

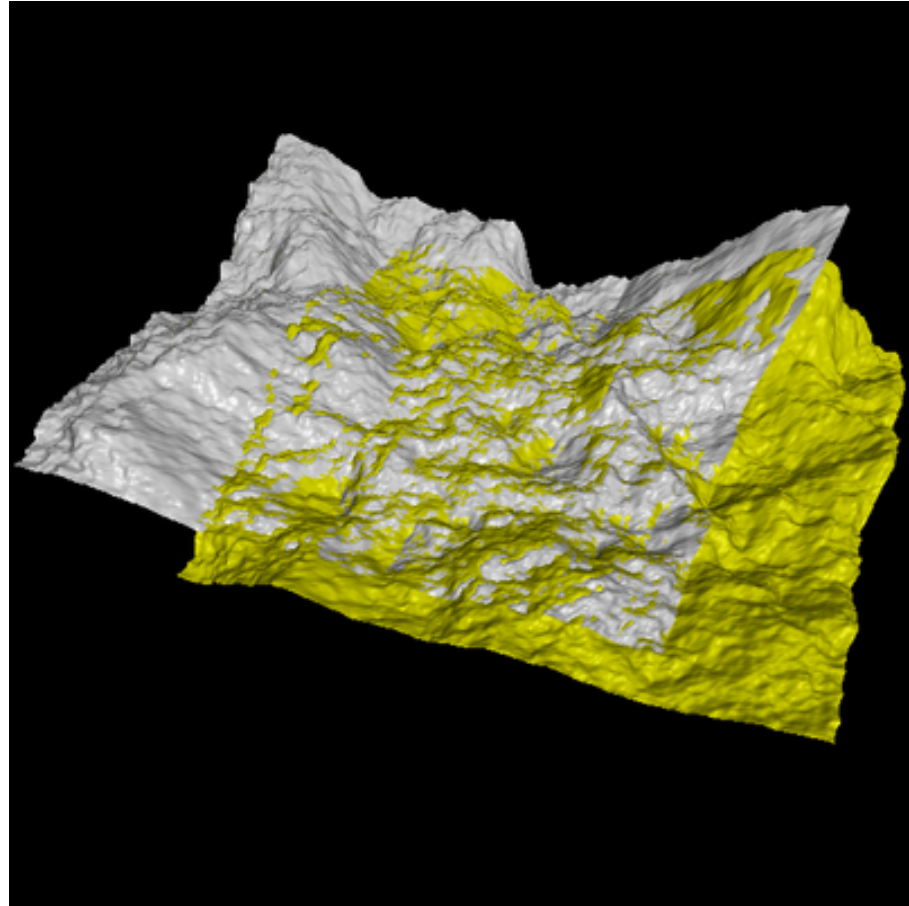
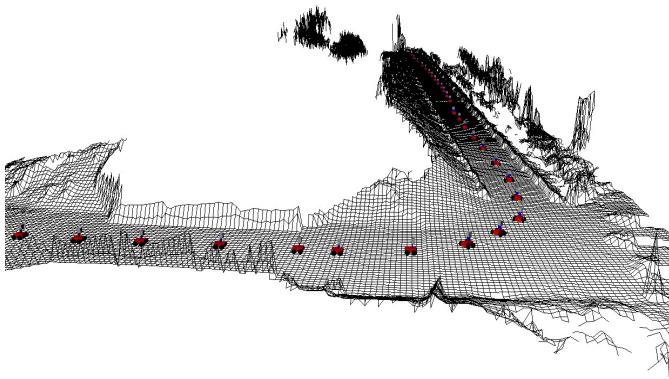
# Introduction to Mobile Robotics

## Iterative Closest Point Algorithm

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Slides adopted from: Wolfram Burgard, Cyrill Stachniss,  
Maren Bennewitz, Kai Arras and Probabilistic Robotics Book

# Motivation



# The Problem

- Given: two corresponding point sets:

