Introduction to Mobile Robotics

Mapping with Known Poses

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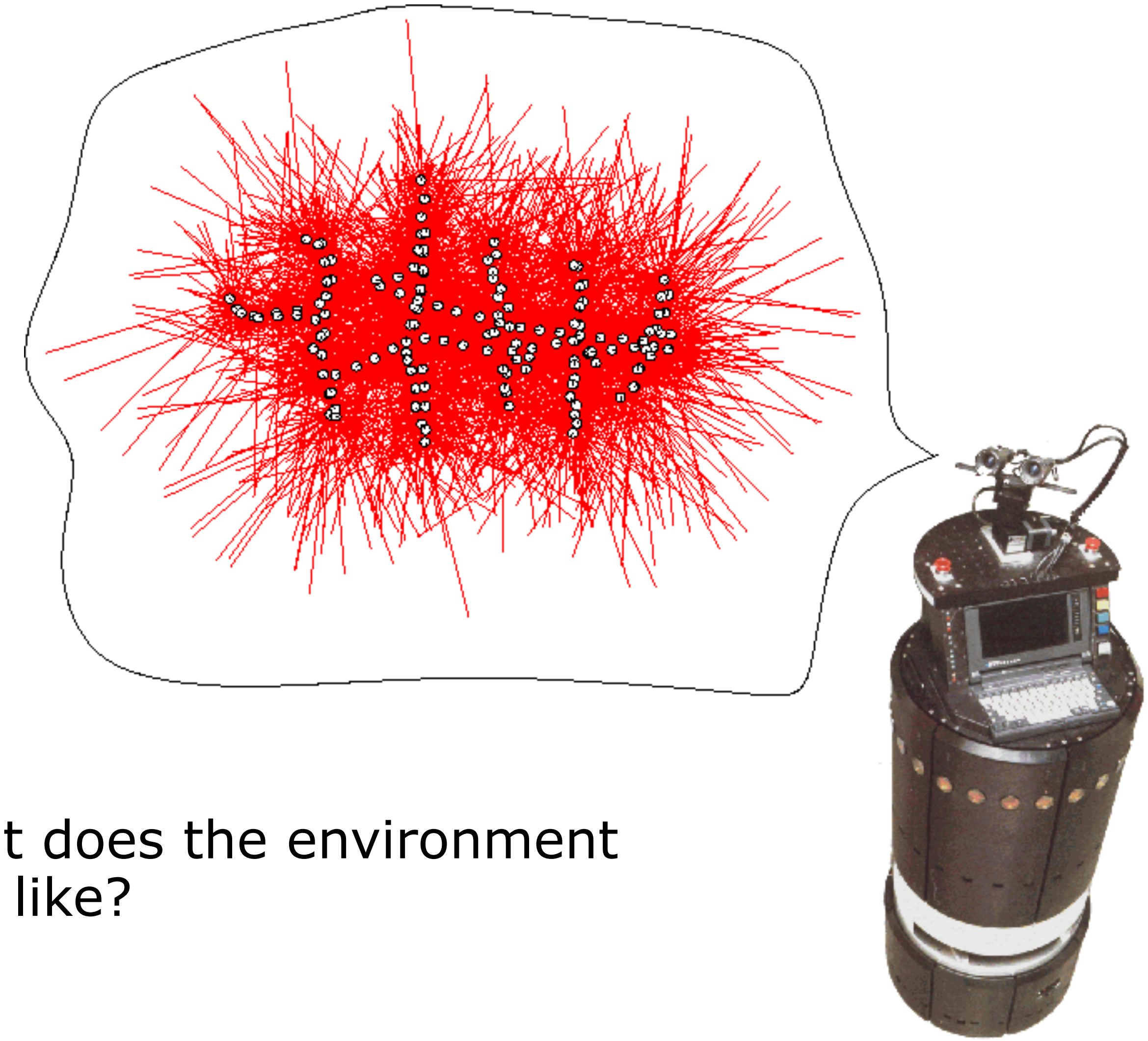
Why Mapping?

- Learning maps is one of the fundamental problems in mobile robotics
- Maps allow robots to efficiently carry out their tasks, allow localization …
- Successful robot systems rely on maps for localization, path planning, activity planning etc.

The General Problem of Mapping



What does the environment look like?

The General Problem of Mapping

- Formally, mapping involves, given the sensor data, and
- Motion data

$$d = \{u_1, z_1, u_2, z_2, \ldots, u_n, z_n\}$$

to calculate the most likely map

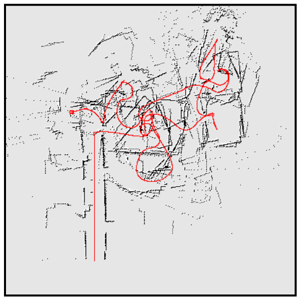
$$m^* = \arg\max_m P(m \mid d)$$

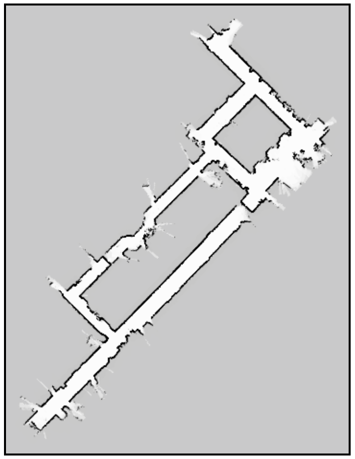
Mapping as a Chicken and Egg Problem

- So far we learned how to estimate the pose of the vehicle given the data and the map.
- Mapping, however, involves to simultaneously estimate the pose of the vehicle and the map.
- The general problem is therefore denoted as the simultaneous localization and mapping problem (SLAM).
- Throughout this section we will describe how to calculate a map given we know the pose of the vehicle.

