

# Advanced Features

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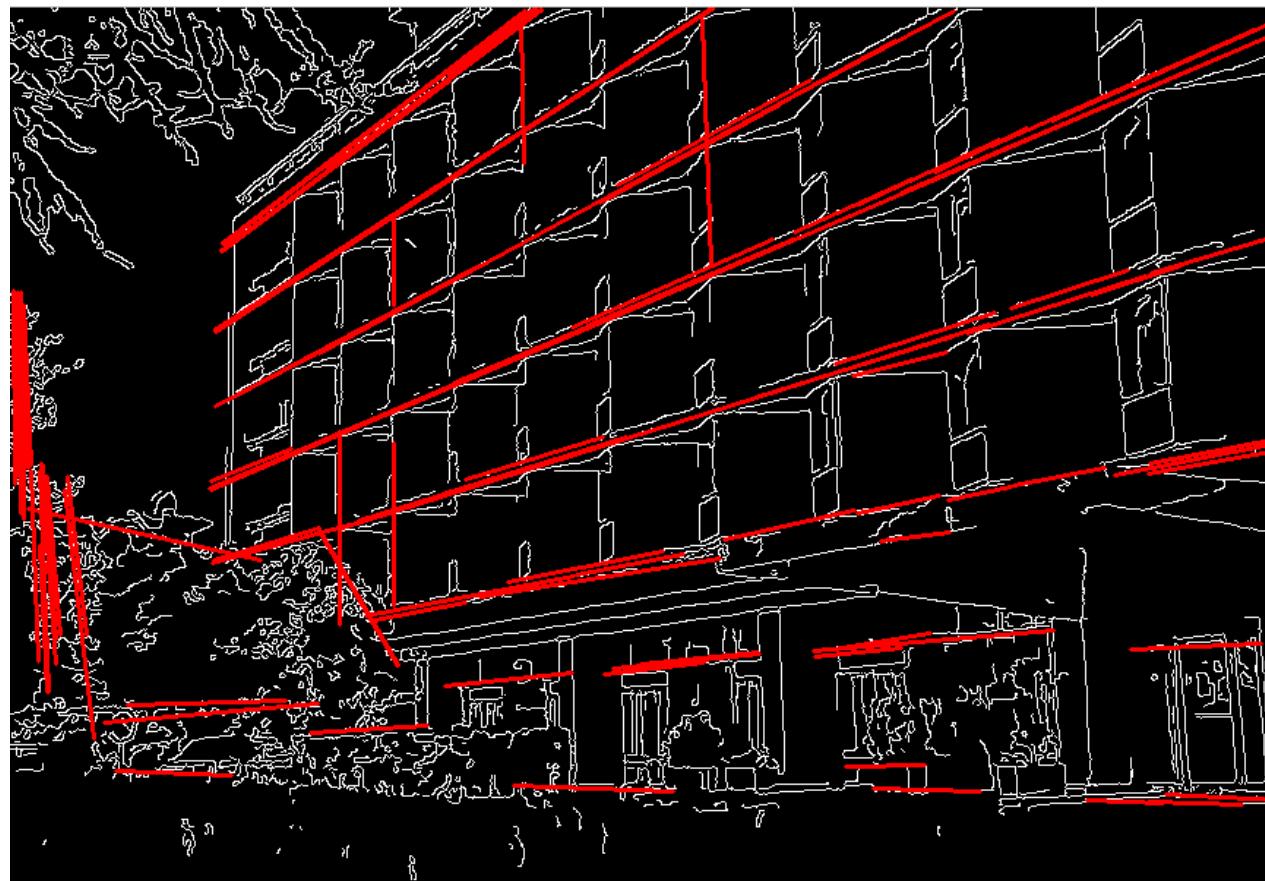
Slides from: S. Thrun, D. Lowe, Forsyth and Ponce

