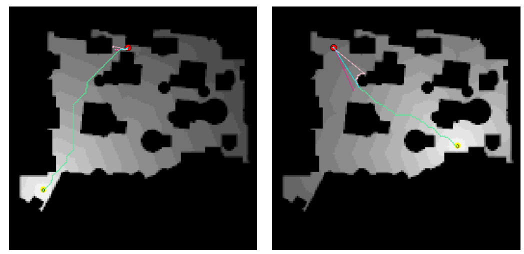
Markov Decisions Processes
Partially observable Markov decision Processes

Value Iteration for Motion Planning



Value Function and Policy Iteration

- Often the optimal policy has been reached long before the value function has converged.
- Policy iteration calculates a new policy based on the current value function and then calculates a new value function based on this policy.
- This process often converges faster to the optimal policy.

Previously

- Variations of MDP's
- Continuous state MDP's – discrete set of actions – Value function approximation
- Today – reinforcement learning
- finite horizon MDP's
- LQR – continuous linear systems MDP's
- Stochastic Policy search
- POMDP's

Value Function Approximation

- Avoid visiting every state - use machine learning methods to approximate value functions
- Pick some function which is easy to compute approximate the value function with fewer parameters – (find the parameters such that the error will be minimized) e.g. linear regression

$$U^\pi(s) = \theta_1 f_1(s) + \ldots + \theta_n f_n(s)$$

- After each trial get the values – solve for parameters

$$U^\pi(s) = \theta_0 + \theta_1 x + \theta_2 y$$

Reinforcement Learning

- Passive learning – policy is fixed (learn the utilities of states)
- Active learning – learn what to do (exploration/exploitation)
- MDP's know transition function and reward function
- Now: do not know either

Types of environments
- deterministic
- stochastic

Reinforcement Learning

- All we can is
  1. act 2. perceive state 3. get reward
  Example trials, using some fixed policy

(1,1) **-0.04** -> (1,2) **-0.04** -> (1,3) **-0.04** -> …. -> (4,3) **+1**

(1,1) **-0.04** -> (1,2) **-0.04** -> (1,3) **-0.04** -> (2,3) **-0.04** -> (3,3) …. -> (4,3) **+1**

(1,1) **-0.04** -> (2,1) **-0.04** -> (3,1) **-0.04** -> (3,2) **-0.04** -> (4,2) **-1**

Use the information about rewards to learn the expected utility of each state

Passive Reinforcement Learning

- Learn the utility – expected sum of rewards

$$U^\pi(s) = E\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) | \pi, s_0 = s\right]$$

- Direct utility estimation (Widrow, Hoff 1960)
- At the end of each sequence calculate the observed reward-to-go for each state and update the utility – one sample
- Keep track of the average utility – over all visits to a particular state
- Does not exploit the information that the utilities of neighboring states are related

$$U^\pi(s) = R(s) + \gamma \sum_{s'} T(s, \pi(s), s') U^\pi(s')$$

Adaptive dynamic programming

- How to update the utilities – such that they will be satisfy the Bellman equation
- We need to know how the states are related
- Need to learn at the same time the transition model
- keep track of frequencies of reaching *x'* from *x* by executing the action *u*
- Use the Bellman equation to determine the next utility value for each state – solve linear system of linear eq.
- Learns the utility function faster, exploits correlations between the states

Temporal Difference Learning

- Another way how to use the Bellman equation

$$U^\pi(s) = R(s) + \gamma \sum_{s'} T(s, \pi(s), s') U^\pi(s')$$
$$U^\pi(1,3) = -0.04 + U^\pi(2,3)$$

After few iterations the above constraint is not satisfied

- Adjust the value function based on difference between the utilities of successive states

$$U^\pi(s) \leftarrow U^\pi(s) + \alpha(R(s) + \gamma U^\pi(s') - U^\pi(s))$$

- Simpler – instead of doing value determination – just update the value

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Active reinforcement learning

- Decide what actions to take – no fixed policy
- At each step – follow optimal policy given the current estimate of the utility function
- Greedy agents it mail fail to learn the correct utilities unless it explores also other states
- Choosing always optimal actions can lead overall to suboptimal results
- Fundamental trade-off exploitation (maximize its reward) and exploitation (maximize overall well being)
- Choose random action 1/t times – otherwise follow optimal policy – alternatively design some function which will tradeoff greed vs curiosity (taking an action which yields lower utility – but has not been tried often)

