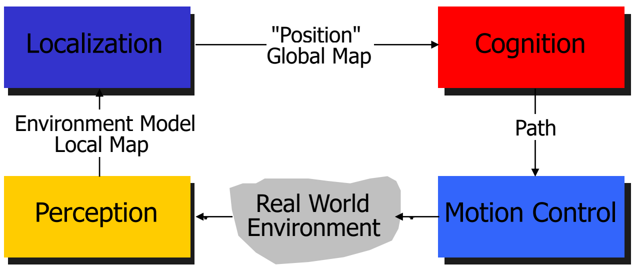
Perception

- Sensors
- Uncertainty
- Features, models of environments



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BibaBot, BlueBotics SA, Switzerland

IMU
Inertial Measurement Unit

Emergency Stop Button

Wheel Encoders

Omnidirectional Camera

Pan-Tilt Camera

Sonar Sensors

Laser Range Scanner

Bumper

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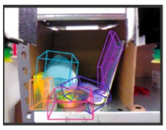
Robotic Navigation

- Stanford Stanley Grand Challenge
- Outdoors unstructured env., single vehicle
- Urban Challenge
- Outdoors structured env., mixed traffic, traffic rules

Robotic Manipulation

- Ability to detect objects and their pose
- Challenges – clutter
- Previously unseen objects

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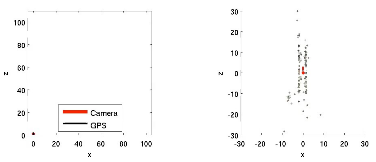
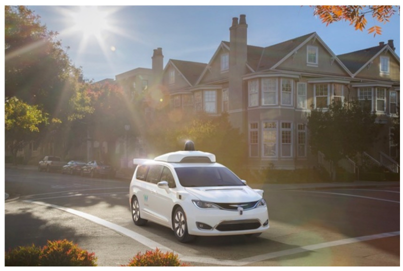
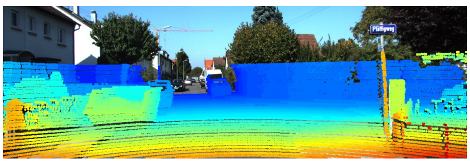
Localization



Mapping


Semantic Understanding

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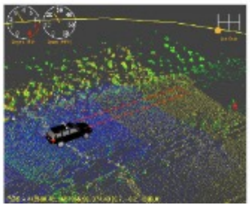
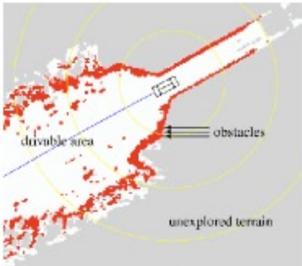
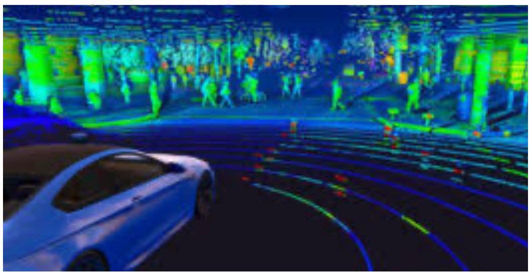
- Terrain mapping using lasers

- Determining obstacle course, scene understanding





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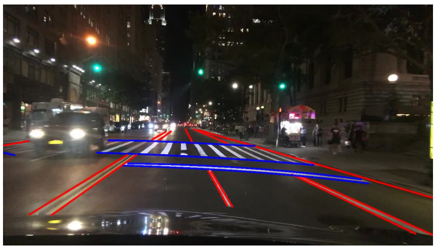
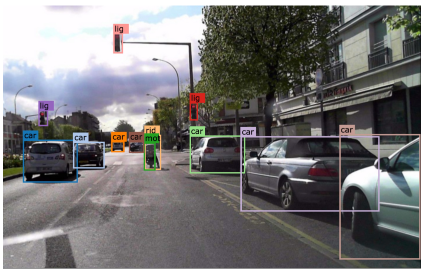
Robust Visual, Multi-modal Perception

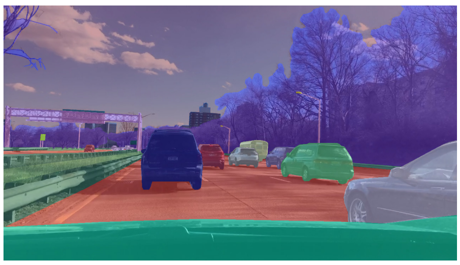
Drivable Areas



Lane Markings



Car detections



Semantic Segmentation

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source: UC Berkeley Deep Drive

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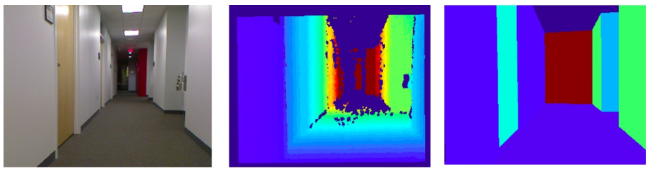
Mapping and Localization

- Exploit the single view parsing of indoor environment satisfying Manhattan constraints
- Assign each pixel in the image to one of the floor or walls
- Single view parses can be temporally inconsistent
- Local structure of the scene changes little between consecutive views: the prior wall layout is used as one of the constraints to infer the layout of the current frame.



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Mapping and Localization

- The basic ingredients: estimation of visual motion and
- 3D structure
- Difficulties: data association, correspondences, invariance
- Choice of features, choice of models, context
- Sparse features, large number of views

- How can learning alleviate some of the difficulties ?
- How to build representations for more sophisticated tasks ?
- How can the use of semantic information help ?

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Graph SLAM and Loop Closure

- Manhattan constraints lead to robust and consistent rotation estimates
- Need to global alignment to account to translation drift
- Graph SLAM optimization on (x, y): the camera has fixed height and the estimated rotations are accurate.
- Keyframe based loop closure detection: a frame with visible junction or T-junction is set to be a keyframe. Two keyframes are matched if the GIST score is less than 0.025 and the distance between to keyframe is less than 5m.