## Fitting

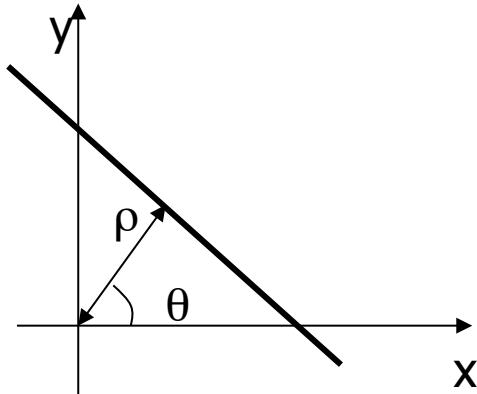
- We've learned how to detect edges, corners, blobs. Now what?
- We would like to form a higher-level, more compact representation of the features in the image by grouping multiple features according to a simple model



# Fitting



# Line fitting



Non-max suppressed gradient magnitude

- Edge detection, non-maximum suppression  
(traditionally Hough Transform – issues of resolution, threshold selection and search for peaks in Hough space)
- Connected components on edge pixels with similar orientation
  - group pixels with common orientation

# Line Fitting

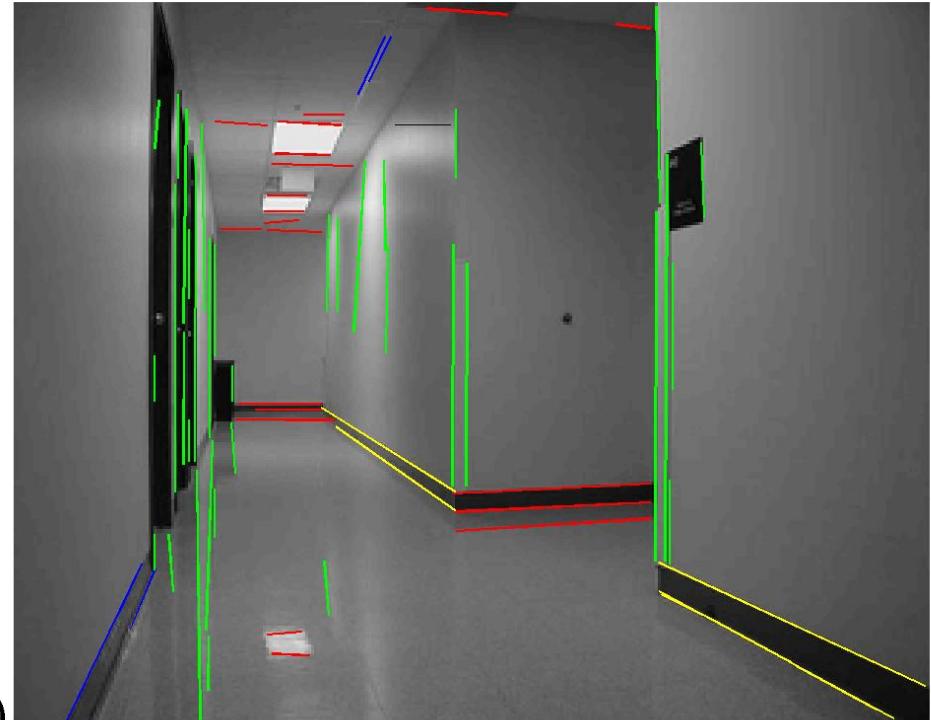
$$A = \begin{bmatrix} \sum x_i^2 & \sum x_i y_i \\ \sum x_i y_i & \sum y_i^2 \end{bmatrix}$$

second moment matrix  
associated with each  
connected component  
 $v_1$  - eigenvector of  $A$

$$v_1 = [\cos(\theta), \sin(\theta)]^T$$

$$\theta = \arctan(v_1(2)/v_1(1))$$

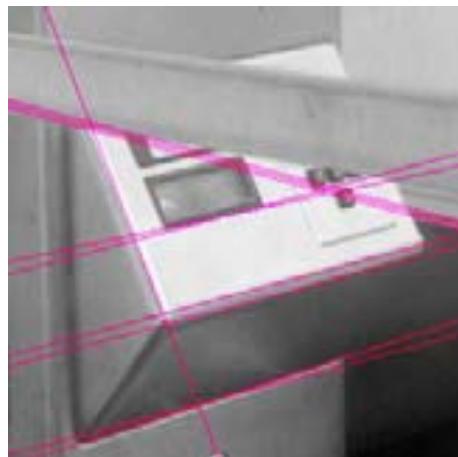
$$\rho = \bar{x} \sin(\theta) - \bar{y} \cos(\theta)$$