$$X = \{x_1, \dots, x_n\}$$

$$P = \{p_1, \dots, p_n\}$$

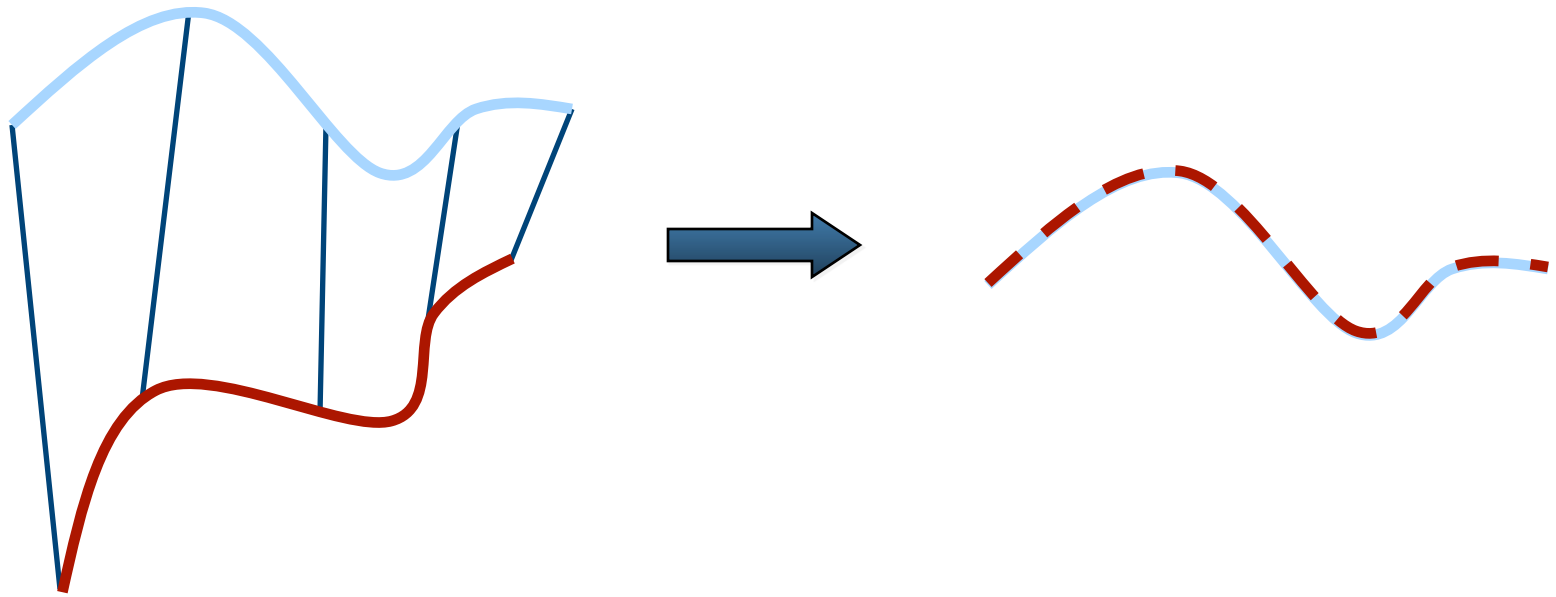
- Wanted: translation  $t$  and rotation  $R$  that minimizes the sum of the squared error:

$$E(R, t) = \frac{1}{N_p} \sum_{i=1}^{N_p} \|x_i - Rp_i - t\|^2$$

Where  $x_i$  and  $p_i$  are corresponding points.

# Key Idea

- If the correct correspondences are known, the correct relative rotation/translation can be calculated in closed form.



# Center of Mass

$$\mu_x = \frac{1}{N_x} \sum_{i=1}^{N_x} x_i \quad \text{and} \quad \mu_p = \frac{1}{N_p} \sum_{i=1}^{N_p} p_i$$

are the centers of mass of the two point sets.

## Idea:

- Subtract the corresponding center of mass from every point in the two point sets before calculating the transformation.
- The resulting point sets are:

$$X' = \{x_i - \mu_x\} = \{x'_i\}$$

and

$$P' = \{p_i - \mu_p\} = \{p'_i\}$$

# SVD

Let 
$$W = \sum_{i=1}^{N_p} x_i' p_i'^T$$

denote the singular value decomposition (SVD) of  $W$  by:

$$W = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V^T$$

where  $U, V \in \mathbb{R}^{3 \times 3}$  are unitary, and

$\sigma_1 \geq \sigma_2 \geq \sigma_3$  are the singular values of  $W$ .