Types of SLAM-Problems

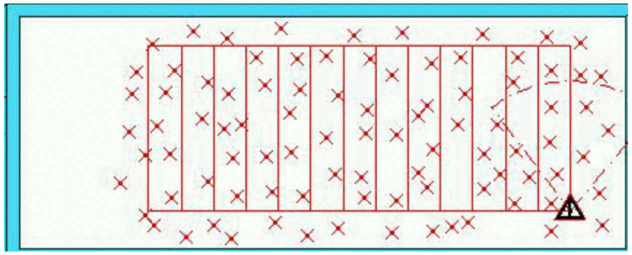
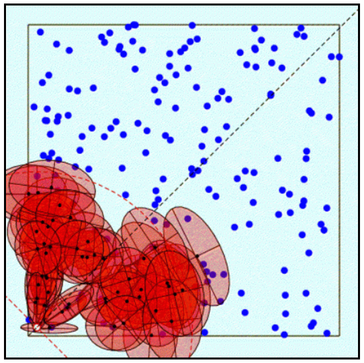
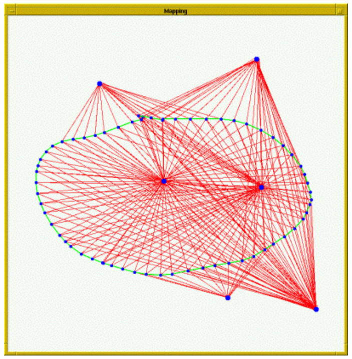
- Grid maps or scans





[Lu & Milios, 97; Gutmann, 98: Thrun 98; Burgard, 99; Konolige & Gutmann, 00; Thrun, 00; Arras, 99; Haehnel, 01;…]

- Landmark-based

[Leonard et al., 98; Castelanos et al., 99: Dissanayake et al., 2001; Montemerlo et al., 2002;…

Problems in Mapping

- Sensor interpretation
  - How do we extract relevant information from raw sensor data?
  - How do we represent and integrate this information over time?
- Robot locations have to be estimated
  - How can we identify that we are at a previously visited place?
  - This problem is the so-called data association problem.

Occupancy Grid Maps

- Introduced by Moravec and Elfes in 1985
- Represent environment by a grid.
- Estimate the probability that a location is occupied by an obstacle.
- Key assumptions
  - Occupancy of individual cells (m[xy]) is independent

$$Bel(m_t) = P(m_t \mid u_1, z_2 \ldots, u_{t-1}, z_t) = \prod_{x,y} Bel(m_t^{[xy]})$$

  - Robot positions are known!

Updating Occupancy Grid Maps

- Idea: Update each individual cell using a binary Bayes filter.

$$Bel(m_t^{[xy]}) = \eta \; p(z_t \mid m_t^{[xy]}) \int p(m_t^{[xy]} \mid m_{t-1}^{[xy]}, u_{t-1}) Bel(m_{t-1}^{[xy]}) \, dm_{t-1}^{[xy]}$$

- Additional assumption: Map is static (drop the time index on m)

$$Bel(m_t^{[xy]}) = \eta \; p(z_t \mid m_t^{[xy]}) Bel(m_{t-1}^{[xy]})$$

- Represent belief as

log odds - computationally elegant-

varies from –infinity to infinity no need to do truncation

$$\overline{B}\left(m_t^{[xy]}\right) := \log odds(m_t^{[xy]})$$

$$odds(x) := \left(\frac{P(x)}{1 - P(x)}\right)$$

Updating Occupancy Grid Maps

- Update the map cells using the inverse sensor model $p(x \mid z_t)$ - specifies distribution over x as a function of measurements (simple state complex measurement)

- Or use the log-odds representation

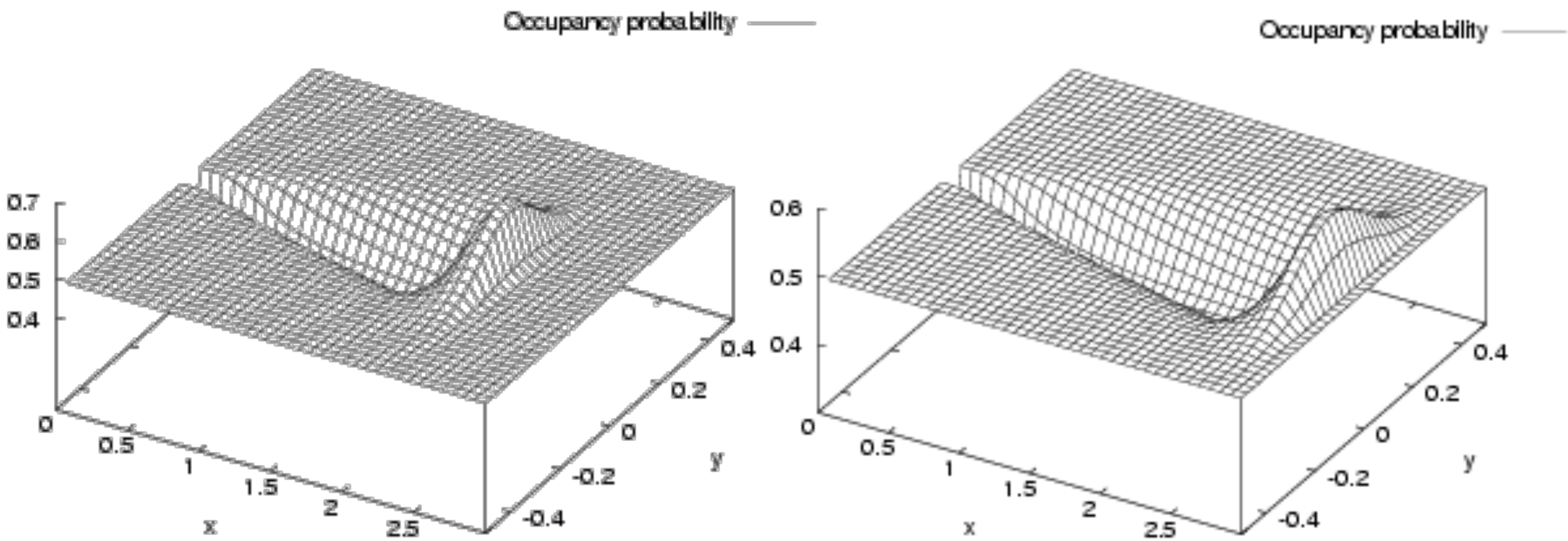
$$
\begin{aligned}
\bar{B}\left(m_t^{[xy]}\right) = \log odds\left(m_t^{[xy]} \mid z_t, u_{t-1}\right) \\
- \log odds\left(m_t^{[xy]}\right) \\
+ \bar{B}\left(m_{t-1}^{[xy]}\right)
\end{aligned}
$$

