twice the rotation angle of the shape pattern S_b and the number (4) of elements of the shape pattern S_a is half the number (8) of elements of the shape pattern S_b. Therefore the first shape pattern S_a is implied as the conjunction of two shape patterns by the law of simplification.

$$S_a \,\&\, S_b \;\Rightarrow\; S_a = \prod_{i=1}^{4} t_2\{e_i, (90^o, a_5)\} \qquad \text{(Law of simplication)}$$

The two shape patterns characterise the Renaissance style, therefore the conjunction of two shapes that is the 90^o rotation pattern characterises this aspects of Renaissance style.

$$S_a \,\&\, S_b \;::>\; [\text{Renaissance style}] \quad |<$$
$$\prod_{i=1}^{4} t_2\{e_i, (90^o, a_5)\} \;::>\; [\text{Renaissance style}]$$

4.2.2 The turning constants into variables rule

A shape description is composed of elements and their relationships. A set of shape descriptions that have the same predicates and some relationships in elements and arguments can be generalised by using the turning constants into variables rule. In shape pattern descriptions, sub-nodes or constants of arguments are turned into variables by generalisation rules. The shape pattern description is composed of elements and their relationships. A set of shape pattern descriptions that have the same predicates and some relationships in elements and arguments can be generalised by this way.

$$S_a = \prod_{i=1}^{n} t_i(e_a, a_a)$$
$$S_b = \prod_{i=1}^{n} t_j(e_b, a_b)$$

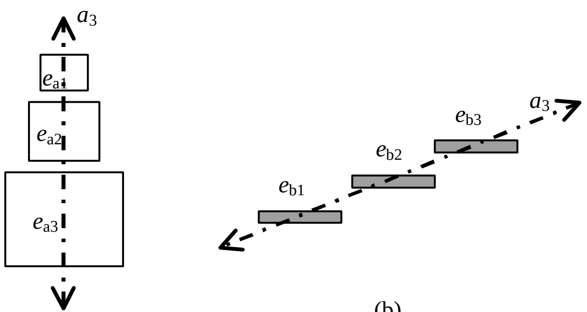
$$\text{If } t_i = t_j \quad \text{Then} \quad S_a \,\&\, S_b \;\; |< \;\; \prod_{i=1}^{n} t_j\,(x_e, x_a) \tag{12}$$

Consider two different shapes, in which the pattern of the first shape is an increase in size and of the second is an increase in floor heights, Figure 7. In the first shape pattern description S_a, the size of rooms increases by scale a_{a4} and they are located on a linear axis a_{a3} with a distance a_{a1}, Figure 7(a). In the second shape pattern description S_b, the heights of floors increase with scale a_{b4} and they are located on a linear axis a_{b3} with a distance a_{b1}, Figure 7(b). The two shape pattern descriptions belong to different domains.

$$S_a = \prod_{i=1}^{3} t_1\{t_4[e_{ai}, a_{a4}], (a_{a1}, a_{a3})\}$$
$$S_b = \prod_{i=1}^{3} t_1\{t_4[e_{bi}, a_{b4}], (a_{b1}, a_{b3})\}$$

Shape pattern description S_a and S_b have the same predicates, the same types of translation axes and the same numbers of sub-elements, therefore two different shape pattern descriptions in different domains can be generalised by the turning constants into variables rule. Sub-elements and arguments are turned into variables.

$$S_a \,\&\, S_b \;\; |< \;\; \prod_{i=1}^{3} t_1\{t_4[x_e, x_{a4}], (x_{a1}, x_{a3})\}$$



(a) (b)

Figure 7 (a) Increase of room sizes, (b) increase of floor heights.

4.2.3 The climbing generalisation tree rule

In structured descriptions, nodes are generalised to the lowest super node. Compared shape descriptions are different, but they belong to a class, which can be a result of the climbing generalisation tree rule.