Q-learning

- Instead of learning utilities - learn action value function Q(s,a)

$$Q(a,s) \leftarrow R(s) + \gamma \max_{a'} Q(a', s')$$

- Active TD Q-learning agent

$$Q(a,s) \leftarrow Q(a,s) + \alpha(R(s) + \gamma \max_{a'} Q(a', s') - Q(s,a))$$

- TD learning – too expensive to store the value functions
- In large models it is very hard to learn visit every state

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Policy search

- Policy - maps states to actions
- We would like to learn directly the policy
- Parameterize the policy by a collection of functions

$$\pi(s) = \max_a Q_\theta(a, s)$$

$$\pi(s) = \text{softmax}_a Q_\theta(a, s)$$

Value of the policy

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POMDPs

- In POMDPs we apply the very same idea as in MDPs.
- **Since the state is not observable**, the agent has to **make its decisions based on** the belief state which is a **posterior distribution over states**.
- Let b be the belief of the agent about the state under consideration.
- POMDPs compute a **value function over belief space**:

$$V_T(b) \;=\; \gamma \max_u \left[r(b,u) + \int V_{T-1}(b') p(b' \mid u, b)\, db' \right]$$

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Problems

- Each belief is a probability distribution, thus, each value in a **POMDP is a function of an entire probability distribution**.
- **This is problematic, since probability distributions are continuous**.
- Additionally, we have to deal with the **huge complexity of belief spaces**.
- For **finite worlds** with finite state, action, and measurement spaces and finite horizons, however, we can **effectively represent the value functions by piecewise linear functions**.

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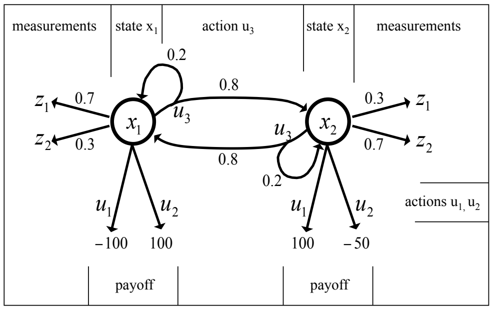
An Illustrative Example



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The Parameters of the Example

- The actions u_1 and u_2 are terminal actions.
- The action u_3 is a sensing action that potentially leads to a state transition.
- The horizon is finite and γ=1.

$$\begin{aligned} r(x_1,u_1) &= -100 & r(x_2,u_1) &= +100 \\ r(x_1,u_2) &= +100 & r(x_2,u_2) &= -50 \\ r(x_1,u_3) &= -1 & r(x_2,u_3) &= -1 \end{aligned}$$

$$\begin{aligned} p(x_1'|x_1,u_3) &= 0.2 & p(x_2'|x_1,u_3) &= 0.8 \\ p(x_1'|x_2,u_3) &= 0.8 & p(z_2'|x_2,u_3) &= 0.2 \end{aligned}$$

$$\begin{aligned} p(z_1|x_1) &= 0.7 & p(z_2|x_1) &= 0.3 \\ p(z_1|x_2) &= 0.3 & p(z_2|x_2) &= 0.7 \end{aligned}$$

Payoff in POMDPs

- In MDPs, the payoff (or return) depended on the state of the system.
- In POMDPs, however, the true state is not exactly known.
- Therefore, we compute the **expected payoff** by **integrating over all states**:

$$\begin{aligned} r(b,u) &= E_x[r(x,u)] \\ &= \int r(x,u)p(x)\,dx \\ &= p_1\, r(x_1,u) + p_2\, r(x_2,u) \end{aligned}$$

Payoffs in Our Example (1)

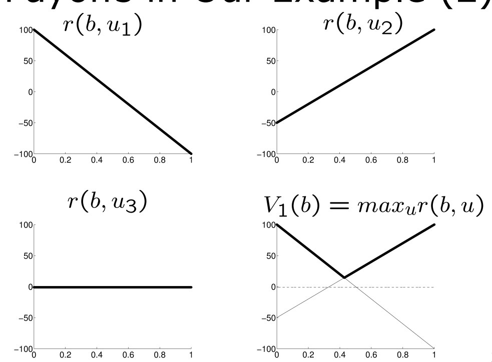
- If we are totally certain that we are in state x_1 and execute action u_1, we receive a reward of -100
- If, on the other hand, we definitely know that we are in x_2 and execute u_1, the reward is +100.
- In between it is the linear combination of the extreme values weighted by the probabilities

$$\begin{aligned} r(b,u_1) &= -100\, p_1 + 100\, p_2 \\ &= -100\, p_1 + 100\,(1-p_1) \end{aligned}$$

$$r(b,u_2) = 100\, p_1 - 50\,(1-p_1)$$

$$r(b,u_3) = -1$$

Payoffs in Our Example (2)