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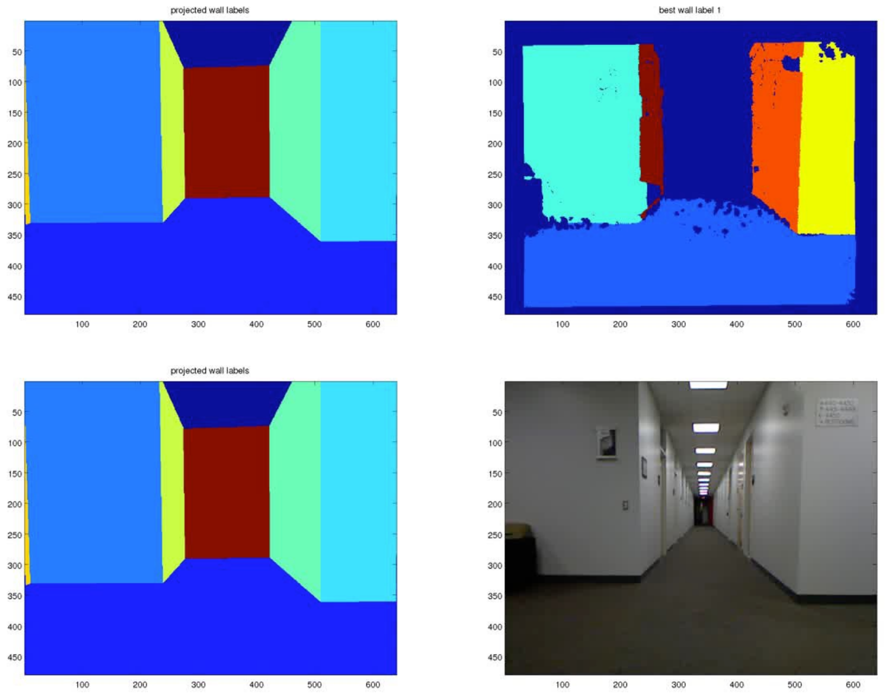

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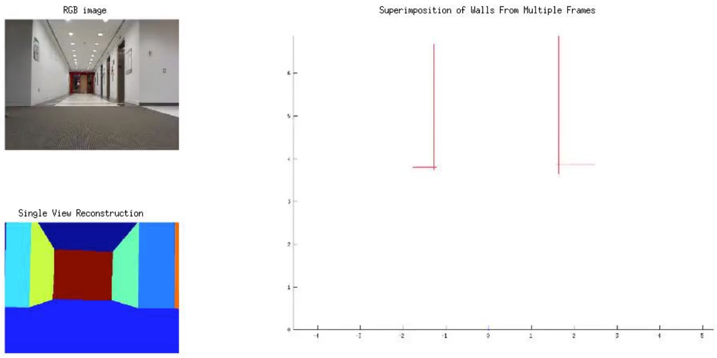

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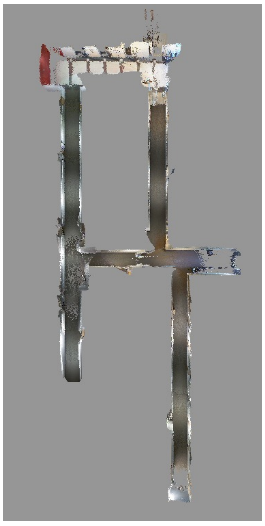
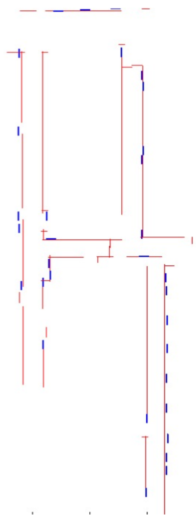
B. GMU sequences





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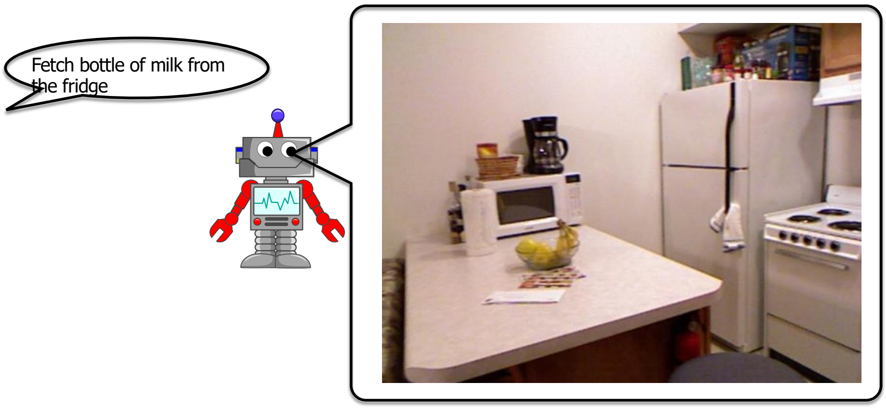
Semantic Scene Understandning



- Need to characterize the entities in the image in term of their semantic and geometric relationship

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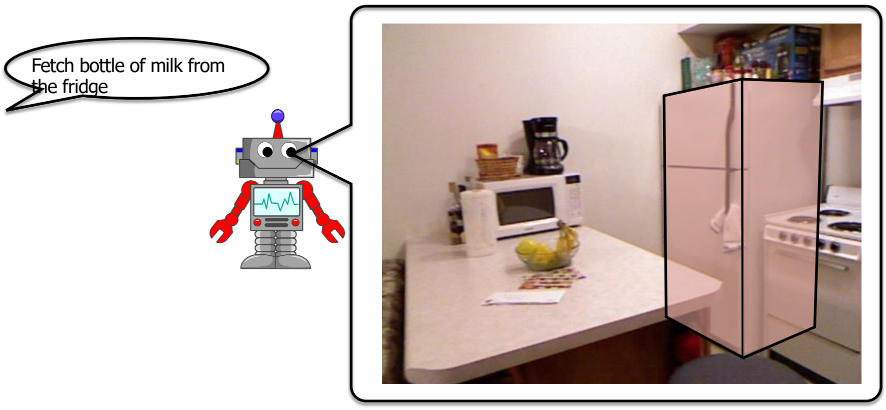
Scene Understanding