# SVD

**Theorem** (without proof):

If  $\text{rank}(W) = 3$ , the optimal solution of  $E(R,t)$  is unique and is given by:

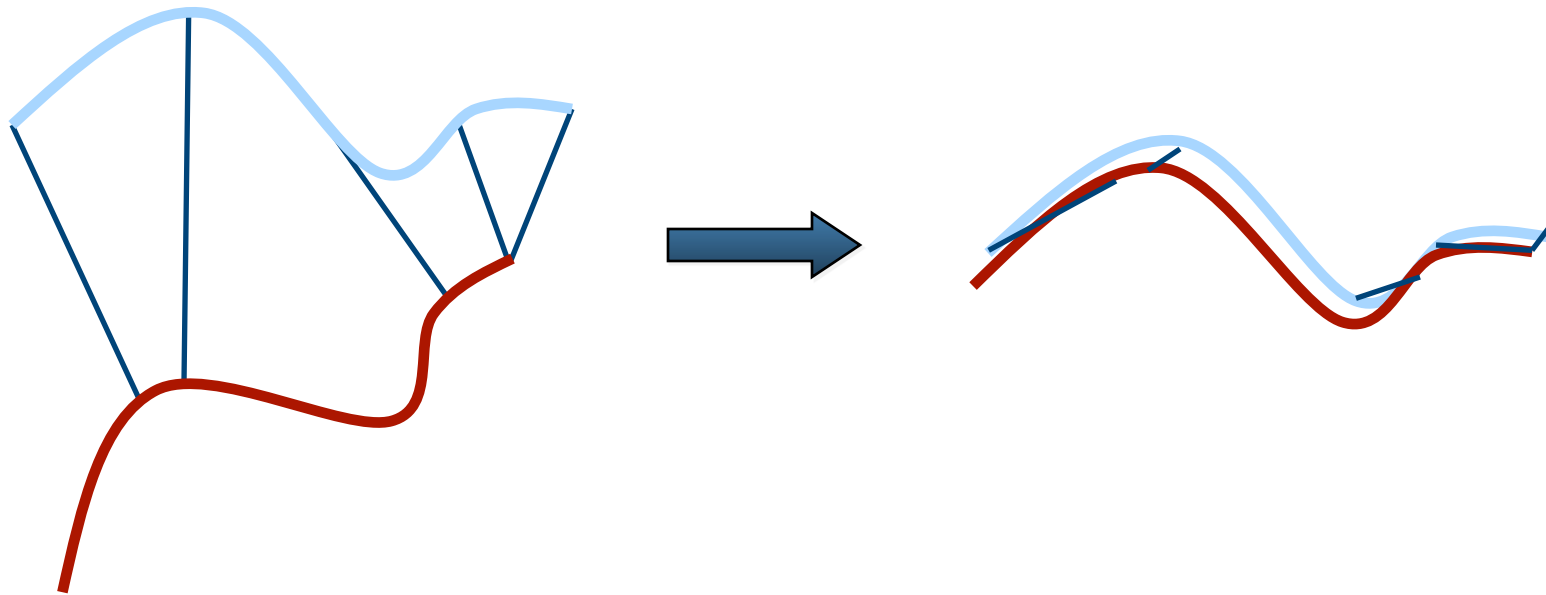
$$R = UV^T$$
$$t = \mu_x - R\mu_p$$

The minimal value of error function at  $(R,t)$  is:

$$E(R, t) = \sum_{i=1}^{N_p} (\|x'_i\|^2 + \|y'_i\|^2) - 2(\sigma_1 + \sigma_2 + \sigma_3)$$

## ICP with Unknown Data Association

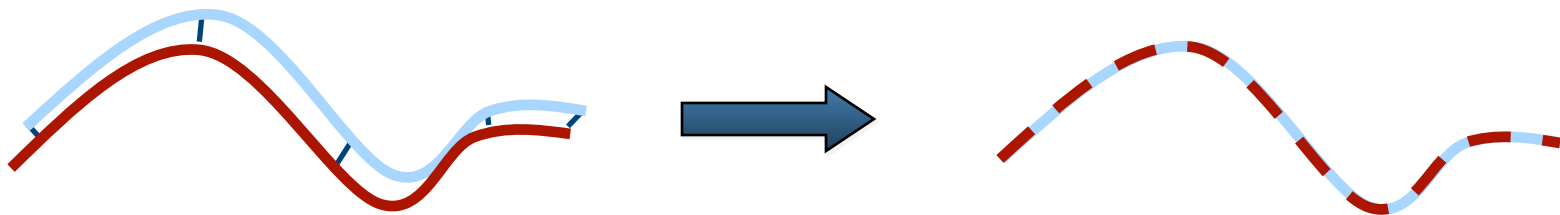
- If correct correspondences are not known, it is generally impossible to determine the optimal relative rotation/translation in one step