The Resulting Policy for T=1

- Given we have a finite POMDP with T=1, we would use $V_1(b)$ to determine the optimal policy.
- In our example, the optimal policy for T=1 is

$$\pi_1(b) = \begin{cases} u_1 & \text{if } p_1 \le \frac{3}{7} \\ u_2 & \text{if } p_1 > \frac{3}{7} \end{cases}$$

- This is the upper thick graph in the diagram.

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Piecewise Linearity, Convexity

- The resulting value function $V_1(b)$ is the maximum of the three functions at each point

$$\begin{aligned} V_1(b) &= \max_u \; r(b,u) \\ &= \max \left\{ \begin{array}{rr} -100\; p_1 & +100\;(1-p_1) \\ 100\; p_1 & -50\;(1-p_1) \\ -1 & \end{array} \right\} \end{aligned}$$

- It is piecewise linear and convex.

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Pruning

- If we carefully consider $V_1(b)$, we see that only the first two components contribute.
- The third component can therefore safely be pruned away from $V_1(b)$.

$$V_1(b) = \max \left\{ \begin{array}{rr} -100\; p_1 & +100\;(1-p_1) \\ 100\; p_1 & -50\;(1-p_1) \end{array} \right\}$$

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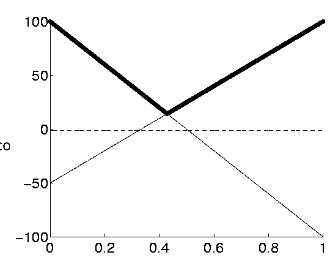
Increasing the Time Horizon

- Assume the robot can make an observation before deciding on an action.



- To be co

$V_1(b)$

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Additional topics

- LQR solving MDP's exactlly
- Finite horizon problems
- Policy search , Reinforce and Pegasus alg.

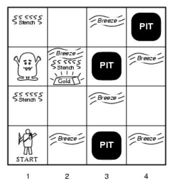
- Robotics Self Assembly http://www.youtube.com/ssrlab0/
- http://msl.cs.uiuc.edu/~lavalle/projects.html Weasle balls – sensorless control http://www.youtube.com/watch?v=P7vfTzbpx5k&lr=1
- Petman Boston Dynamics http://www.youtube.com/watch?v=Dl40uEjcP3o&feature=fvst

- Sand Flea jumping robots http://www.youtube.com/watch?v=6b4ZZQkcNEo
- Sand swimming robot http://news.discovery.com/tech/snake-like-robot-swims-rescue-110513.html
- http://youtu.be/_-p08o_oTO4 Autonomous Helicopters

- Medical robotics – needle steering
- http://www.youtube.com/watch?feature=endscreen&NR=1&v=yFbUvmsNXX4
- Laundry folding http://www.youtube.com/watch?v=gy5g33S0Gzo

Wumpus World PEAS description

- Performance measure
  - gold +1000, death -1000
  - -1 per step, -10 for using the arrow
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing i
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square

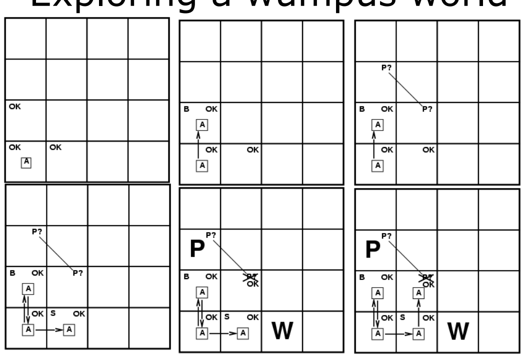


- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

Wumpus world characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete Yes
- Single-agent? Yes

Exploring a wumpus world



No stench or breeze in [1 1], nearby states are ok