$$\bar{B}\left(m_t^{[xy]}\right) := \log odds(m_t^{[xy]})$$

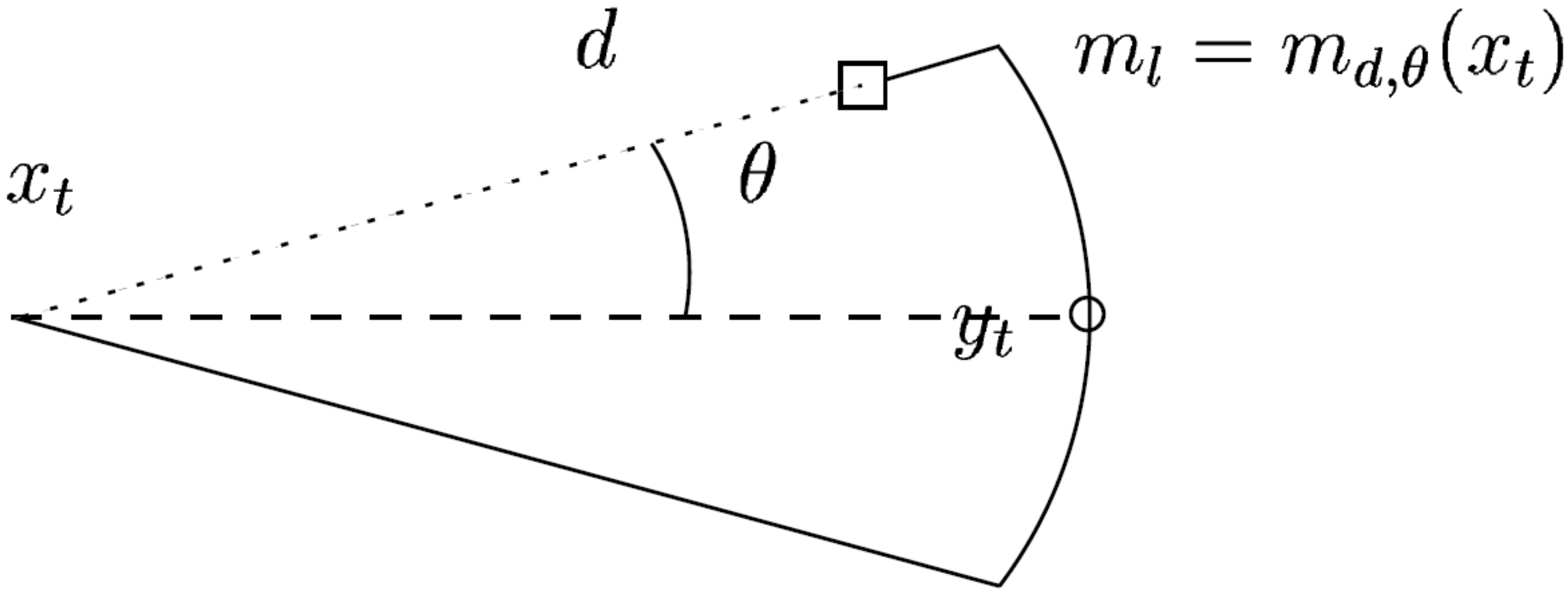
$$odds(x) := \left(\frac{P(x)}{1 - P(x)}\right)$$

Typical Sensor Model for Occupancy Grid Maps

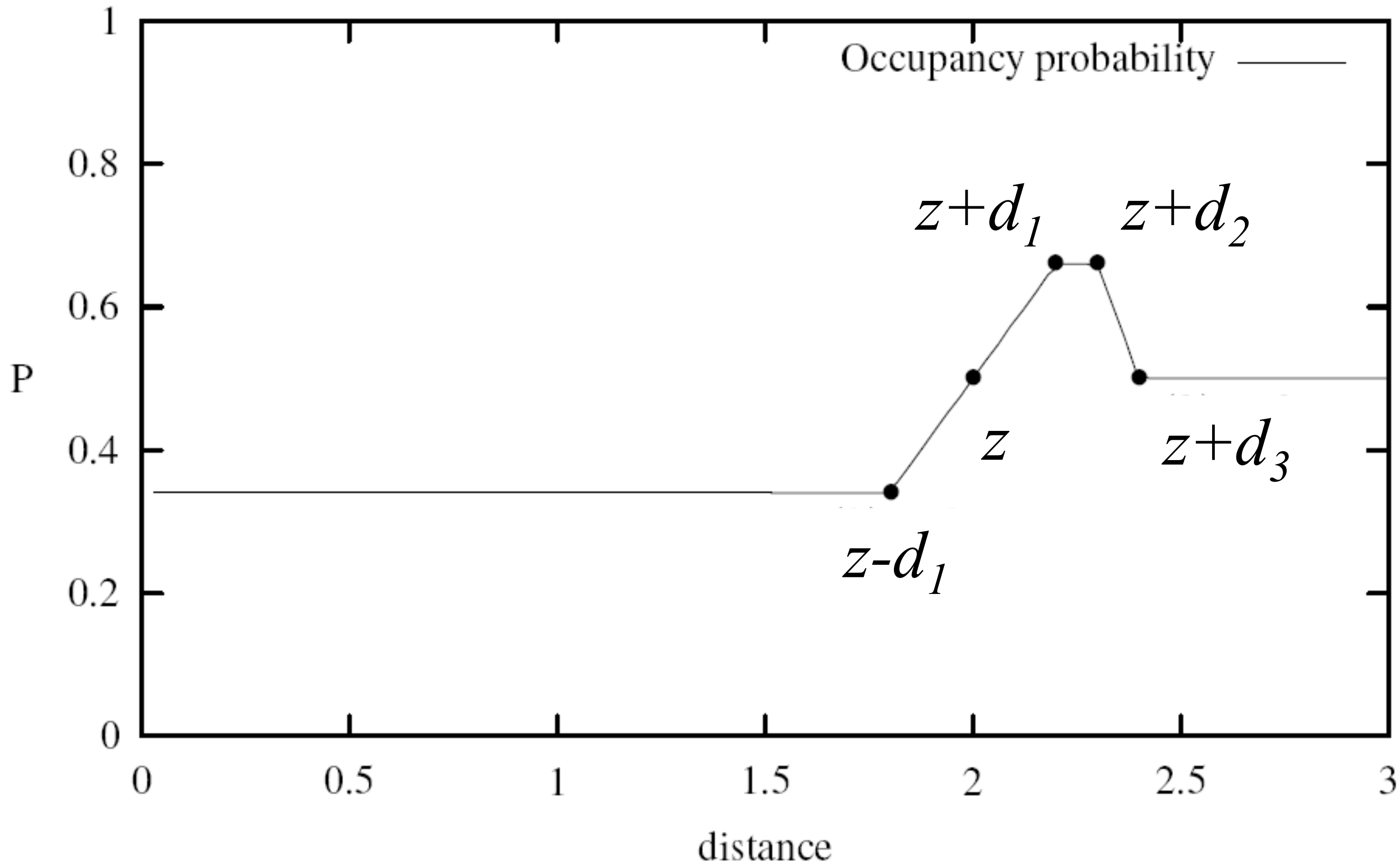
Combination of a linear function and a Gaussian:



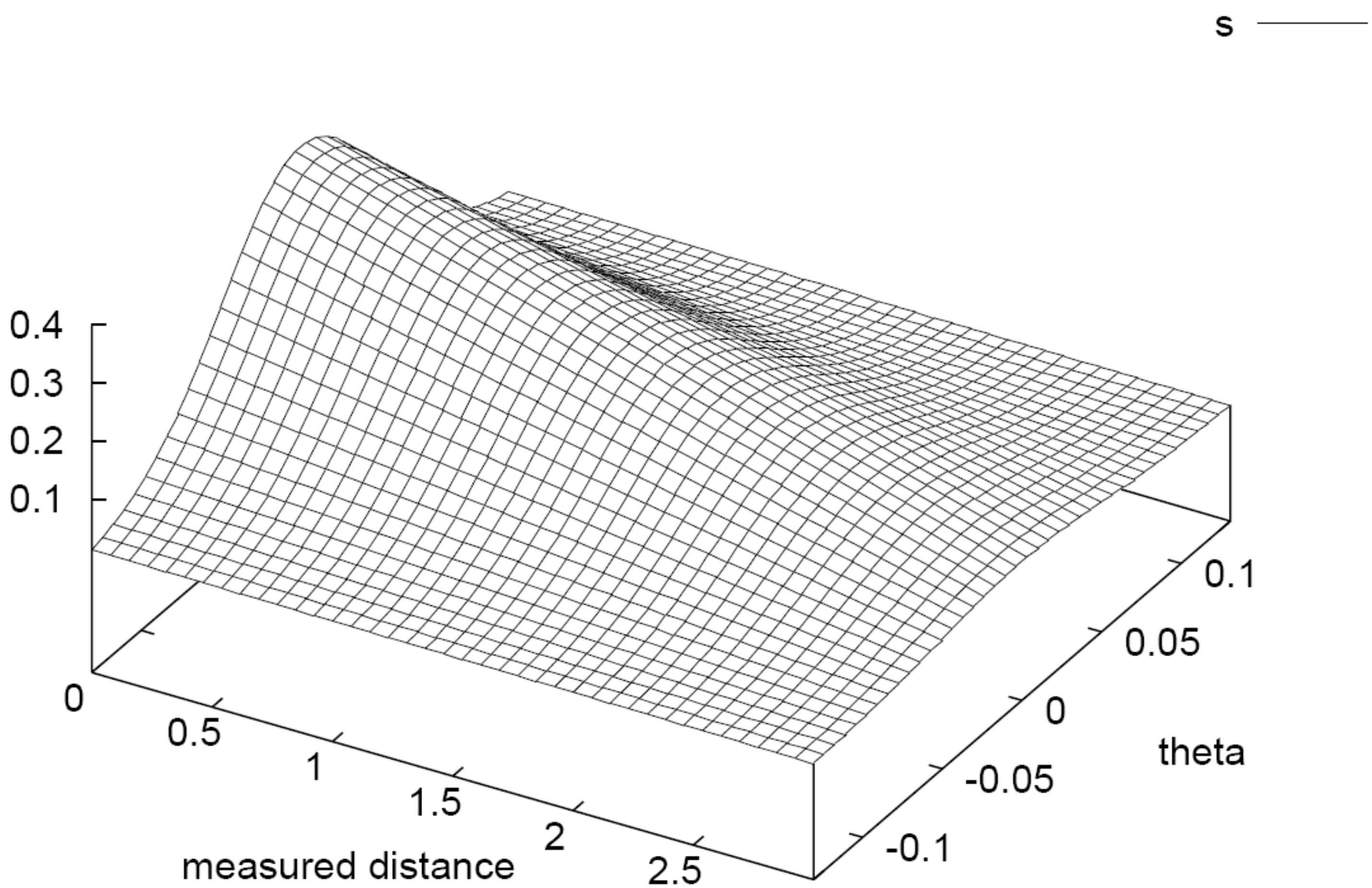
Key Parameters of the Model



Occupancy Value Depending on the Measured Distance



Deviation from the Prior Belief
(the sphere of influence of the sensors)