$S_1 = a$
$S_2 = b$
. . .
$S_j = k$

If $S_1, S_2, \ldots, S_j \in$ A_class

$$\text{Then} \quad S_1 \,\&\, S_2 \,\&\, \ldots \,\&\, S_j :: > \; K \quad |< \quad \text{A\_class} \; :: > \; K \tag{13}$$

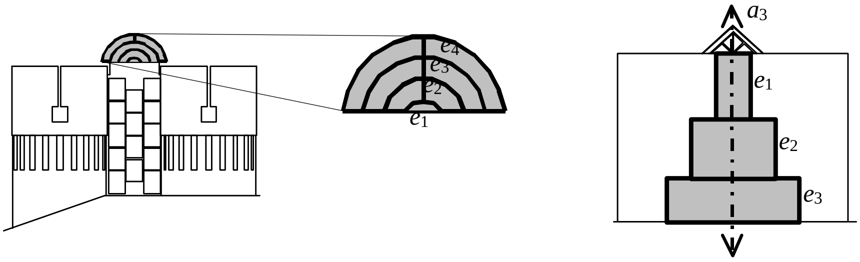
Suppose there are two shape objects that imply some aspects of Botta's style, examplified in the Origlio house and Pregassona house. The first shape pattern S_a is a gradation pattern in which the size of four half circles increases by scale a_4, Figure 8(a). The second shape pattern S_b is also a gradation pattern in which the size of three rectangles increases by scale a_4, translated on an axis a_3, Figure 8(b).

$$S_a = \prod_{i=1}^{4} t_4\{e_i, a_4\}$$
$$S_b = \prod_{i=1}^{3} t_1\{t_4[e_i, (a_1, a_3)], a_4\}$$

The two shape patterns are not embedded in each other, but they belong to a class that is a gradation of shapes. Thus they can be generalised into the class that is the gradation pattern ($\prod_{i=1}^{n} t_4\{x_{ai}, x_{a4}\}$), and their sub-elements and arguments are turned into variables by the turning constants into variables rule. This gradation of shapes implies the aspects of the Botta style.

$$S_a \,\&\, S_b \Rightarrow \prod_{i=1}^{n} t_4\{x_{ei}, x_{a4}\}$$

$$S_a \,\&\, S_b \ {::>}\ \text{Botta style} \quad |< \quad \prod_{i=1}^{n} t_4\{x_{ei}, x_{a4}\} \ {::>}\ \text{Botta style}$$

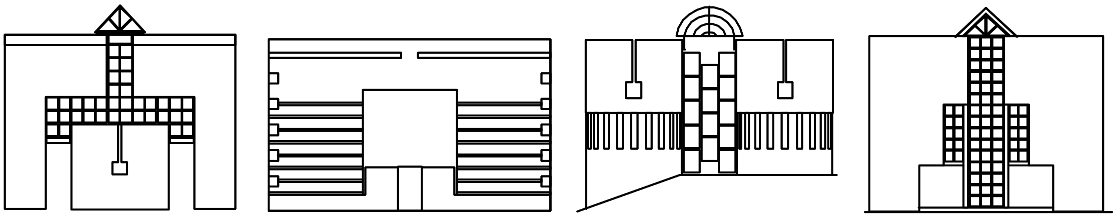


(a) (b)

Figure 8 Gradation of shapes in Botta style: (a) Origlio house, (b) Pregassona house.

5 Mario Botta Style

From four of Botta's architectural buildings in Figure 9, we can induce replicate form elements and schemas that characterise Botta style. L-shapes, squares, rectangles and key holes are recognised as form elements for Botta style, and reflection, a set of squares, self-symmetry, movement, gradation, translation relationships are identified as schemas for Botta style.



Stabio house TGV Station Origlio house Pregassona house

Figure 9 Architectural buildings of Mario Botta.

T_{Botta} = **Botta**{(Stabio House, TGV Station, Origlio House, Pregassona House),
(**Reflection**,
A set of squares,
Self-symmetry,
Movement,
Gradation,
Translation)}

Some schemas are constructed from low level elements or relationships. In the reflection schema, its subelements are various, such as, L-shapes, translation, gradation, reflection in Figure 9.