Detecting Fridge and its 3D orientation

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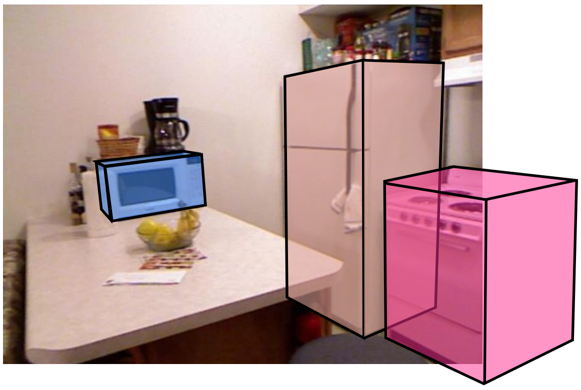
3D Pose Estimation

- Definition: Estimating rotation and translation for each object



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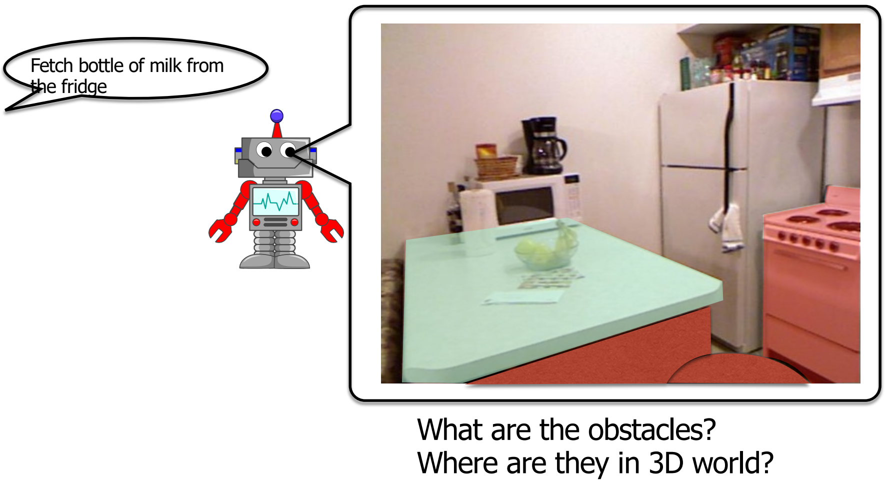
Scene Understanding



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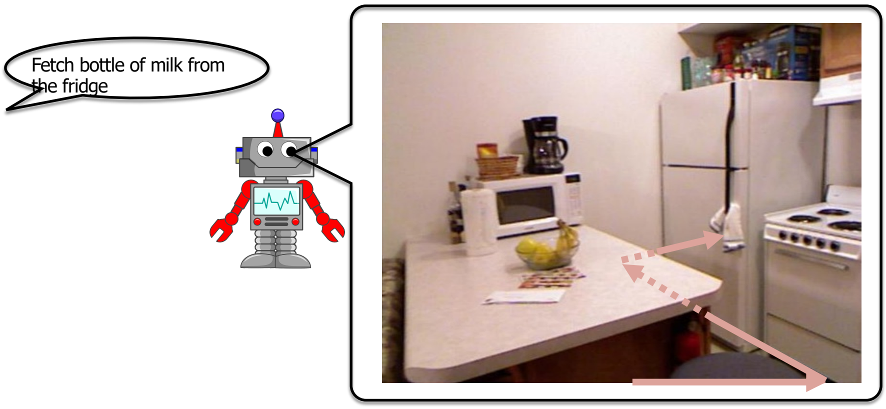
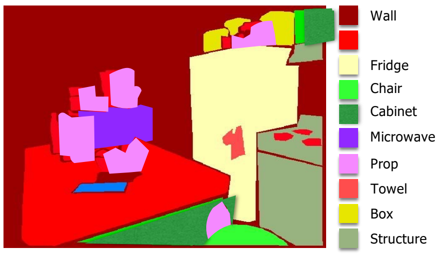
Scene Understanding