- Line fitting lines determined from eigenvalues and eigenvectors of A
  - Candidate line segments - associated line quality

## Fitting

- Choose a parametric model to represent a set of features



simple model: lines



simple model: circles



complicated model: car

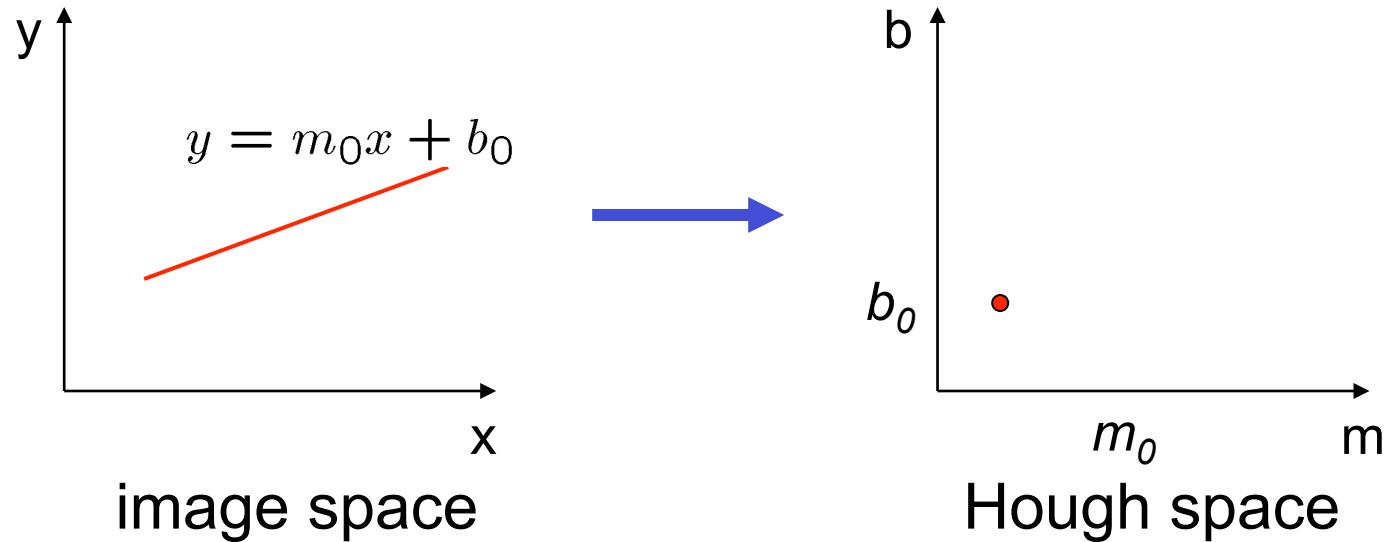


Source: K. Grauman

## Finding lines in an image

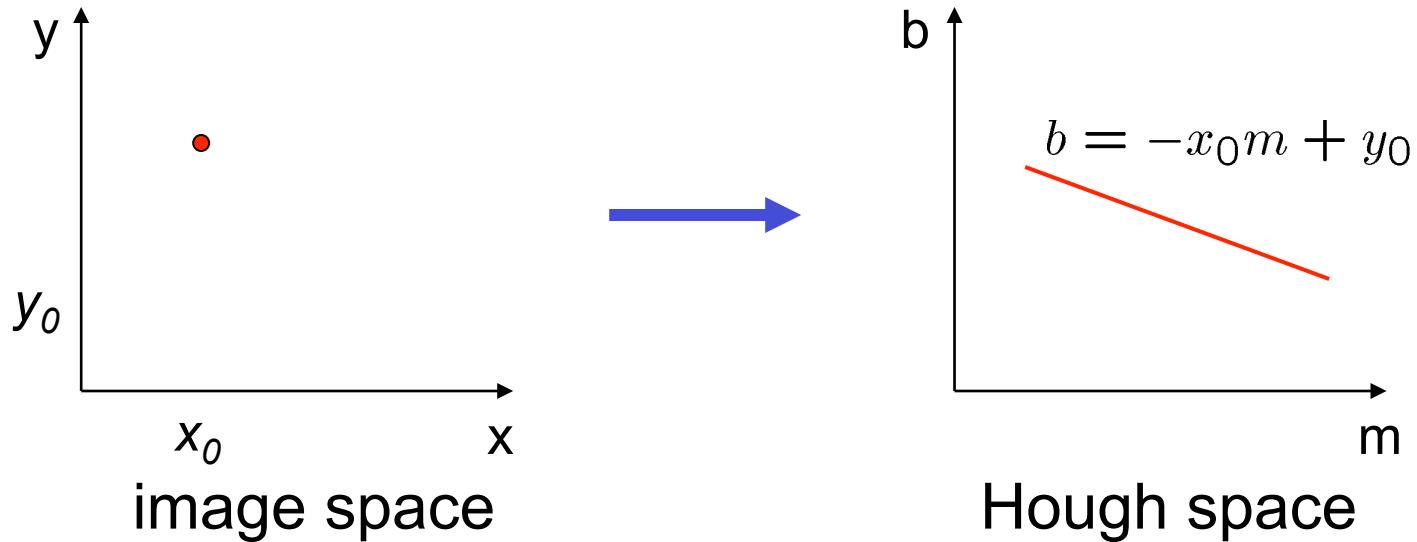
- Option 1:
  - Search for the line at every possible position/orientation
  - What is the cost of this operation?
- Option 2:
  - Use a voting scheme: Hough transform

## Finding lines in an image



- Connection between image  $(x,y)$  and Hough  $(m,b)$  spaces
  - A line in the image corresponds to a point in Hough space
  - To go from image space to Hough space:
    - given a set of points  $(x,y)$ , find all  $(m,b)$  such that  $y = mx + b$

# Finding lines in an image



- Connection between image  $(x,y)$  and Hough  $(m,b)$  spaces
  - A line in the image corresponds to a point in Hough space
  - To go from image space to Hough space:
    - given a set of points  $(x,y)$ , find all  $(m,b)$  such that  $y = mx + b$
  - What does a point  $(x_0, y_0)$  in the image space map to?
    - A: the solutions of  $b = -x_0 m + y_0$
    - this is a line in Hough space

# Hough transform algorithm

- Typically use a different parameterization

$$d = x\cos\theta + y\sin\theta$$

- $d$  is the perpendicular distance from the line to the origin
- $\theta$  is the angle this perpendicular makes with the x axis
- Why?

Idea – keep an accumulator array (Hough space)  
and let each edge pixel contribute to it

