Kinematics, Kinematics Chains

Previously

- Representation of rigid body motion
- Two different interpretations
  - as transformations between different coordinate frames
  - as operators acting on a rigid body
- Representation in terms of homogeneous coordinates
- Composition of rigid body motions
- Inverse of rigid body motion

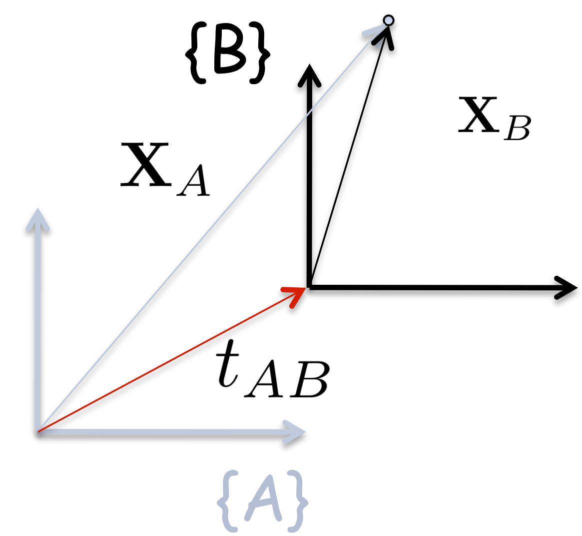
Rigid Body Transform

Translation only t_{AB} is the origin of the frame B expressed in the Frame A

$$\mathbf{X}_A = \mathbf{X}_B + t_{AB}$$

Composite transformation:

$$\mathbf{X}_A = R_{AB}\mathbf{X}_B + t_{AB}$$



Transformation: $T = (R_{AB}, t_{AB})$

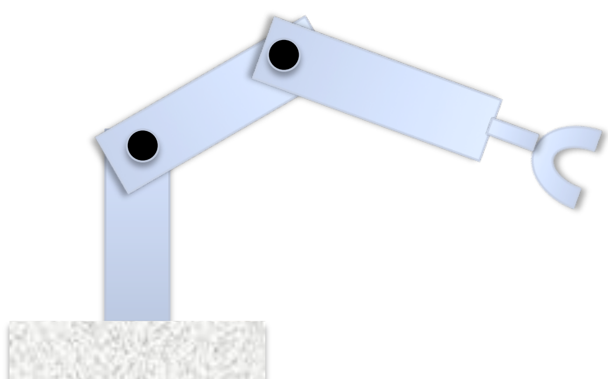
Homogeneous coordinates

$$\mathbf{X}_A = \begin{bmatrix} R_{AB} & t_{AB} \\ 0 & 1 \end{bmatrix} \mathbf{X}_B$$

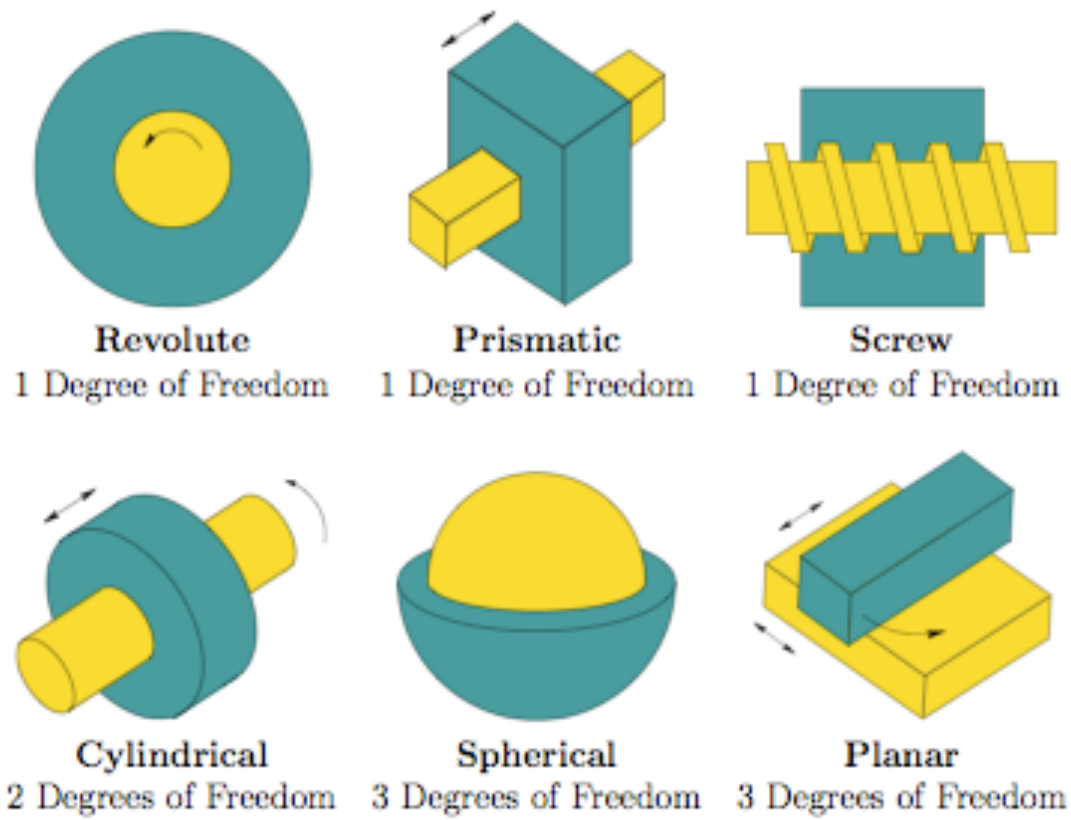
The points from frame A to frame B are transformed by the inverse of $T = (R_{AB}, t_{AB})$ (see example next slide)