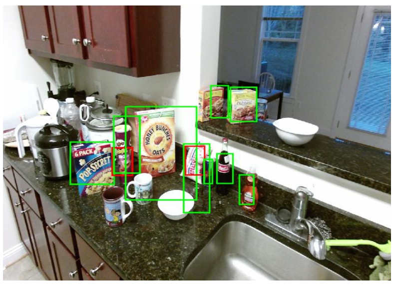


Plan a safe path

Semantic Scene Understanding

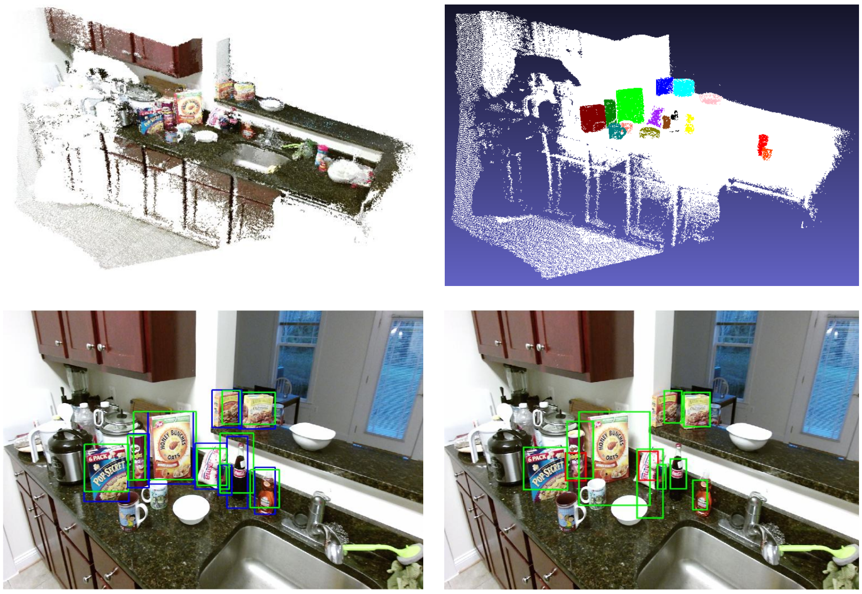


Semantic Labelling



Object Detection, 3D Pose

Mapping and Localization



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4.1.1

Classification of Sensors

- Proprioceptive sensors
  - measure values internally to the system (robot),
  - e.g. motor speed, wheel load, heading of the robot, battery status
- Exteroceptive sensors
  - information from the robots environment
  - distances to objects, intensity of the ambient light, unique features.
- Passive sensors
  - energy coming for the environment (cameras)
- Active sensors
  - emit their proper energy and measure the reaction
  - better performance, but some influence on environment

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Role of Perception in Robotics

- Where am I relative to the world?
  - sensors: vision, stereo, range sensors, acoustics
  - problems: scene modeling/classification/recognition
  - integration: localization/mapping algorithms (e.g. SLAM)
  - pose estimation
- What is around me?
  - sensors: vision, stereo, range sensors, acoustics, sounds, smell
  - problems: object recognition, 3D reconstruction, semantic understanding
  - integration: collision avoidance/navigation, learning

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Role of Perception in Robotics

- How can I safely interact with environment (including people!)?
  - sensors: vision, range, haptics (force+tactile)
  - problems: 3D structure/range estimation, tracking, materials, size, weight,
  - integration: navigation, manipulation, control, learning
- How can I solve "new" problems (generalization)?
  - sensors: vision, range, haptics, undefined new sensor
  - problems: categorization by function/shape/context/??
  - integrate: inference, navigation, manipulation, control, learning

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Visual Perception Topics

Techniques

- Single view geometry
- Feature detection and matching
- 3D reconstruction

Applications in Robotics:

- range sensing, Obstacle detection, environment interaction
- Mapping, registration, localization, recognition
- Manipulation

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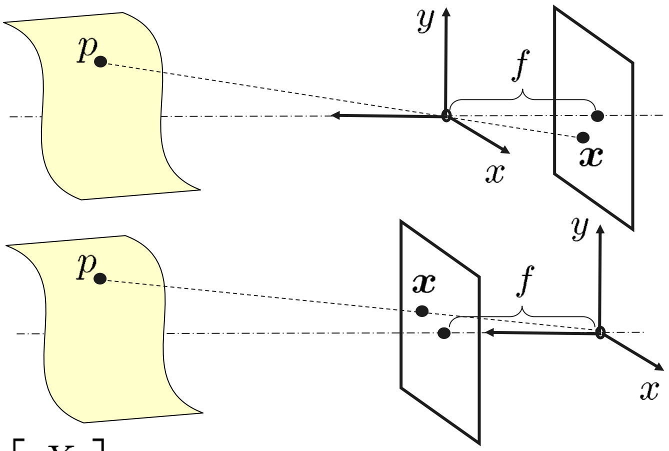
Image Formation

Pinhole

Frontal pinhole



$$\boldsymbol{X} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \rightarrow \boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{Z}\begin{bmatrix} X \\ Y \end{bmatrix}$$

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Pinhole Camera Model

- Image coordinates are nonlinear function of world coordinates
- Relationship between coordinates in the camera frame and sensor plane

2-D coordinates $$\boldsymbol{x} = \begin{bmatrix} x \\ y \end{bmatrix} = \frac{f}{Z} \begin{bmatrix} X \\ Y \end{bmatrix}$$

Homogeneous coordinates

$$\boldsymbol{x} \rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \frac{1}{Z} \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix}, \quad \boldsymbol{X} \rightarrow \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix},$$

$$Z \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{K_f} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\Pi_0} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

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Image Coordinates

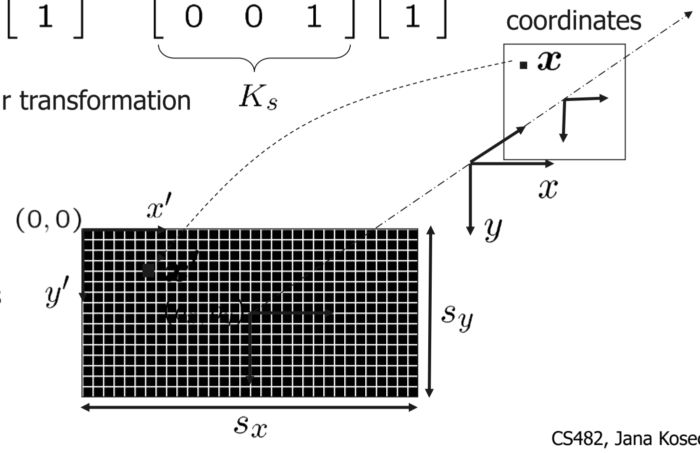
- Relationship between coordinates in the sensor plane and image

$$\boldsymbol{x}' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} s_x & s_\theta & o_x \\ 0 & s_y & o_y \\ 0 & 0 & 1 \end{bmatrix}}_{K_s} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Linear transformation K_s

metric coordinates

pixel coordinates



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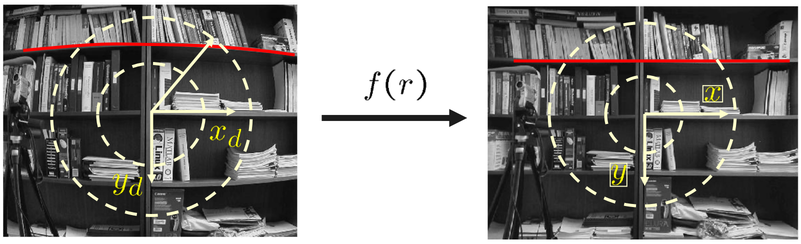
Camera parameters – Radial Distortion