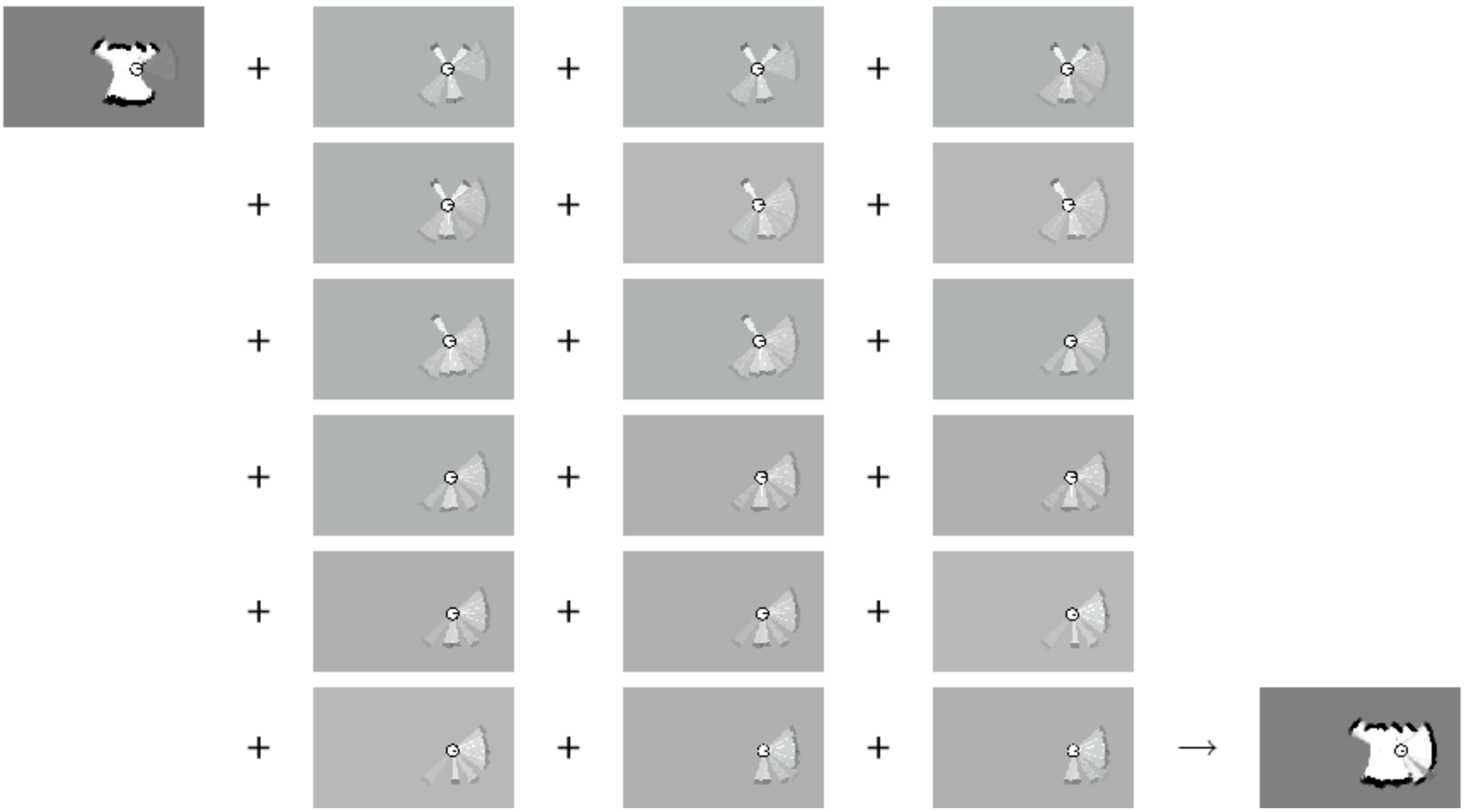


Calculating the Occupancy Probability Based on Single Observations

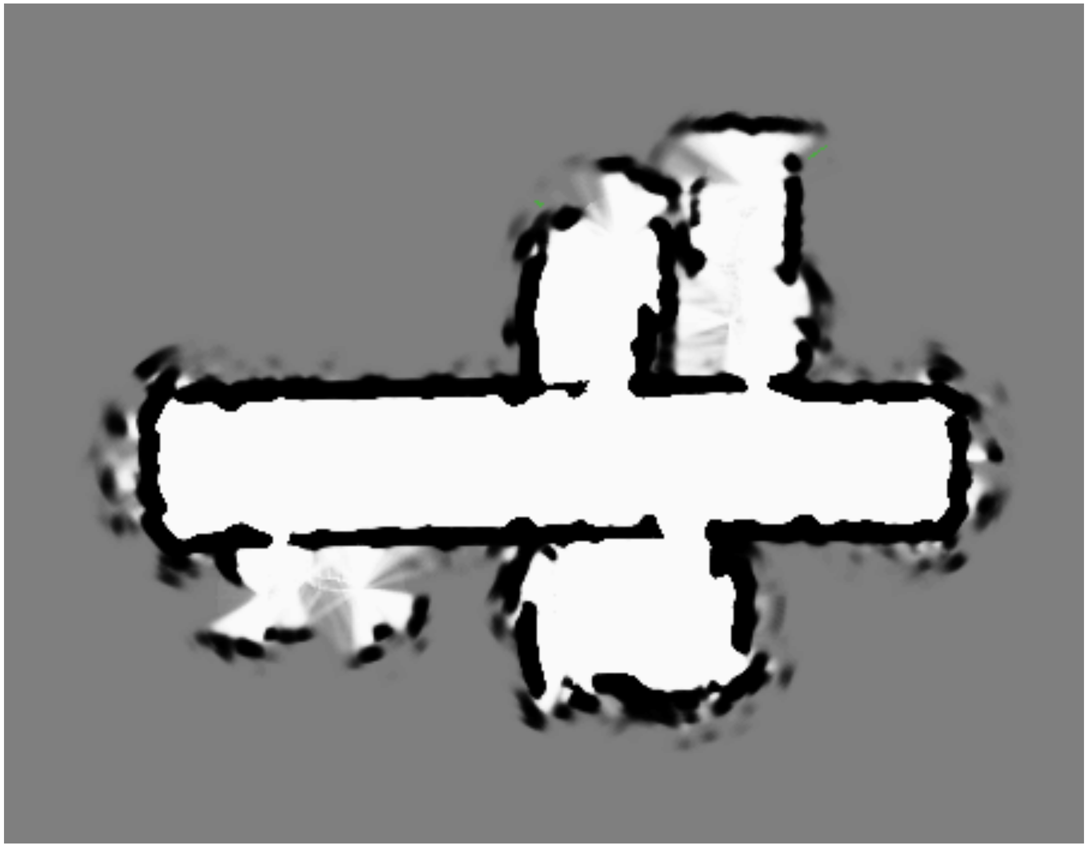
$$P(m_{d,\theta}(x(k)) \mid y(k), x(k)) = P(m_{d,\theta}(x(k)))$$
$$+ \begin{cases} -s(y(k),\theta) & d < y(k) - d_1 \\ -s(y(k),\theta) + \frac{s(y(k),\theta)}{d_1}\,(d - y(k) + d_1) & d < y(k) + d_1 \\ s(y(k),\theta) & d < y(k) + d_2 \\ s(y(k),\theta) - \frac{s(y(k),\theta)}{d_3 - d_2}\,(d - y(k) - d_2) & d < y(k) + d_3 \\ 0 & \text{otherwise.} \end{cases}$$

Given the measurement y, what is the probability of cell at d, theta to be updated