Kinematic Chains

- We will focus on mobile robots (brief digression)
- In general robotics - study of multiple rigid bodies lined together (e.g. robot manipulator)
- Kinematics – study of position, orientation, velocity, acceleration regardless of the forces
- Simple examples of kinematic model of robot manipulator and mobile robot
- Components – links, connected by joints

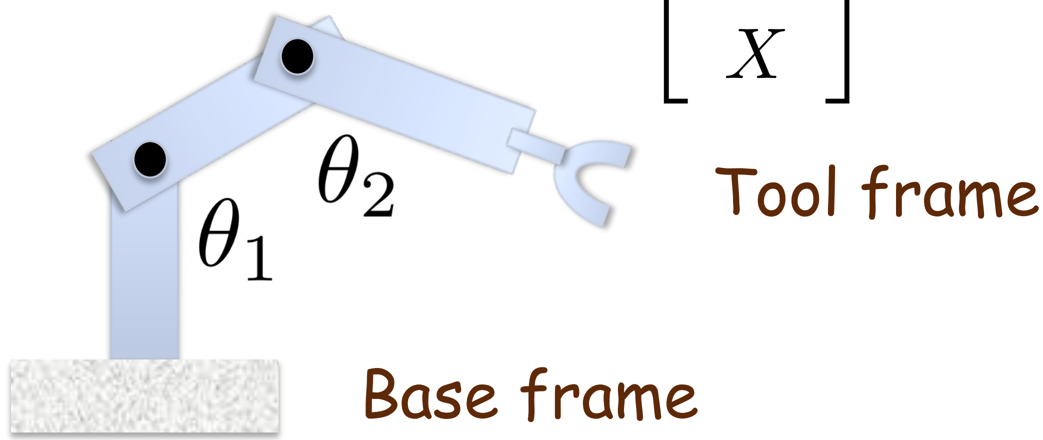


Various joints



Kinematic Chains

$$\begin{bmatrix} X \\ Y \\ X \end{bmatrix}$$

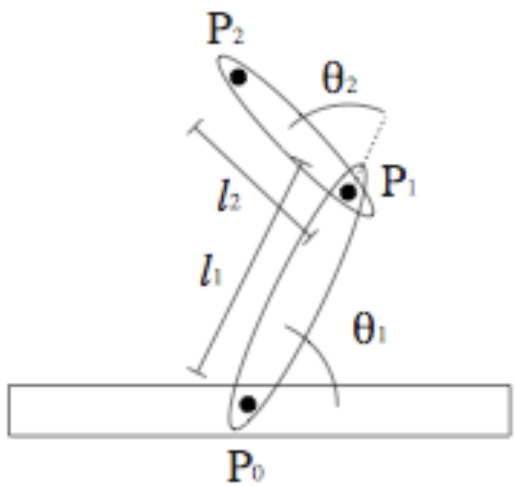


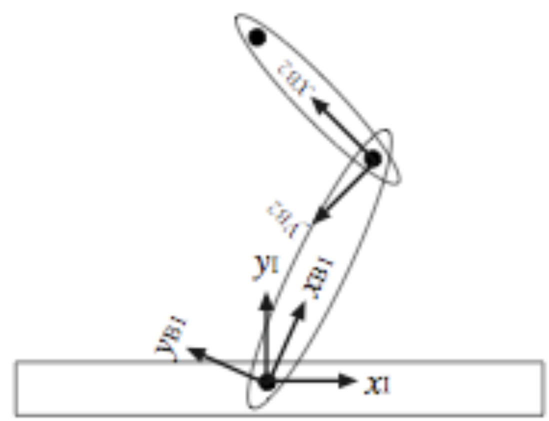
- Given θ_1, θ_2 determine what is X, Y, Z
- Given $\dot{\theta}_1, \dot{\theta}_2$ determine what is $\dot{X}, \dot{Y}, \dot{Z}$
- We can control θ_1, θ_2, want to understand how it affects position of the tool frame
- How does the position of the tool frame change as the manipulator articulates
- Actuators change the joint angles