Nonlinear transformation along the radial direction



$$\begin{aligned} \boldsymbol{x} &= c + f(r)(\boldsymbol{x}_d - c), \quad r = \|\boldsymbol{x}_d - c\| \\ f(r) &= 1 + a_1 r + a_2 r^2 + a_3 r^3 + a_4 r^4 + \cdots \end{aligned}$$

Distortion correction: make lines straight

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Calibration Matrix and Camera Model

- Relationship between coordinates in the world frame and image
- Intrinsic parameters

Pinhole camera   Pixel coordinates

$$\lambda \boldsymbol{x} = K_f \Pi_0 \boldsymbol{X} \qquad \boldsymbol{x}' = K_s \boldsymbol{x}$$

- Adding transformation between camera coordinate systems and world coordinate system
- Extrinsic Parameters

$$\lambda \boldsymbol{x}' = \begin{bmatrix} fs_x & fs_\theta & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\lambda \boldsymbol{x} = K_f \Pi_0 g \boldsymbol{X} = \Pi \boldsymbol{X}$$

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Image of a Point

Homogeneous coordinates of a 3-D point p

$$\boldsymbol{X} = [X, Y, Z, W]^T \in \mathbb{R}^4, \quad (W = 1)$$

Homogeneous coordinates of its 2-D image

$$\boldsymbol{x} = [x, y, z]^T \in \mathbb{R}^3, \quad (z = 1)$$

Projection of a 3-D point to an image plane

$$\lambda \boldsymbol{x} = \Pi \boldsymbol{X}$$

$$\lambda \in \mathbb{R}, \ \Pi = [R, T] \in \mathbb{R}^{3\times 4}$$

$$\lambda \boldsymbol{x}' = \Pi \boldsymbol{X}$$

$$\lambda \in \mathbb{R}, \ \Pi = [KR, KT] \in \mathbb{R}^{3\times 4}$$

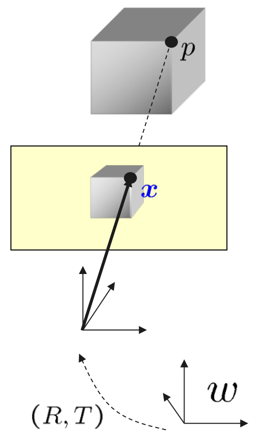

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Image of a Line

Homogeneous representation of a 3-D line L

$$\boldsymbol{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} X_o \\ Y_o \\ Z_o \\ 1 \end{bmatrix} + \mu \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ 0 \end{bmatrix}, \quad \mu \in \mathbb{R}$$

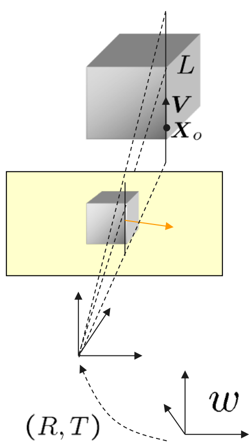
Homogeneous representation of its 2-D image

$$\boldsymbol{l} = [a, b, c]^T \in \mathbb{R}^3$$

Projection of a 3-D line to an image plane

$$\boldsymbol{l}^T \boldsymbol{x} = \boldsymbol{l}^T \Pi \boldsymbol{X} = 0$$

$$\Pi = [KR, KT] \in \mathbb{R}^{3\times 4}$$



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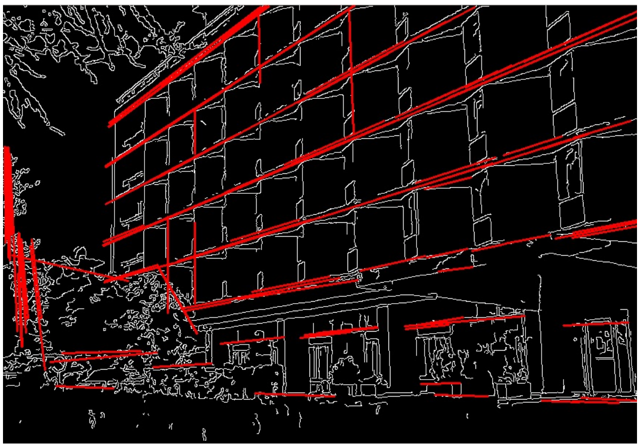
Fitting



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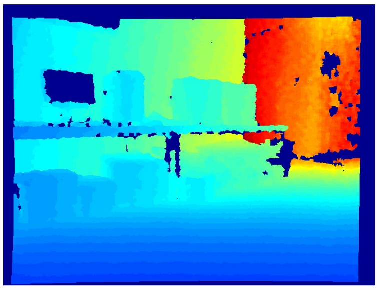
Plane Fitting



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Fitting: Overview

- If we know which points belong to the line, how do we find the "optimal" line parameters?
  - Least squares
- What if there are outliers?
  - Robust fitting, RANSAC
- What if there are many lines?
  - Voting methods: RANSAC, Hough transform
- What if we're not even sure it's a line?
  - Model selection

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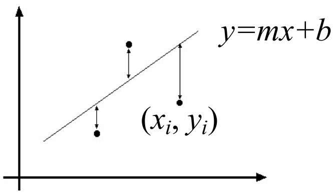
Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m x_i + b$
- Find (m, b) to minimize