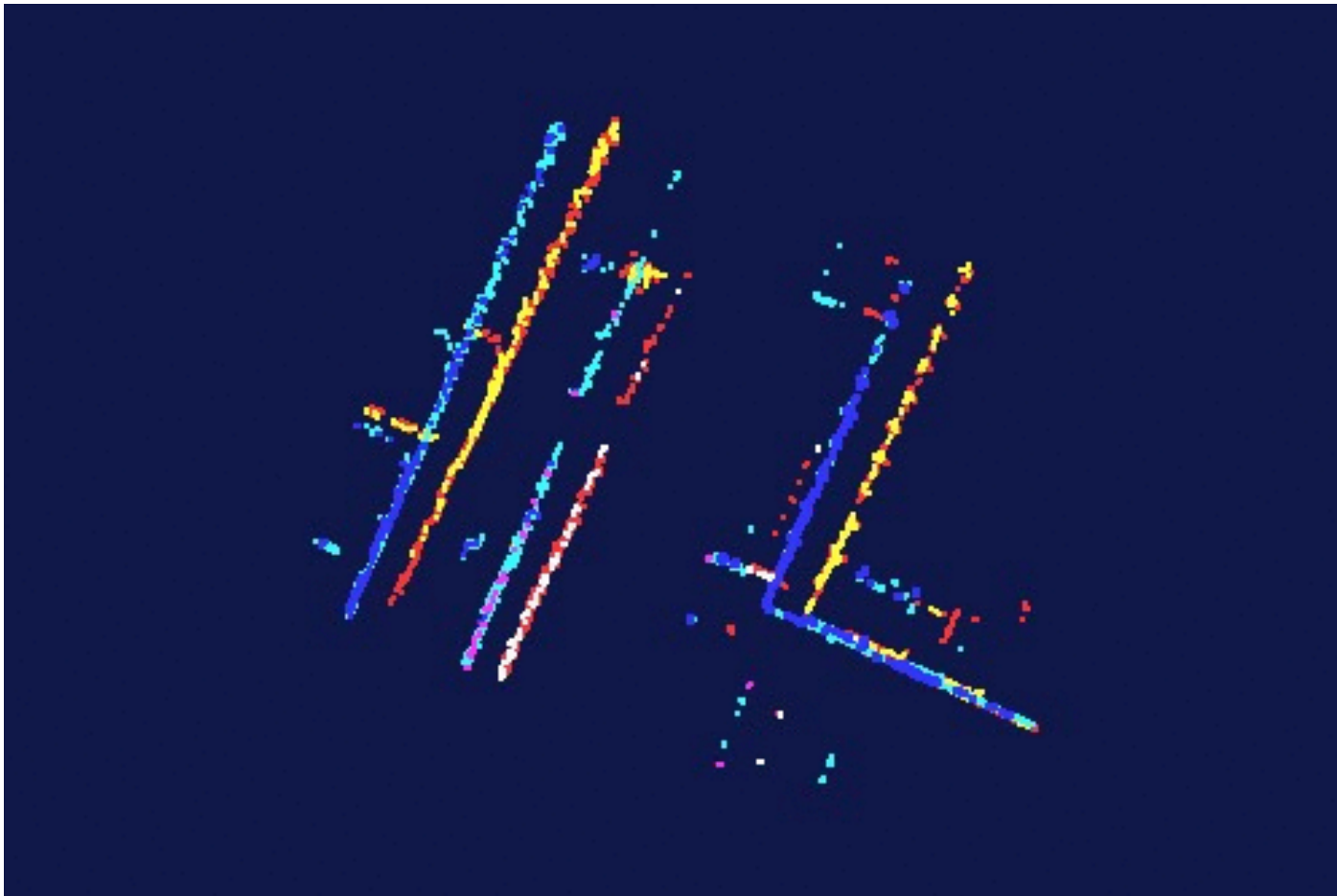


# ICP-Algorithm

- Idea: iterate to find alignment
- Iterated Closest Points (ICP)  
[Besl & McKay 92]
- Converges if starting positions are  
“close enough”



# Iteration-Example




# ICP-Variants

- Variants on the following stages of ICP have been proposed:
  1. Point subsets (from one or both point sets)
  2. Weighting the correspondences
  3. Data association
  4. Rejecting certain (outlier) point pairs

# Performance of Variants

- Various aspects of performance:
  - Speed
  - Stability (local minima)
  - Tolerance wrt. noise and/or outliers
  - Basin of convergence  
(maximum initial misalignment)
- Here: properties of these variants

