CS483 - Final Exam (Version 1)
Wednesday, May 2nd
Prof. Jana Košecká
(100 out of 120 points)

Please turn in your crib sheet with your exam.

Name: ....

Question	Score	Points
1	10	
2	10	
3	10	
4	20	
5	5	
6	10	
7	30	
8	10	
9	15	
Total	120	

1. (10) Give a Huffman code for this string. Write down both the tree and the actual binary encoding of the string.

   AAABAABAACAABAACACAACBAACBAACA

2. (10) Given an adjacency list representation of a graph how long does it take to compute the out-degree of all vertices? How long does it take to compute in-degree of all vertices? How long does is take to compute in-degree and out-degree of a single vertex ?

3. (10) What is the running time of Dijkstra's algorithm when the priority queue is maintained as
   a) a simple linear array
   b) an ordinary binary heap
   c) Is Dijkstra's algorithm a greedy algorithm ?

```
Dijkstra(G, w, s)
Initialize-single-source(G,s)
S <- 0
Q <- 0
while Q ≠ 0
     do u <- Extract-Min(Q)
     S <- S ∪ u
     for each vertex v ∈ Adj[u]
        do Relax(u,v,w)
```

4. (20) We are running one of these three algorithms on the graph below, where the algorithm has already processed the bold-face edges. (ignore the directions on the edges for Prim's and Kruskal's).

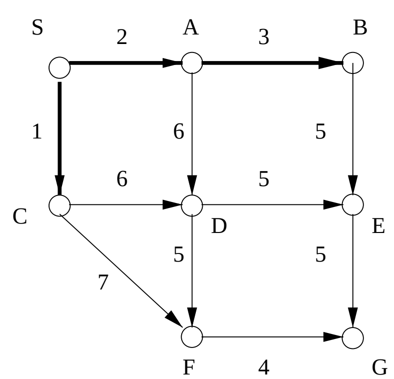
   a) Prim's for the minimum spanning tree
   b) Kruskal's for the minimum spanning tree
   c) Dijkstra's shortest paths from s

   a) Which edge would be added next in Prim's algorithm.

   b) Which edge would be added next in Kruskal's algorithm.

   c) Which vertex would be marked next in Dijkstra's algorithm?
   (i.e. deleted from the top of the heap).

   d) Which final edge would Dijkstra's algorithm choose as part of the shortest path to **this** vertex. (i.e. which edge connects to **this** vertex as a part of the path of the shortest path from s. (**this** refers to vertex found in the previous step).



5. (5) You are given a computer network where some pairs are connected by a 2-way communication line. You know which computers have links directly connecting them and you want to find out for a specific computer c_o its shortest route to every other computer c_i in the network. The distance is defined in tems of number of hops the message must travel. There are C computers and L links. Suggest an efficient algorithm to find the best route to all other computers in the network.

6. (10) Suppose you have to find a route in the network which has enough bandwith for the connection. Your network has n nodes and $b_{ij} = b_{ji}$ is the bandwidth between nodes i and j. The bandwidth of the connection depends on (i.e. is) the lowest bandwith link in the route.

   a) Suppose that you are given two nodes i and j and the needed bandwidth B. How would you efficiently find a route from j to j which has bandwidth $\geq B$.

   b) Suppose that you want a route of maximum bandwidth from i to j. How would you efficiently find such a route ?

7. (30) Answer True or False. Justify your answer. Each answer is for 3 points.

The problem of determining an optimal order for multiplying a chain of matrices can be solved by a greedy algorithm, since it displays the optimal substructure and overlapping subproblems properties .

The topological sort of an arbitrary directed graph $G(V, E)$ can be computed in linear time.

Kruskal's algorithm for minimum weight spanning trees is an example of dynamic programming algorithm.

Shortest path between two vertices is unique if all edge weights are distinct.

The paths returned by Bellman-Ford at iteration $|V| - 3$ can be the shortest paths.

A arbitrary graph with $G(V, E)$, with $|E| = |V| - 1$ edges is a tree.

Directed graph is strongly connected if and only if a DFS started from any vertex will visit every vertex in the graph without needed to be restarted.

If problem A reduces (is polynomial-time reducible) to problem B and B is NP-complete then A is NP-complete.

Every problem in NP is NP-complete.

If problem A reduces to problem B and B is in P, then A is in P.

8. (10) Consider following matrix, which corresponds to the initialized distance matrix of the all-pairs-shortest-path algorithm.

$$\begin{pmatrix} 0 & 2 & 4 & 3 \\ 3 & 0 & \infty & 3 \\ 5 & \infty & 0 & 3 \\ \infty & 1 & 4 & 0 \end{pmatrix}$$

a) Draw the corresponding graph.
b) Execute two iterations of Floyd-Warshal algorithm.
c) What is the running time of the Floyd-Warshal algorithm.
d) In the following formula for updating costs (which you can use), what does k corresponds to ?

$$d_{ij}^{k} = min(d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1})$$

9. (15) Let 2-CNF-SAT be a set of satifiable boolean formulas in CNF with exactly 2 literals per clause. Show that 2-CNF-SAT $\in$ P. Suggest an efficient algorithm. (Hint: Observe that $x \vee y$ is equivalent to $\neg x \rightarrow y$). Reduce 2-CNF-SAT to a problem in a directed graph that is efficiently solvable.