$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \quad X = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \quad B = \begin{bmatrix} m \\ b \end{bmatrix}$$

$$E = \|Y - XB\|^2 = (Y - XB)^T (Y - XB) = Y^T Y - 2(XB)^T Y + (XB)^T (XB)$$

$$\frac{dE}{dB} = 2X^T XB - 2X^T Y = 0$$

$$X^T XB = X^T Y$$

Normal equations: least squares solution to $XB=Y$

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Problem with "vertical" least squares

- Not rotation-invariant
- Fails completely for vertical lines

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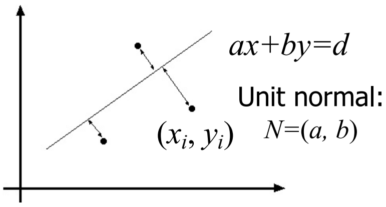
Total least squares

- Distance between point (x_i, y_i) and line $ax+by=d$ $(a^2+b^2=1)$: $|ax_i + by_i - d|$



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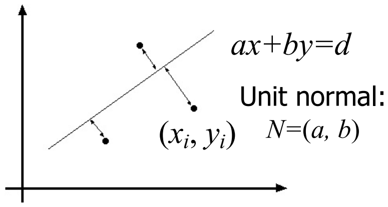
Total least squares

- Distance between point (x_i, y_i) and line $ax+by=d$ $(a^2+b^2=1)$: $|ax_i + by_i - d|$
- Find (a, b, d) to minimize the sum of squared perpendicular distances



$$E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

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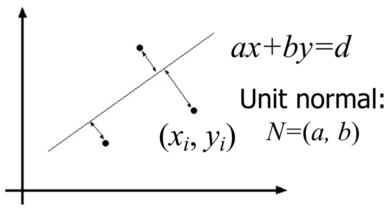
Total least squares

- Distance between point (x_i, y_i) and line $ax+by=d$ $(a^2+b^2=1)$: $|ax_i + by_i - d|$
- Find (a, b, d) to minimize the sum of squared perpendicular distances



$$E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

$$\frac{\partial E}{\partial d} = \sum_{i=1}^{n} -2(ax_i + by_i - d) = 0 \qquad d = \frac{a}{n}\sum_{i=1}^{n} x_i + \frac{b}{n}\sum_{i=1}^{n} y_i = a\bar{x} + b\bar{y}$$

$$E = \sum_{i=1}^{n} (a(x_i - \bar{x}) + b(y_i - \bar{y}))^2 = \left\| \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 = (UN)^T (UN)$$

$$\frac{dE}{dN} = 2(U^T U)N = 0$$

Solution to $(U^T U)N = 0$, subject to $\|N\|^2 = 1$: eigenvector of $U^T U$ associated with the smallest eigenvalue (least squares solution to *homogeneous linear system* $UN = 0$)

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Total least squares

$$U = \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \quad U^T U = \begin{bmatrix} \sum_{i=1}^{n} (x_i - \bar{x})^2 & \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \\ \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) & \sum_{i=1}^{n} (y_i - \bar{y})^2 \end{bmatrix}$$

second moment matrix

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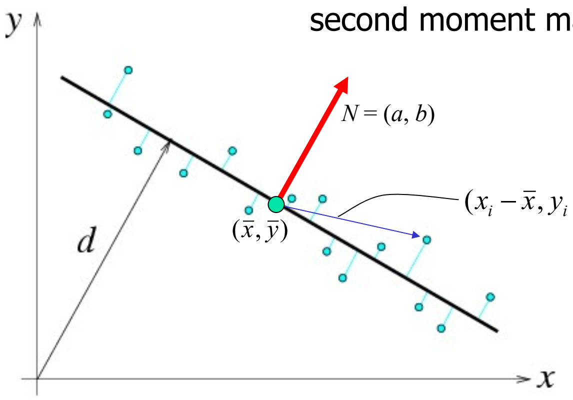
Total least squares

$$U = \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \vdots \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \quad U^T U = \begin{bmatrix} \sum_{i=1}^{n} (x_i - \bar{x})^2 & \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \\ \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) & \sum_{i=1}^{n} (y_i - \bar{y})^2 \end{bmatrix}$$

second moment matrix



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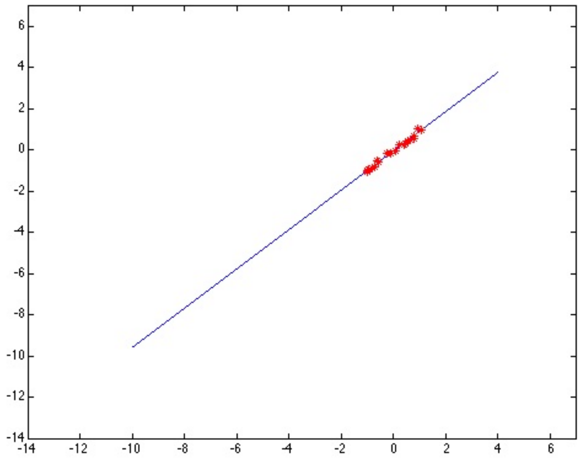
Least squares: Robustness to noise

- Least squares fit to the red points:



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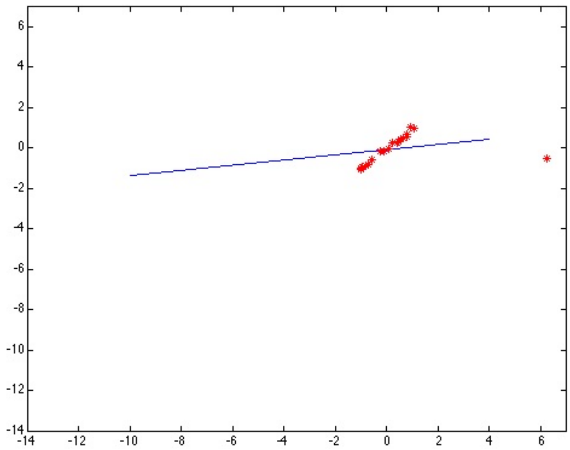
Least squares: Robustness to noise

