CS485 - Homework 1
Due date: February 16th

Be as concise as possible.

1. (5) Consider rigid body transformations in the plane. Draw a right triangle defined by three points $A = (2,1), B = (4,1), C = (4,6)$.

   - Consider a rotation matrix

   $$T_1 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

   a. What is the determinant of the matrix ?

   - Consider transformation matrix

   $$T_2 = \begin{bmatrix} \sin\theta & \cos\theta \\ \cos\theta & -\sin\theta \end{bmatrix}$$

   a. Is the matrix orthonormal ? What is the determinant of the matrix ?

   c. Is T_2 rigid body transformation ? What is the difference between T_1 and T_2, how are the results different?

2. (5) Point $P_A = [p_1, p_2, p_3]^T$ expressed in a stationary frame A is rotated about axis Z_A by θ degrees and then rotated around axis X_A by ϕ degrees. Give a rotation matrix that accomplishes these two rotations. Both of the rotations are around stationary frame.

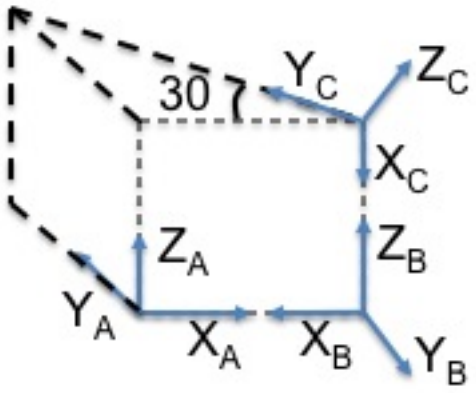
3. Let $R \in SO(3)$ be a rotation matrix generated by rotating about a unit vector ω by θ radians that satisfies $R = exp(\hat{\omega}\theta)$.

   Consider following rotation matrix:

   $$R = \begin{bmatrix} 0.1729 & -0.1468 & 0.9739 \\ 0.9739 & 0.1729 & -0.1468 \\ -0.1468 & 0.9739 & 0.1729 \end{bmatrix}$$

   Use the formulas given in class to compute the rotation axis and the associated angle. b) Use Matlab function `eig` to compute the eigenvalues and eigenvectors of the above rotation matrix R. What is the eigenvector associated with unit eigenvalue ? Can you explain it's physical meaning ?

4. (5) Determine the values of the following transformations T_{AB}, T_{BC} and T_{CB} between the frames A, B, C attached to a wedge in the figure below. The notation T_{AB} denotes a transformation which will transform points in the coordinate frame B to coordinate frame A.



5. (5) Consider and example of a single two link leg manipulator given in class. The forward kinematics equations of the leg is:

$$\begin{aligned} x &= l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ y &= l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{aligned}$$

   Compute the determinant of the Jacobian and determine when it is singular (i.e equals 0).

6. (10) Write a Matlab program to simulate the motion a differential drive robot.

   - The function should take as an input vector ξ_0 specifying the initial pose $[x_0, y_0, \theta_0]$ and velocities v, ω and time t denotes number of time steps and δt the length of the time step. You should return resulting path as three vectors each $1 \times n$ long where n is the number of time steps. The output will correspond to pose indexed by time.

     ```
     [x,y,theta] = diffDrive([x0, y0, theta0], v, omega, t, delta)
     ```

   - For the following example assume that at time $t = 0$ the configuration (pose) of the robot is $\xi_0 = [x, y, \theta] = [100, 50, 45^\circ]$. Robot starts moving with some angular and linear velocity $\omega = 2^\circ/s$ and $v = 1\,m/s$. How is the path affected by the choice of δt ? Hand in the plot of the code and the plot of the path.