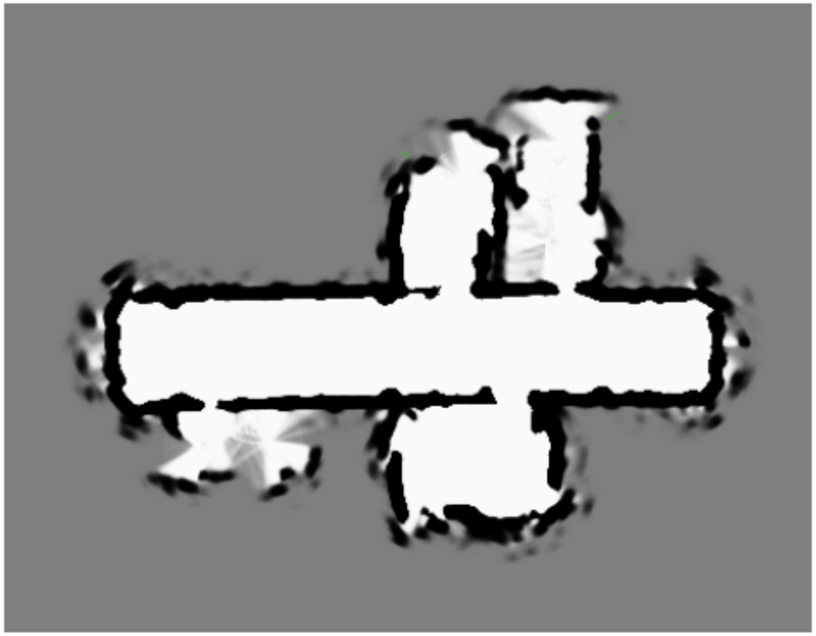
Incremental Updating of Occupancy Grids (Example)

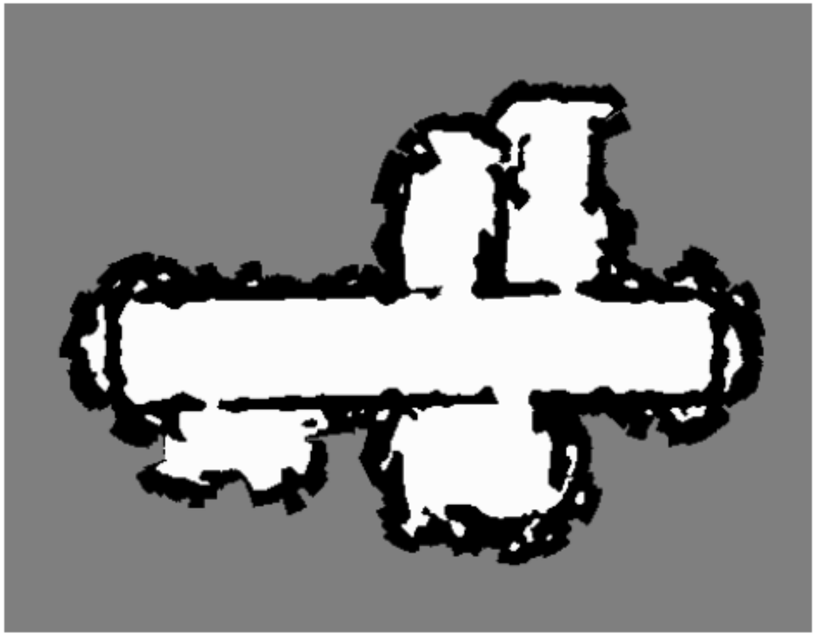


Resulting Map Obtained with Ultrasound Sensors



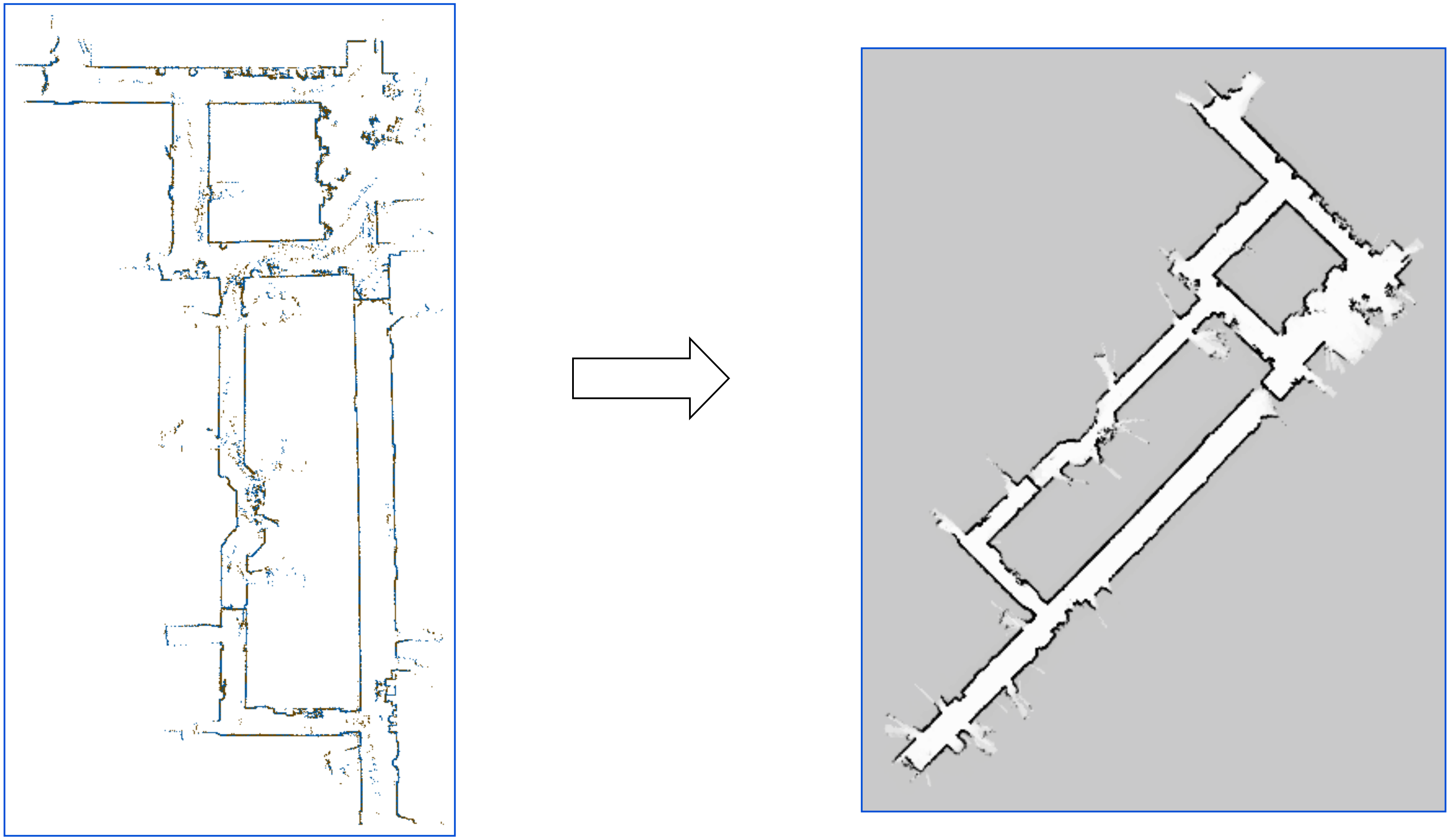
Resulting Occupancy and Maximum Likelihood Map