- Least squares fit with an outlier:



Problem: squared error heavily penalizes outliers

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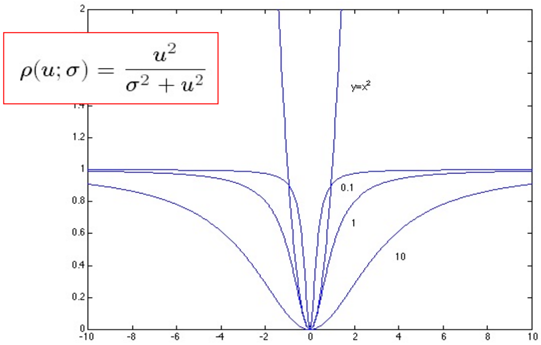
Robust estimators

- General approach: find model parameters θ that minimize

$$\sum_i \rho\left(r_i\left(x_i,\theta\right);\sigma\right)$$

$r_i(x_i, \theta)$ – residual of ith point w.r.t. model parameters θ
ρ – robust function with scale parameter σ



The robust function ρ behaves like squared distance for small values of the residual u but saturates for larger values of u

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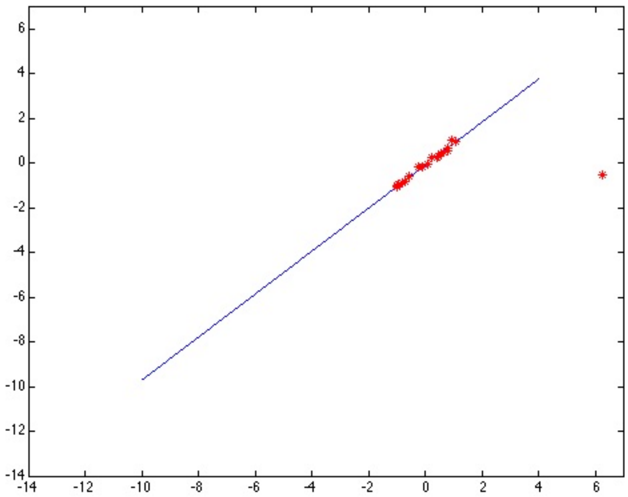
Choosing the scale: Just right



The effect of the outlier is minimized

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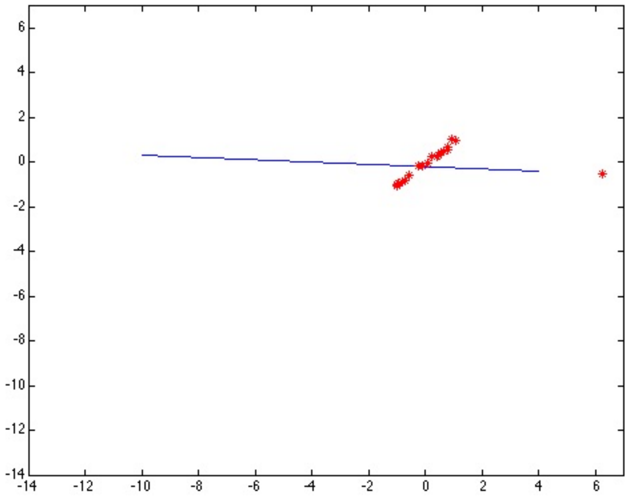
Choosing the scale: Too small



The error value is almost the same for every point and the fit is very poor

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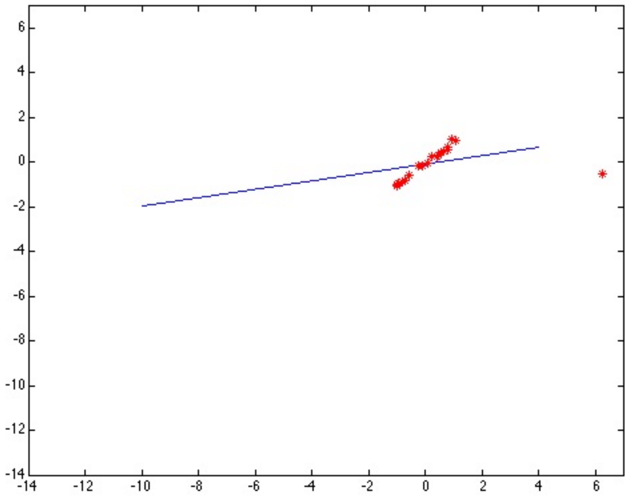
Choosing the scale: Too large



Behaves much the same as least squares

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Robust estimation: Details

- Robust fitting is a nonlinear optimization problem that must be solved iteratively
- Least squares solution can be used for initialization
- Adaptive choice of scale: approx. 1.5 times median residual (F&P, Sec. 15.5.1)

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RANSAC

- Robust fitting can deal with a few outliers – what if we have very many?
- Random sample consensus (RANSAC): Very general framework for model fitting in the presence of outliers
- Outline
  - Choose a small subset of points uniformly at random
  - Fit a model to that subset
  - Find all remaining points that are "close" to the model and reject the rest as outliers
  - Do this many times and choose the best model

M. A. Fischler, R. C. Bolles. Random Sample Consensus: A Paradigm for Model Fitting with Applications to Image Analysis and Automated Cartography. Comm. of the ACM, Vol 24, pp 381-395, 1981.

