

CS483 - Practice problems 2 *Jana Košecká*

- 3.8. Using an assumption that n is power of 2, we can get an estimate for $W(n)$ that will be of the same order. We can expand the recurrence and see the pattern after few terms

$$W(n) = cn + W(n/2) = cn + cn/2 + W(n/4) = c + cn/2 + cn/4 + W(n/8) \quad (1)$$

$$= cn \sum_{i=0}^{\lg n - 1} = cn \frac{(1/2)^{\lg n} - 1}{1/2 - 1} = cn \frac{1 - 1/n}{1/2} \quad (2)$$

$$= 2cn(1 - 1/n) = 2c(n - 1) \quad (3)$$

So $W(n)$ is $\theta(n)$.

- 3.10. b) case 3 Master Theorem
c) case 2 Master Theorem
d) None of the cases of Master Theorem applies - see eq. (3.14)
e) case 3 Master Theorem.
- 4.6. $1, n, n - 1, \dots, 2;$
 $n, n - 1, n - 1 \dots 3, 2, 1$.
- 4.12. Write down the recurrence equations which would corresponds to the new divide and conquer algorithm and solve it. The solution falls in between the cases of Master theorem, see eq. 3.14. Justification for the solution of given by eq (3.14) is bit harder, but you can give it a try (it won't be required).
- 4.15. $n(n-1)/2$ key comparisons, $n-1$ interchanges.
- 4.19 just execute few steps of the alg by hand and observe the behavior.
- 4.35 Y, T, X, P, O, E, M, I, C, L. 14 key comparisons.