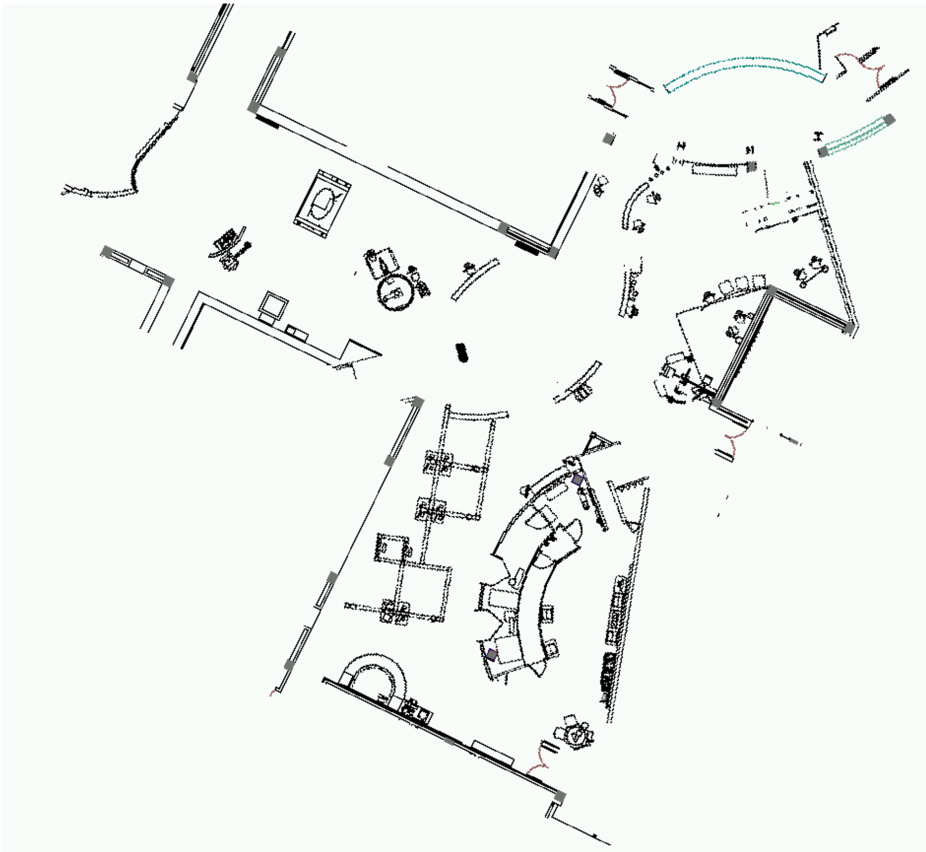


The maximum likelihood map is obtained by clipping the occupancy grid map at a threshold of 0.5