Reflection = { Reflection of L-shapes,
Reflection of **Translation**,
Reflection of Gradation,
Reflection of Reflection }

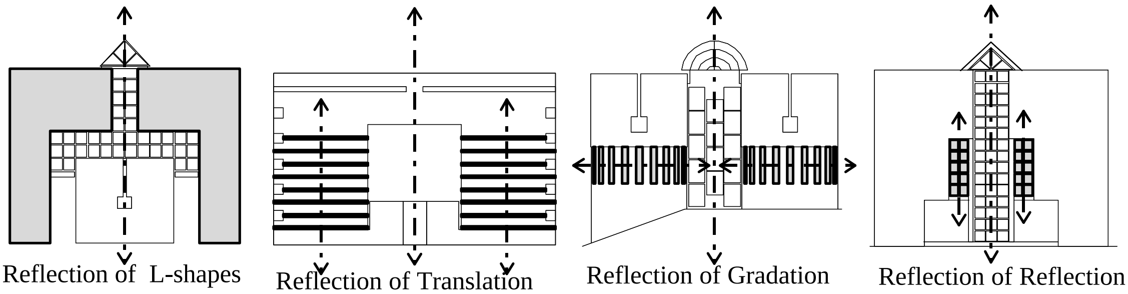

Reflection of L-shapes
Reflection of Translation
Reflection of Gradation
Reflection of Reflection

Figure 9 Reflection symmetry constructed from various elements.

Among these, the translation schema is readily decomposable. Sets of squares in Stabio house and Pregassona house specify the translation relationship in Figure 10, the translation symmetry in TGV Station is identified from sets of horizontal lines. Two groups of Rectangles in Origlio house are in a translation symmetry relationship.

Translation = { Translation of **Squares**,
Translation of **Horizontal lines**,
Translation of **Rectangles** }

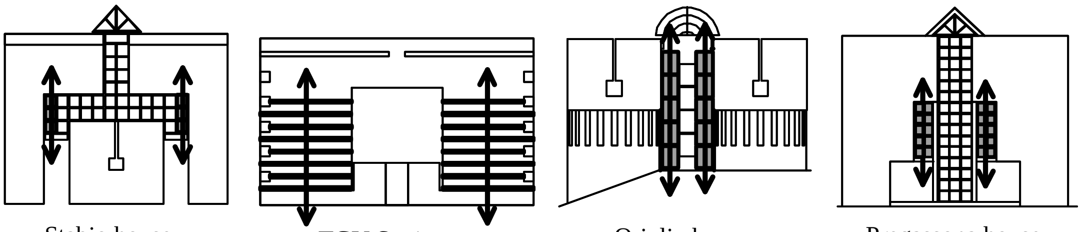

Stabio house
TGV Station
Origlio house
Pregassona house

Figure 10 Translation schemas in Mario Botta style.

6 Prototype style and family style

Style categories are organised around not only the necessary and sufficient critical criteria (conjunction or prototype), but also family resemblance (disjunction). In the classical view of categorisation (Markman, 1989), only common features that cover all the objects in a class identify style. It is a clear and straightforward way of learning style; for example, floral curve shapes appear in all Art Nouveau architectural buildings, and self-reflection symmetry appears in all Palladian villas. Prototype categories are described by the conjunctive generalisation that discriminates the given class from all other possible classes (Michalski, 1983). However, this is not enough to explain style. Wittgenstein (1960) suggested that family resemblance as well as conjunction is linked to determine a category; for example, in a group of letters AB, BC, CD, DE, there are no common letters in the items, but they are linked by at least one or several elements in common with one or more other items. Disjunction of class members also should be considered to explain style.

Conjunctive generalisation in this paper is that generalisation which occurs from the conjunction of two or more descriptions. Conjunction is identified by similarity in terms of predicates, elements and arguments. Conjunction is generalised by generalisation rules. Symbolic descriptions of objects are composed of sub-elements and their relationships in a hierarchical structure. Sub-elements may be related without any relationships between each other. Sub-elements and relationships belong to different layers, and a set of layers describes an object.

Good examples for a style are maximally similar to members of their category and minimally similar to members of other categories (Markman, 1989). Also commonalities in a number of different dimensions characterise style; for example, similarity in materials, similarity in shapes, similarity in space organisation. Here only shapes and shape patterns are used for identifying style.

Integration of prototype style and family style

One of the purposes of this research is to lay the foundation to provide the computer with an ability to produce shape objects using learned design knowledge. This section introduces a possible application of shape pattern descriptions in shape generation. There are many possible generation methods, including instantiation, analogical reasoning, shape grammar and genetic algorithm.

Variables of abstract shape patterns can be instantiated into shapes or patterns. In analogical reasoning, shape pattern schemas learned from one domain can be instantiated or applied in different domains. For example, a pattern learned from nature can be transferred and applied in an architectural façade design. Learned shape patterns can be used as rules for shape generation, and learned shapes and shape patterns can be initial shape elements for shape grammar generation. Genetic algorithms can use learned shapes and shape patterns as genotypes or phenotypes. Using genetic algorithms, many shape patterns can be produced. All these methods are useful for generating shapes that contain most of the important properties of previous shape objects. Thus generated shape objects are regarded as members of a class to which previous shape objects belong.

