Edge Detection

original image

gradient magnitude

Canny edge detector
- Compute image derivatives
- if gradient magnitude > τ and the value is a local maximum along gradient direction – pixel is an edge candidate

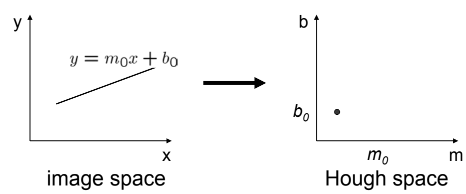
An edge is not a line...

- How can we detect ***lines*** ?

Finding lines in an image

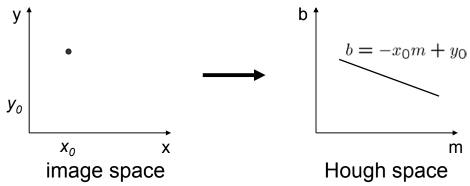
- Option 1:
  - Search for the line at every possible position/orientation
  - What is the cost of this operation?
- Option 2:
  - Use a voting scheme: Hough transform

Finding lines in an image



- Connection between image (x,y) and Hough (m,b) spaces
  - A line in the image corresponds to a point in Hough space
  - To go from image space to Hough space:
    - given a set of points (x,y), find all (m,b) such that y = mx + b

Finding lines in an image



- Connection between image (x,y) and Hough (m,b) spaces
  - *A line in the image corresponds to a point in Hough space*
  - To go from image space to Hough space:
    - given a set of points (x,y), find all (m,b) such that y = mx + b
  - What does a point (x_0, y_0) in the image space map to?
    - A: the solutions of $b = -x_0 m + y_0$
    - this is a line in Hough space

Hough transform algorithm

- Typically use a different parameterization

  $$d = x cos\theta + y sin\theta$$

  - d is the perpendicular distance from the line to the origin
  - θ is the angle this perpendicular makes with the x axis
  - Why?

Idea – keep an accumulator array (Hough space) and let each edge pixel contribute to it
Line candidates are the maxima in the accumulator array

Typical Hough Transform

Basic Hough transform algorithm
1. Initialize H[d, θ]=0
2. For each edge point I[x,y] in the image
3. For θ = 0 to 180

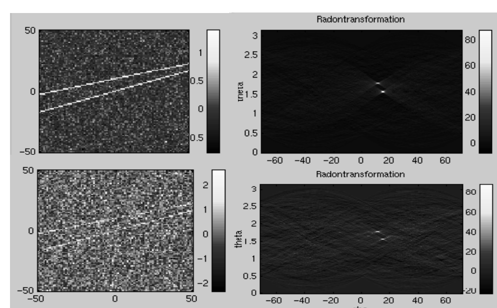
   H[d, θ] += 1 where $d = x cos\theta + y sin\theta$
   - point is now a sinusoid in Hough space
   - Find the value(s) of (d, θ) where H[d, θ] is maximum

The detected line in the image is given by

$$d = x cos\theta + y sin\theta$$

What's the running time (measured in # votes)?

Radon transform



Extensions

- Extension 1: Use the image gradient
  1. Initialize
  2. for each edge point I[x,y] in the image
     compute unique (d, θ) based on image gradient at (x,y)
     H[d, θ] += 1
  3. Find the value(s) of (d, θ) where H[d, θ] is maximum

1. Extension 2
   - give more votes for stronger edges
   - The same procedure can be used with circles, squares, or any other shape

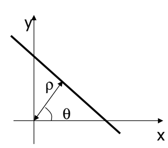
Hough Transform for Curves

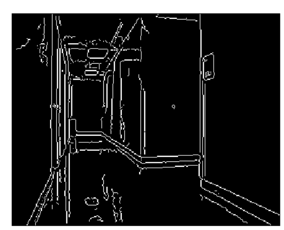
- The H.T. can be generalized to detect any curve that can be expressed in parametric form:
  - Y = f(x, a1,a2,…ap)
  - a1, a2, … ap are the parameters
  - The parameter space is p-dimensional
  - The accumulating array is LARGE!

H.T. Summary

- H.T. is a "voting" scheme
  - points vote for a set of parameters describing a line or curve.
- The more votes for a particular set
  - the more evidence that the corresponding curve is present in the image.
- Can detect MULTIPLE curves in one shot.
- Computational cost increases with the number of parameters describing the curve.

Line fitting





Non-max suppressed gradient magnitude

- Edge detection, non-maximum suppression
  (traditionally Hough Transform – issues of resolution, threshold selection and search for peaks in Hough space)
- Connected components on edge pixels with similar orientation
  - group pixels with common orientation

Line Fitting

$$A = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i \\ \sum x_i y_i & \sum y_i^2 \end{bmatrix}$$

second moment matrix associated with each connected component
v_1 - eigenvector of A

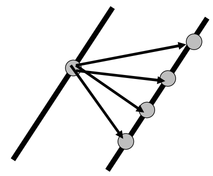
$$v_1 = [\cos(\theta), \sin(\theta)]^T$$
$$\theta = \arctan(v_1(2)/v_1(1))$$
$$\rho = \bar{x}\sin(\theta) - \bar{y}\cos(\theta)$$

- Line fitting lines determined from eigenvalues and eigenvectors of A
- Candidate line segments - associated line quality

Corner detection

Corners contain more edges than lines.

- A point on a line is hard to match.



Finding Corners

Intuition:

- Right at corner, gradient is ill defined.
- Near corner, gradient has two different values.

Formula for Finding Corners

We look at matrix:

Sum over a small region, the hypothetical corner

Gradient with respect to x, times gradient with respect to y

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

Matrix is symmetric

First, consider case where:

$$C = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

This means all gradients in neighborhood are:

(k,0) or (0, c) or (0, 0) (or off-diagonals cancel).

What is region like if:

1. l1 = 0?
2. l2 = 0?
3. l1 = 0 and l2 = 0?
4. l1 > 0 and l2 > 0?

General Case:

From Linear Algebra, it follows that because C is symmetric:

$$C = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

With R a rotation matrix.

So every case is like one on last slide.

So, to detect corners

- Filter image.
- Compute magnitude of the gradient everywhere.
- We construct C in a window.
- Use Linear Algebra to find $\lambda 1$ and $\lambda 2$.
- If they are both big, we have a corner.

Point Feature Extraction

$$G = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

- Compute eigenvalues of G
- If smalest eigenvalue σ of G is bigger than τ - mark pixel as candidate feature point

- Alternatively feature quality function (Harris Corner Detector)

$$C(G) = \det(G) + k \cdot \text{trace}^2(G)$$

Harris Corner Detector - Example

CS482, Jana Kosecka