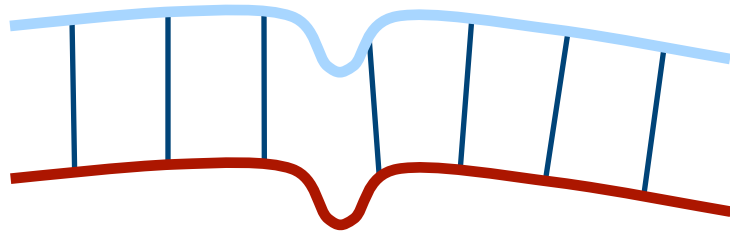
# ICP Variants

- 
1. Point subsets (from one or both point sets)
  2. Weighting the correspondences
  3. Data association
  4. Rejecting certain (outlier) point pairs

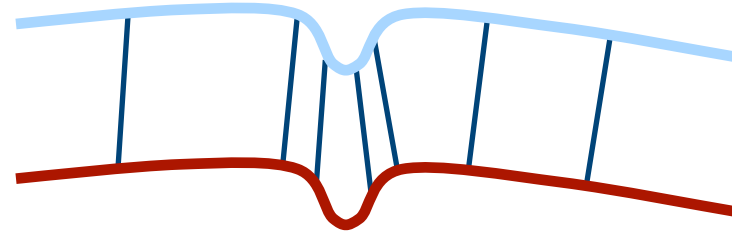
# Selecting Source Points

- Use all points
- Uniform sub-sampling
- Random sampling
- Feature based Sampling
- Normal-space sampling
  - Ensure that samples have normals distributed as uniformly as possible

# Normal-Space Sampling



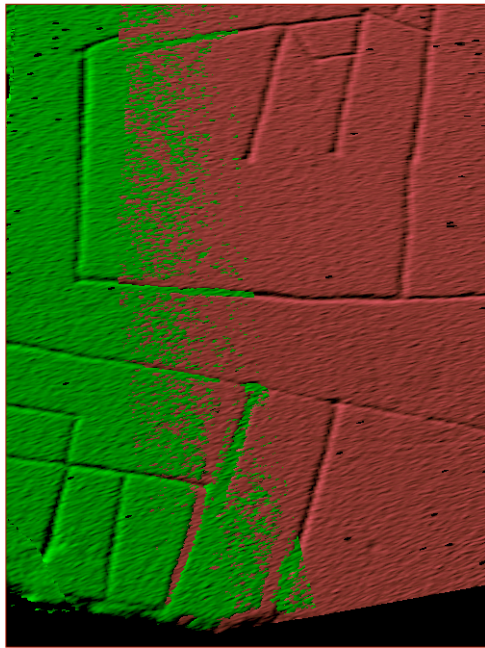
uniform sampling



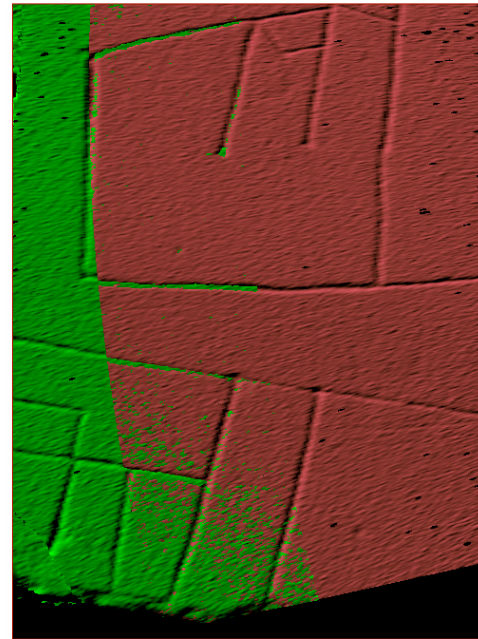
normal-space sampling

# Comparison

- Normal-space sampling better for mostly-smooth areas with sparse features [Rusinkiewicz et al.]