Base shapes for shape generation can be constructed from the combination of properties of family style. Grammars for shapes and shape pattern schemas may be used to construct base shapes. Even though the base shape has all the necessary and sufficient properties of a style, it may still need more modification. The addition of family shapes and modifications using family pattern schemas give the base shape more detail and produce results that have more properties of the style.

A base shape for shape generalisation can be constructed from the combination of prototype schemas and shapes. Grammars for shapes and schemas may be used to construct a base shape. Even though the base shape has all the necessary and sufficient properties of a style, it still need more modifications. Addition of family shapes and modification using family schemas gives the base shape more details and produces results that have more properties of the style.

Learned style of Mario Botta is a set of schemas and shapes. With this style knowledge, a base shape can be constructed in Figure 11.

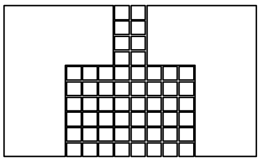
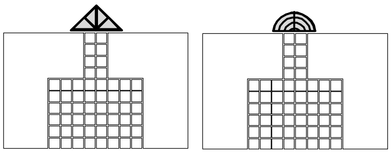


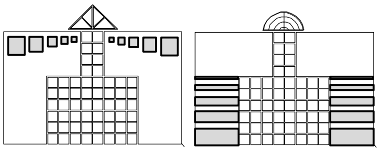
Figure 11 A constructed base shape from prototype style.

Family style and shapes of Mario Botta are added or applied onto the base shape, then various shapes can be produced that are in a category of Botta Style. Results that have more family style properties are more similar to members of Mario Botta style.

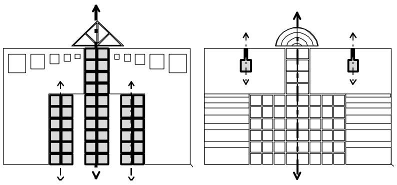
Top triangle or circle



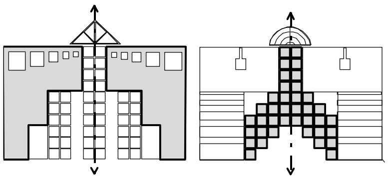
Gradation



Local Symmetry



Movement



Translation

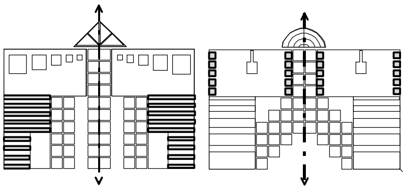


Figure 12 Generation of Botta style from prototype style and family style.

7 Conclusion

In this research, inductive generalisation rules have been exploited for style learning based on shape pattern representation, subsequently shapes and shape pattern knowledge of Mario Botta as style have been investigated.

Style as a shape pattern schema is constructed based on feature similarity and multiple representation using inductive generalisation processes. One form of learning of style occurs when the machine obtains conjunctive generalisations from existing classes of objects, and is called conceptual inductive learning (Michalski, 1983). The process of conceptual inductive learning is that observational statements are constructed in terms of predefined knowledge from a given class of objects, then the conjunctive generalisations that satisfy the predefined knowledge are induced with generalisation rules. The observational statements and conjunctive generalisations are conceptual descriptions that describe specific phenomena of objects, and are in the form of a hierarchical tree structure with variables, predicates and functions. Generalisation rules such as dropping condition rule, turning constants into variables rule**,** climbing generalisation tree rule, are elaborated using shape pattern repersentations and generalised shape pattern descriptions are produced as style knowledge that characterise the set of shape objects.

Those knowledge are design knowledge such as spatial relationships or patterns that cannot be easily recognised in terms of physical properties. The integration of prototype and family style can produce various shape objects that are regarded as members of a class to which a previous shape belonged. Learned style descriptions provide important design knowledge for generation processes, such as instantiation, parametric desgin, analogical design, metaphor design, generative shape grammar and genetic algorithm. The processes described give the computer an ability to learn design knowledge, for supporting design computation and creative design by either humans or computers.

Acknowledgements

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