Line candidates are the maxima in the accumulator  
array

# Typical Hough Transform

Basic Hough transform algorithm

1. Initialize  $H[d, \theta] = 0$
2. For each edge point  $I[x, y]$  in the image

3. For  $\theta = 0$  to  $180^\circ$

$$H[d = x\cos\theta + y\sin\theta]$$

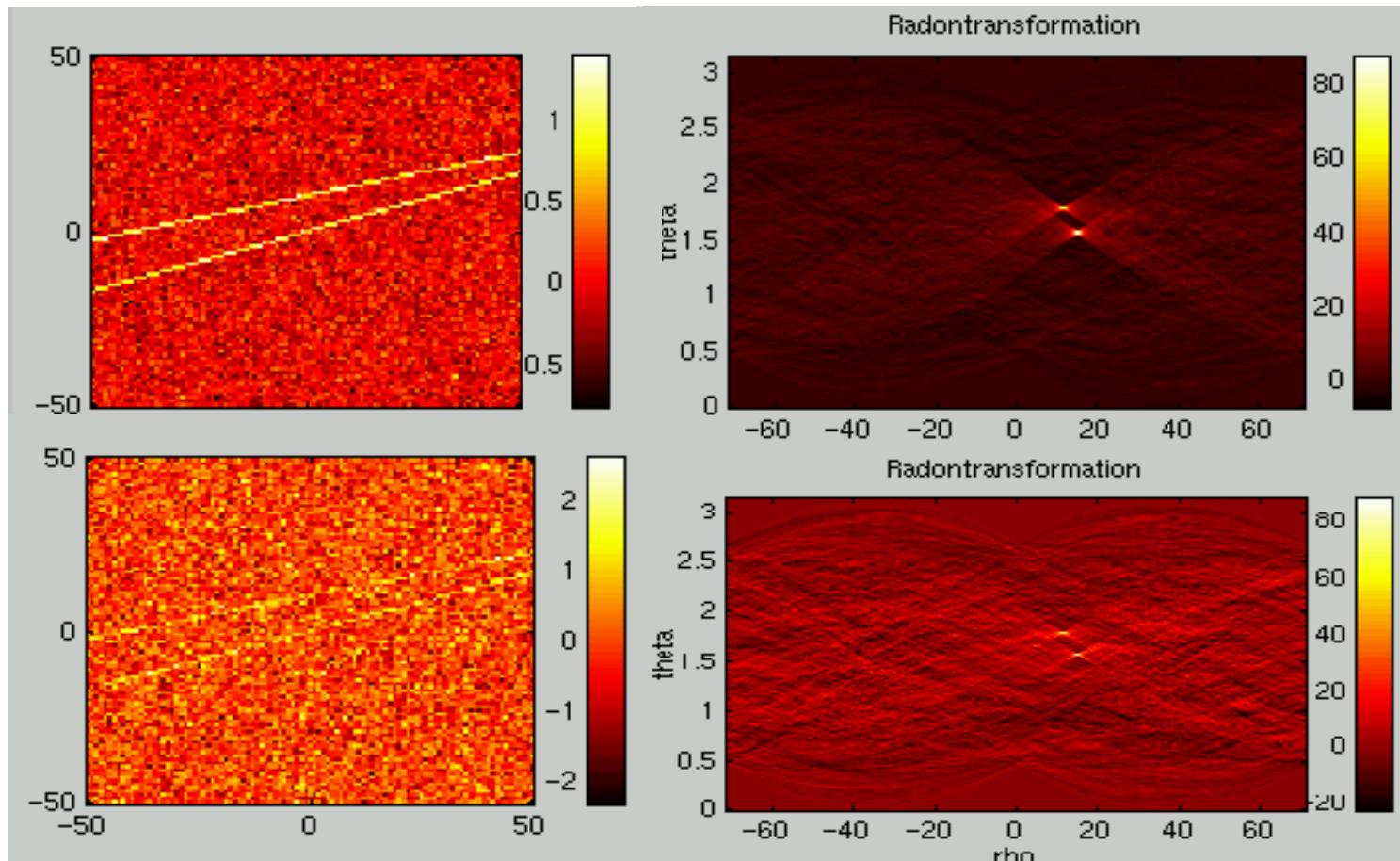
point is now a sinusoid in Hough space

Find the value(s) of  $(d, \theta)$  where  $H[d, \theta]$  is maximum

The detected line in the image is given by maxima in the Hough space

What's the running time (measured in # votes)?

# Radon Transform



- Projection of an image along radial line

## Hough Transform for Curves

- The H.T. can be generalized to detect any curve that can be expressed in parametric form:
  - $Y = f(x, a_1, a_2, \dots, a_p)$
  - $a_1, a_2, \dots, a_p$  are the parameters
  - The parameter space is  $p$ -dimensional
  - The accumulating array is LARGE!