Forward kinematics for a 2D arm

- Find position of the end effector as a function of the joint angles

$$f(\theta_1, \theta_2) = \begin{bmatrix} X \\ Y \end{bmatrix}$$





- Blackboard example

Kinematic Chains in 3D

- More joints possible (spherical, screw)
- Additional offset parameters, more complicated
- Same idea: set up frame with each link
- Define relationship between links
- Two rules:

  - use Z-axis as an axis of a revolute joint

  - connect two axes shortest distance

In 2D we need only link length and joint angle to specify the transform

In 3D $d_i, \theta_i, a_{i-1}, \alpha_{i-1}$ Denavit-Hartenberg parameters (see LaValle (chapter [3])

Inverse kinematics

- In order to accomplish tasks, we need to know given some coordinates in the tool frame, how to compute the joint angles
- Blackboard example (see handout)

Jacobians

- Kinematics enables us study what space is reachable
- Given reachable points in space, how well can be motion of an arm controlled near these points
- We would like to establish relationship between velocities in joint space and velocities in end-effector space
- Given kinematics equations for two link arm

$$\begin{aligned} x &= f_x(\theta_1, \theta_2) \\ y &= f_y(\theta_1, \theta_2) \end{aligned}$$

- The relationship between velocities is
- manipulator Jacobian $J(\theta_1, \theta_2)$

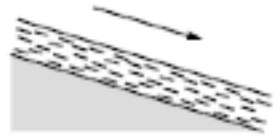
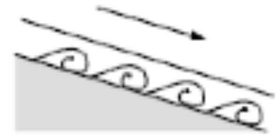
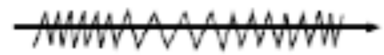
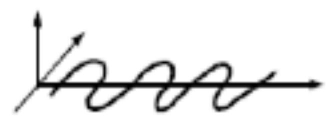
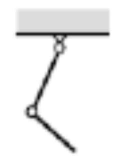
$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = J(\theta_1, \theta_2) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Manipulator Jacobian

- Determinant of the Jacobian
- If determinant is 0, there is a singularity
- Manipulator kinematics: position of end effector can be determined knowing the joint angles
- Actuators: motors that drive the joint angles
- Motors can move the joint angles to achieve certain position

- Mobile robot actuators: motors which drive the wheels
- Configuration of a wheel does not reveal the pose of the robot, history is important

Locomotion concepts

Type of motion	Resistance to motion	Basic kinematics of motion
Flow in a Channel 	Hydrodynamic forces	Eddies 
Crawl 	Friction forces	 Longitudinal vibration
Sliding 	Friction forces	 Transverse vibration
Running 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Jumping 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Walking 	Gravitational forces	Rolling of a polygon (see figure 2.2) 