Random sampling

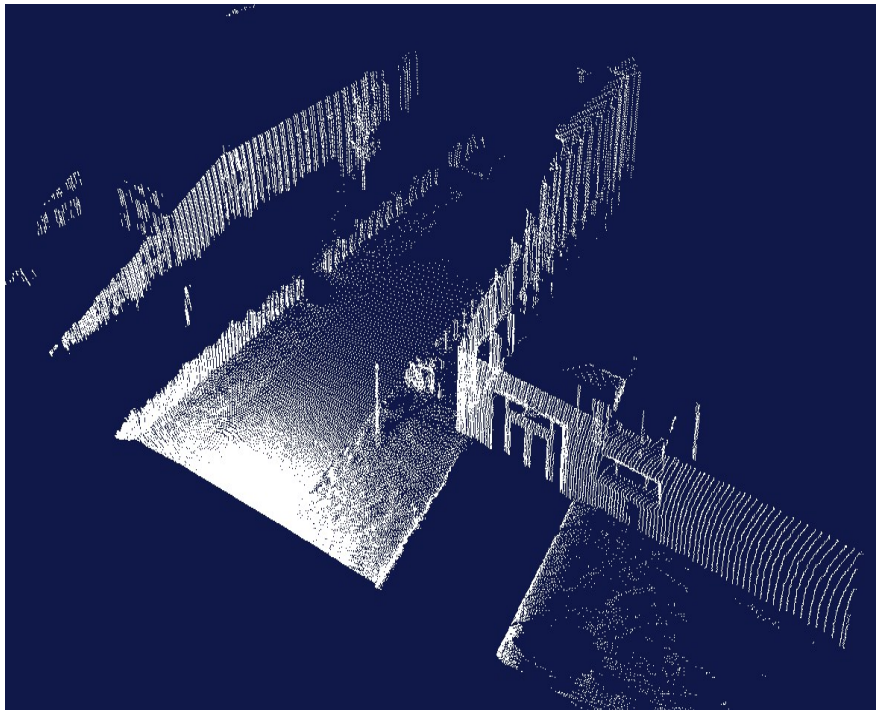


Normal-space sampling

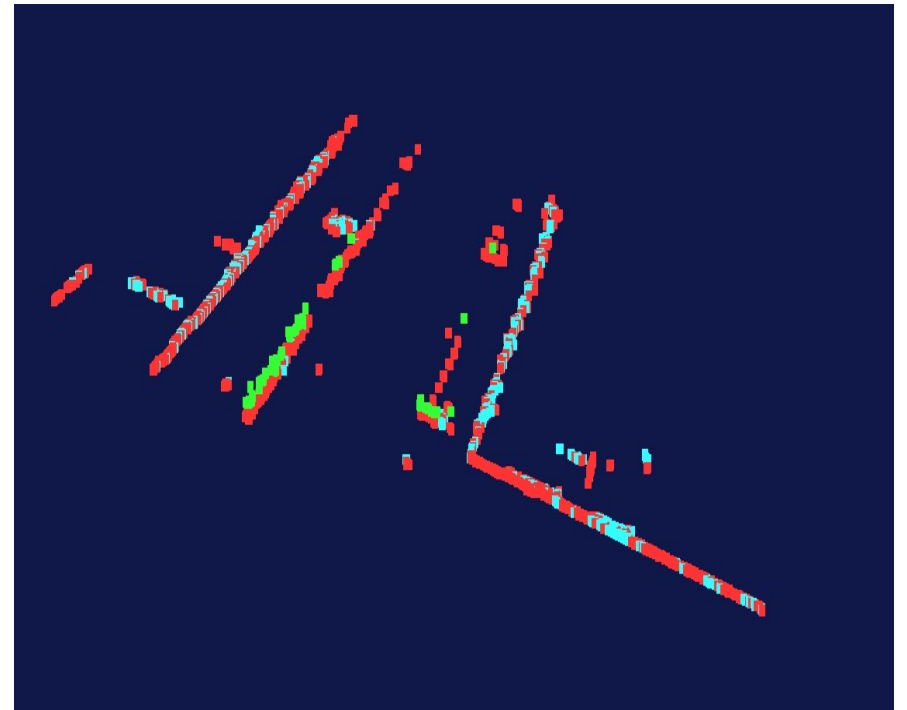


# Feature-Based Sampling

- try to find “important” points
- decrease the number of correspondences
- higher efficiency and higher accuracy
- requires preprocessing

