1. (5) Consider LCS algorithm discussed in class. What is the worst-case running time of the brute force algorithm, which checks all possible subsequences?

2. (15) Coin changing page 402 (353 old book). (The proof in part a) is extra - additional 5 point credit can be obtained for the proof that the algorithm is correct)

3. (10) The incidence matrix of a directed graph $G = (V, E)$ without self-loops (i.e. without edges $(v, v)$) is a $|V| \times |E|$ matrix $B = (b_{ij})$ such that

$$b_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves vertex } i \\ +1 & \text{if edge } j \text{ enters vertex } i \\ 0 & \text{otherwise} \end{cases}$$

Describe what the entries of the matrix product $BB^T$ represent, where $B^T$ is the transpose of the matrix $B$. $BB^T$ is called the Laplacian of the graph.

4. (20) Give linear time algorithm, based on DFS, for the following problem.
   a) Determining whether a given directed graph has a cycle
   b) Determining whether a given undirected graph has a cycle. Your algorithm in this case should run in time $O(n)$, where $n$ is the number of nodes in the graph and independent of the number of edges.