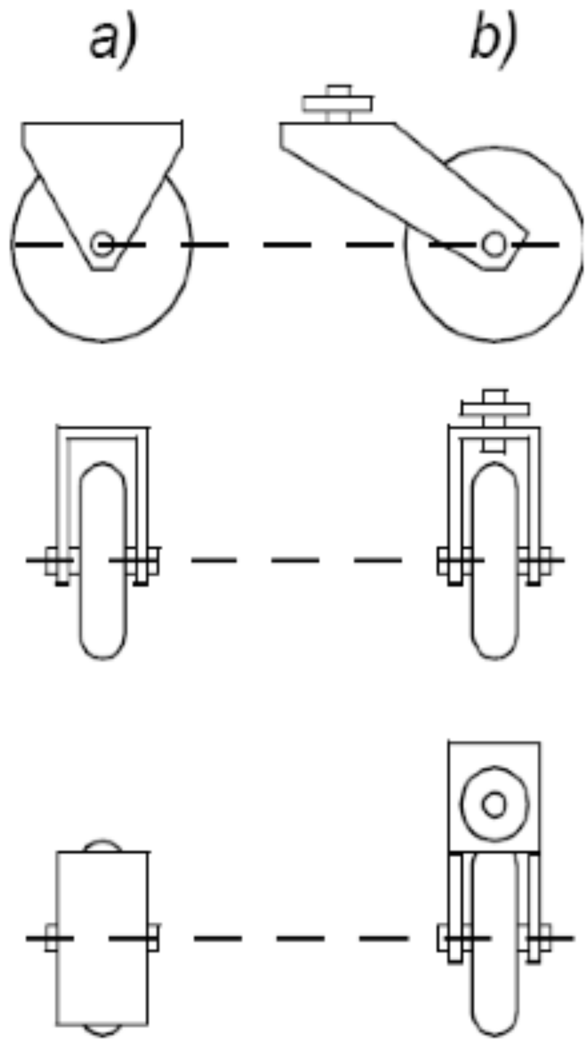
Mobile robot kinematics

- Depends on the type of robot

Position and type of the wheels

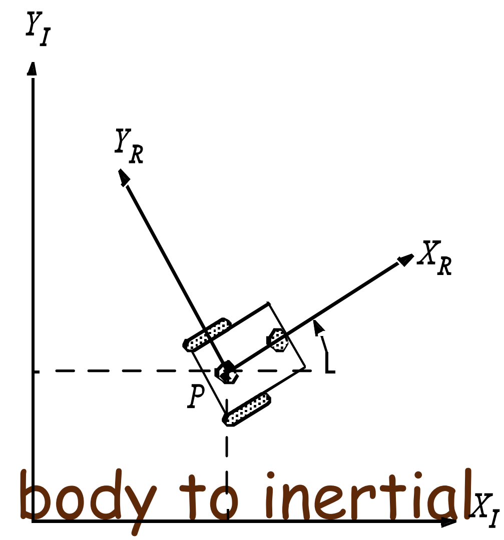
Two types of wheels

a) Standard – rotation around (motorized) wheel axel and the contact point
b) Castor wheel – rotation around wheel axes, contact point and castor axel
c) Swedish wheels
d) Ball wheels



Representing Mobile Robot Position

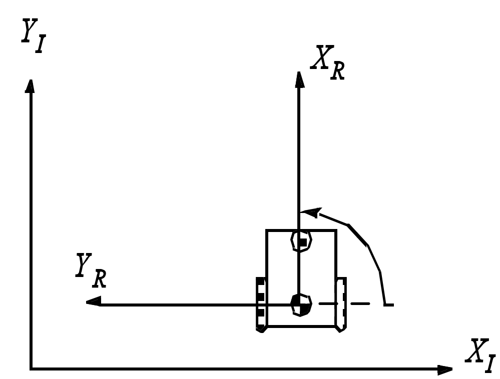
- Representing to robot within an arbitrary initial frame
  - Initial frame: $\{X_I, Y_I\}$
  - Robot frame: $\{X_R, Y_R\}$
  - Robot pose: $\xi_I = [x \quad y \quad \theta]^T$



  - Mapping between the two frames
  - transforms points/velocities from body to inertial frame

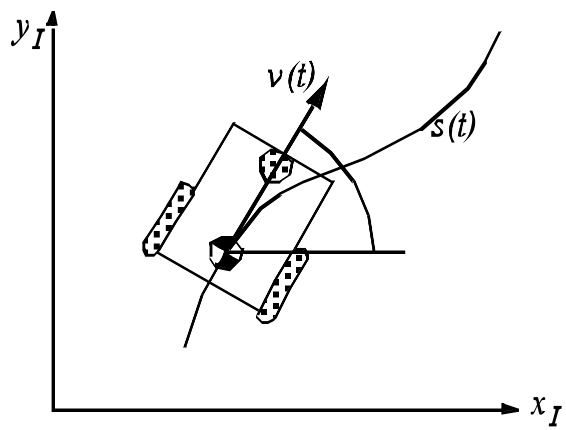
$$T(\theta, x, y) = \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix}$$

  - Example: Robot aligned with Y_I



Mobile robot kinematics

- Differential drive mobile robot
- Two wheels, with radius r, point P centered
- Between two wheels is the origin of the robot frame
- Distance between the wheels l



Mobile Robot Kinematic Models

- Manipulator case – given joint angles, we can always tell where the end effector is
- Mobile robot basis – given wheel positions we cannot tell where the robot is
- We have to remember the history how it got there
- Need to find relationship between velocities and changes in pose
- Presented on blackboard (see handout)
- How is the wheel velocity affecting velocity of the chassis

Differential Drive Kinematics

- Blackboard derivation
- Kinematics in the robot frame

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_R = \begin{bmatrix} \frac{v_l + v_r}{2} \\ 0 \\ \frac{v_r - v_l}{l} \end{bmatrix} = \begin{bmatrix} v \\ 0 \\ \omega \end{bmatrix}$$

- Relationship between robot frame and inertial frame

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}_I$$