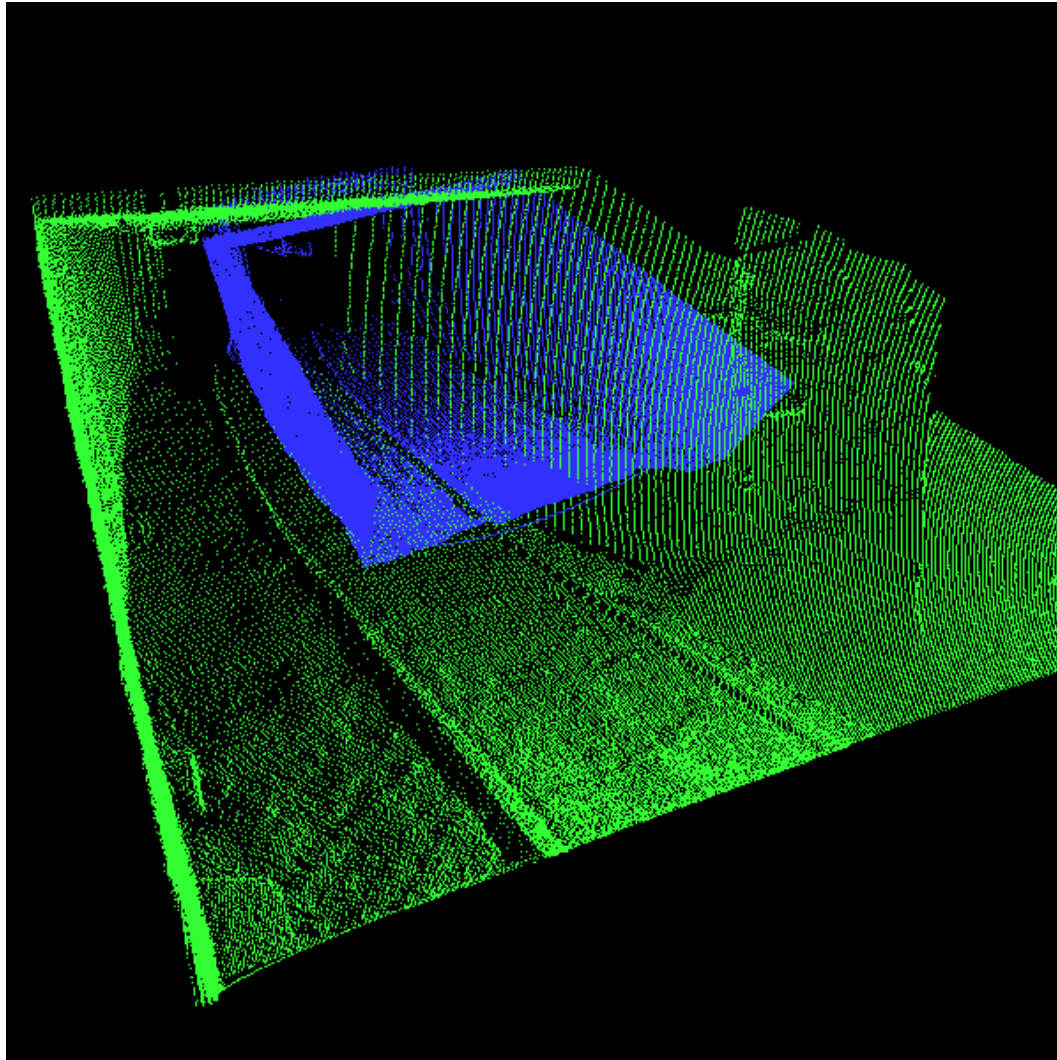


3D Scan (~200.000 Points)



Extracted Features (~5.000 Points)

# Application



[Nuechter et al., 04]

# ICP Variants

1. Point subsets (from one or both point sets)



2. **Weighting the correspondences**

3. Data association

4. Rejecting certain (outlier) point pairs

# Selection vs. Weighting

- Could achieve same effect with weighting
- Hard to guarantee that enough samples of important features except at high sampling rates
- Weighting strategies turned out to be dependent on the data.
- Preprocessing / run-time cost tradeoff (how to find the correct weights?)

# ICP Variants

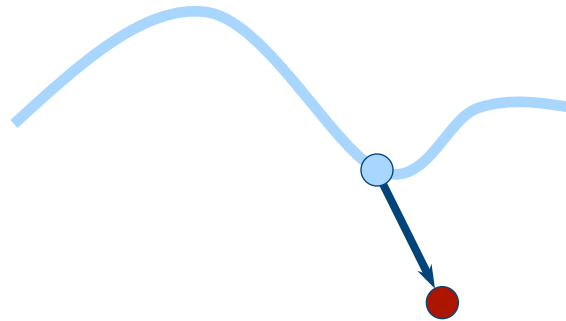
1. Point subsets (from one or both point sets)
2. Weighting the correspondences
- 3. **Data association**
4. Rejecting certain (outlier) point pairs

# Data Association

- has greatest effect on convergence and speed
- Closest point
- Normal shooting
- Closest compatible point
- Projection
- Using kd-trees or oc-trees