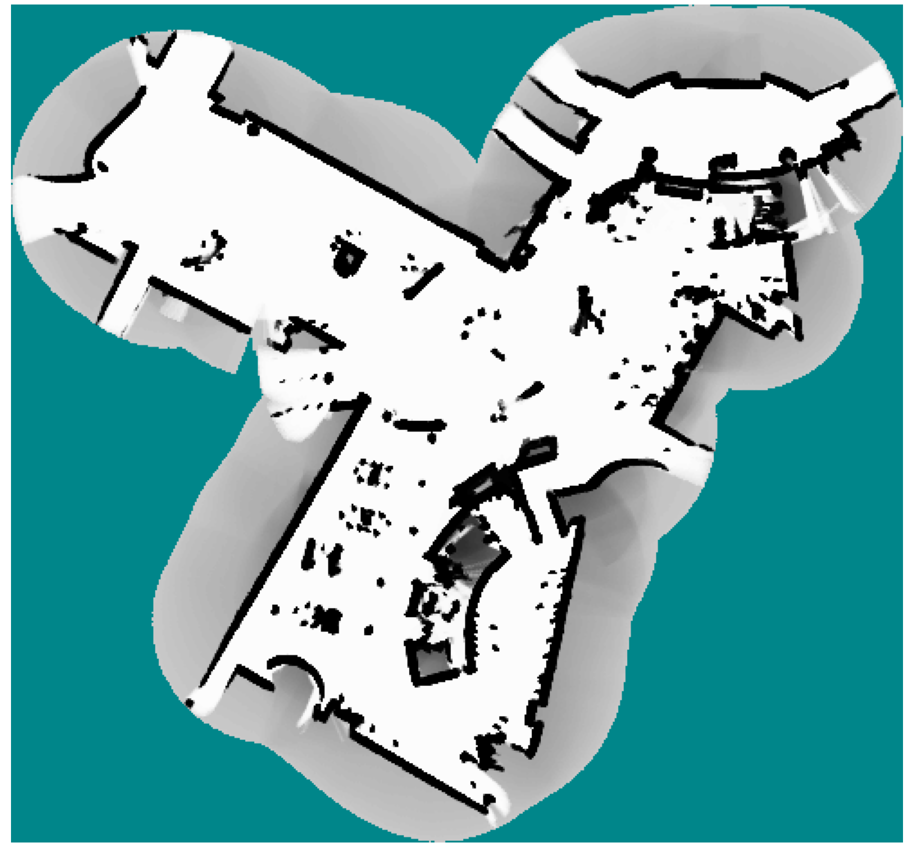
Occupancy Grids: From scans to maps



Tech Museum, San Jose



CAD map



occupancy grid map

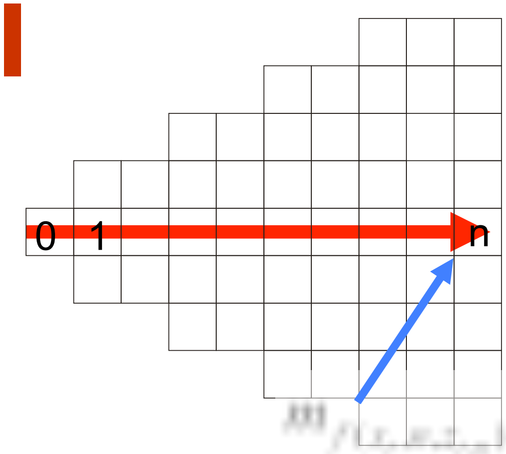
Alternative: Simple Counting

- For every cell count
  - hits(x,y): number of cases where a beam ended at <x,y>
  - misses(x,y): number of cases where a beam passed through <x,y>

$$Bel(m^{[xy]}) = \frac{\text{hits}(x,y)}{\text{hits}(x,y) + \text{misses}(x,y)}$$

- Value of interest: $P(reflects(x,y))$

The Measurement Model



1. pose at time *t*: x_t
2. beam *n* of scan *t*: $z_{t,n}$
3. maximum range reading: $\varsigma_{t,n} = 1$
4. beam reflected by an object: $\varsigma_{t,n} = 0$

$$p(z_{t,n} \mid x_t, m) = \begin{cases} \prod_{k=0}^{z_{t,n}-1} (1 - m_{f(x_t,n,k)}) & \text{if } \varsigma_{t,n} = 1 \\ m_{f(x_t,n,z_{t,n})} \prod_{k=0}^{z_{t,n}-1} (1 - m_{f(x_t,n,k)}) & \text{if } \varsigma_{t,n} = 0 \end{cases}$$

Computing the Most Likely Map

- Compute values for m that maximize

$$m^* = \underset{m}{\arg\max}\, P(m \mid z_1, \ldots, z_t, x_1, \ldots, x_t)$$

- Assuming a uniform prior probability for p(m), this is equivalent to maximizing (applic. of Bayes rule)

$$m^* = \underset{m}{\arg\max}\, P(z_1, \ldots, z_t \mid m, x_1, \ldots, x_t)$$

$$= \underset{m}{\arg\max} \prod_{t=1}^{T} P(z_t \mid m, x_t)$$

$$= \underset{m}{\arg\max} \sum_{t=1}^{T} \ln P(z_t \mid m, x_t)$$

Computing the Most Likely Map

$$m^* = \arg\max_m \left[\sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{n=1}^{N} \Big(I(f(x_t, n, z_{t,n}) = j) \cdot (1 - \varsigma_{t,n}) \cdot \ln m_j \right.$$

$$\left. + \sum_{k=0}^{z_{t,n}-1} I(f(x_t, n, k) = j) \cdot \ln(1 - m_j) \Big) \right]$$

Suppose

$$\alpha_j = \sum_{t=1}^{T} \sum_{n=1}^{N} I(f(x_t, n, z_{t,n}) = j) \cdot (1 - \varsigma_{t,n})$$

$$\beta_j = \sum_{t=1}^{T} \sum_{n=1}^{N} \left[\sum_{k=0}^{z_{t,n}-1} I(f(x_t, n, k) = j) \right]$$

Meaning of α_j and β_j

$$\alpha_j = \sum_{t=1}^{T} \sum_{n=1}^{N} I(f(x_t, n, z_{t,n}) = j) \cdot (1 - \varsigma_{t,n})$$

corresponds to the number of times a beam that is not a maximum range beam ended in cell j (*hits(j)*)

$$\beta_j = \sum_{t=1}^{T} \sum_{n=1}^{N} \left[\sum_{k=0}^{z_{t,n}-1} I(f(x_t, n, k) = j) \right]$$

corresponds to the umber of times a beam intercepted cell j without ending in it (*misses(j)*).

Computing the Most Likely Map

We assume that all cells m_j are independent:

$$m^* = \underset{m}{\arg\max} \left(\sum_{j=1}^{J} \alpha_j \ln m_j + \beta_j \ln(1 - m_j) \right)$$

If we set

$$\frac{\partial m}{\partial m_j} = \frac{\alpha_j}{m_j} - \frac{\beta_j}{1 - m_j} = 0$$

we obtain

$$m_j = \frac{\alpha_j}{\alpha_j + \beta_j}$$

Computing the most likely map amounts to counting how often a cell has reflected a measurement and how often it was intercepted.

Difference between Occupancy Grid Maps and Counting

- The counting model determines how often a cell reflects a beam.
- The occupancy model represents whether or not a cell is occupied by an object.
- Although a cell might be occupied by an object, the reflection probability of this object might be very small.

Example Occupancy Map

Example Reflection Map

Example

- Out of 1000 beams only 60% are reflected from a cell and 40% intercept it without ending in it.
- Accordingly, the reflection probability will be 0.6.
- Suppose *p(occ | z)* = 0.55 when a beam ends in a cell and *p (occ | z)* = 0.45 when a cell is intercepted by a beam that does not end in it.
- Accordingly, after n measurements we will have

$$\left(\frac{0.55}{0.45}\right)^{n*0.6} * \left(\frac{0.45}{0.55}\right)^{n*0.4} = \left(\frac{11}{9}\right)^{n*0.6} * \left(\frac{11}{9}\right)^{-n*0.4} = \left(\frac{11}{9}\right)^{n*0.2}$$

- Whereas the reflection map yields a value of 0.6, the occupancy grid value converges to 1.

Summary

- Occupancy grid maps are a popular approach to represent the environment of a mobile robot given known poses.
- In this approach each cell is considered independently from all others.
- It stores the posterior probability that the corresponding area in the environment is occupied.
- Occupancy grid maps can be learned efficiently using a probabilistic approach.
- Reflection maps are an alternative representation.
- They store in each cell the probability that a beam is reflected by this cell.
- We provided a sensor model for computing the likelihood of measurements and showed that the counting procedure underlying reflection maps yield the optimal map.