# Closest-Point Matching

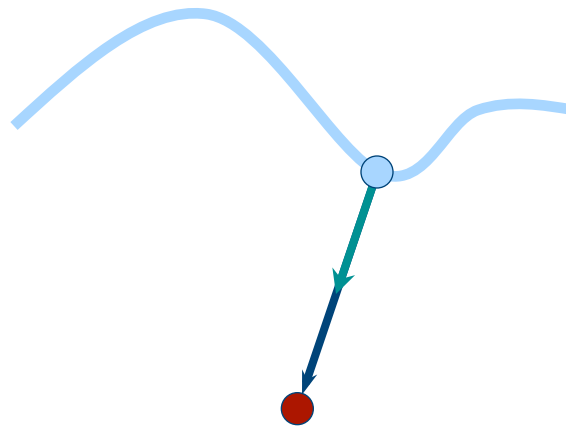
- Find closest point in other the point set



Closest-point matching generally stable,  
but slow and requires preprocessing

# Normal Shooting

- Project along normal, intersect other point set

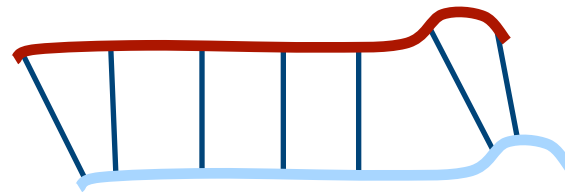
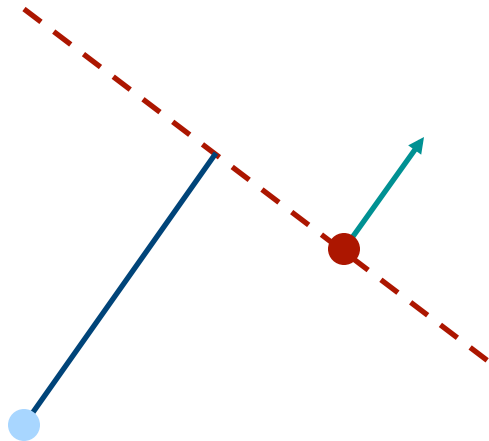


Slightly better than closest point for smooth structures, worse for noisy or complex structures



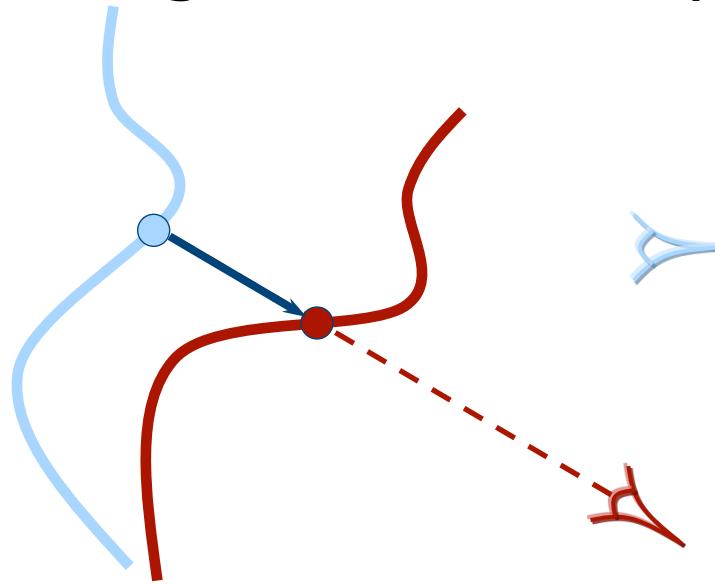
# Point-to-Plane Error Metric

- Using point-to-plane distance instead of point-to-point lets flat regions slide along each other [Chen & Medioni 91]



# Projection

- Finding the closest point is the most expensive stage of the ICP algorithm
- Idea: simplified nearest neighbor search
- For range images, one can project the points according to the view-point [Blais 95]



# Projection-Based Matching

- Slightly worse alignments per iteration
- Each iteration is one to two orders of magnitude faster than closest-point
- Requires point-to-plane error metric

# Closest Compatible Point

- Improves the previous two variants by considering the **compatibility** of the points
- Compatibility can be based on normals, colors, etc.
- In the limit, degenerates to feature matching

# ICP Variants

1. Point subsets (from one or both point sets)
2. Weighting the correspondences
3. Nearest neighbor search
4. **Rejecting certain (outlier) point pairs**



# Rejecting (outlier) point pairs

- sorting all correspondences with respect to their error and deleting the worst  $t\%$ , Trimmed ICP (TrICP) [Chetverikov et al. 2002]
- $t$  is to Estimate with respect to the Overlap



**Problem:** Knowledge about the overlap is necessary or has to be estimated

# ICP-Summary

- ICP is a powerful algorithm for calculating the displacement between scans.
- The major problem is to determine the correct data associations.
- Given the correct data associations, the transformation can be computed efficiently using SVD.