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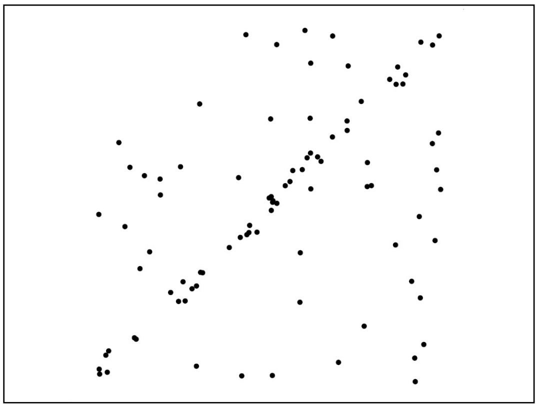
RANSAC for line fitting example



Source: R. Raguram

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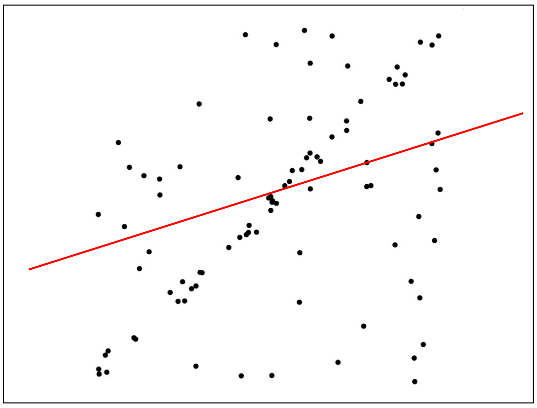
RANSAC for line fitting example



Least-squares fit

Source: R. Raguram

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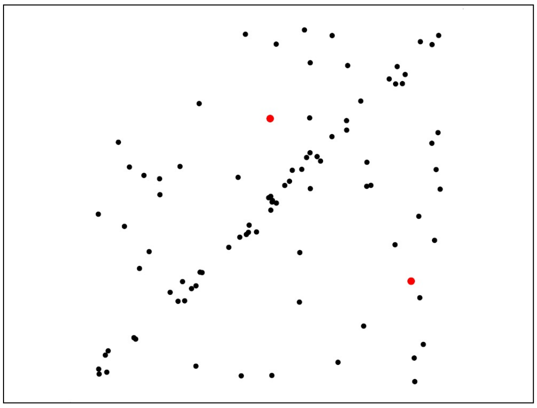
RANSAC for line fitting example



1. Randomly select minimal subset of points

Source: R. Raguram

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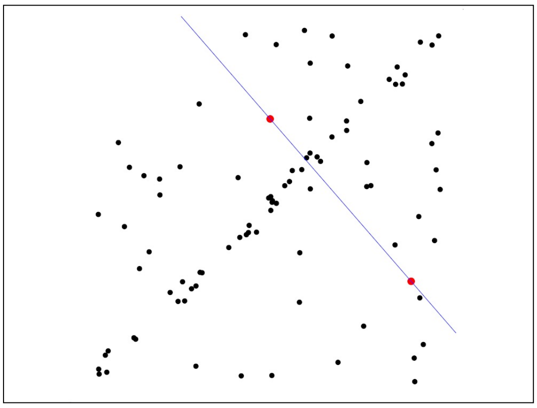
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model

Source: R. Raguram

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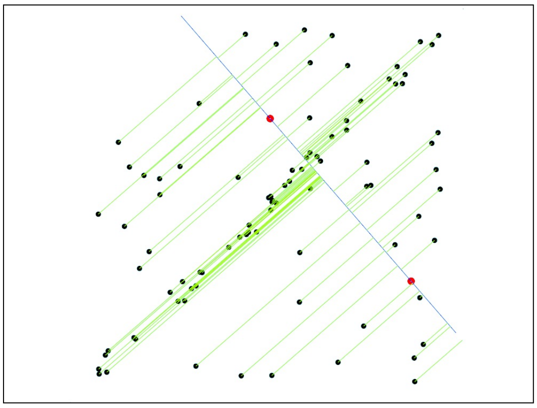
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function

Source: R. Raguram

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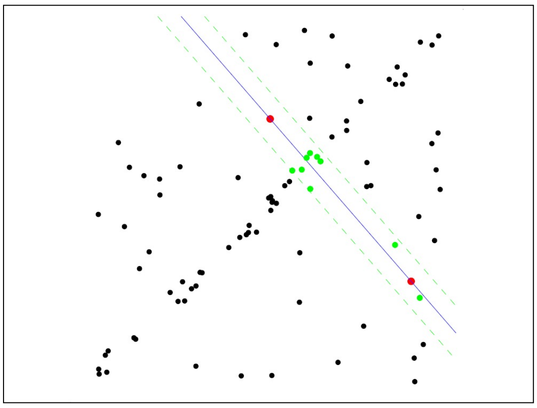
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model

Source: R. Raguram

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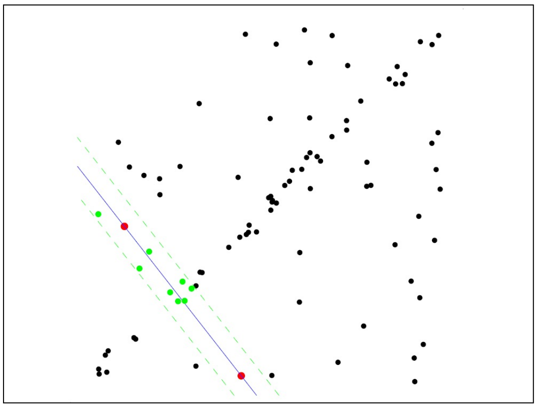
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

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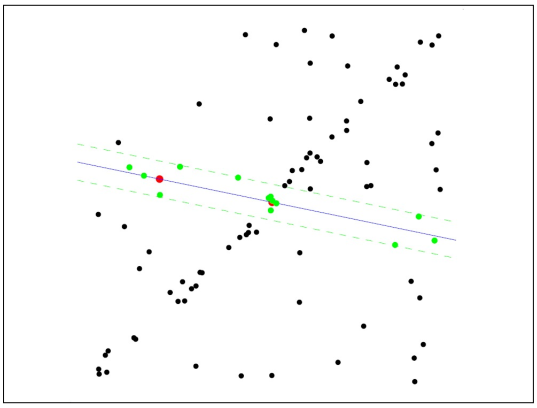
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

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Source: R. Raguram

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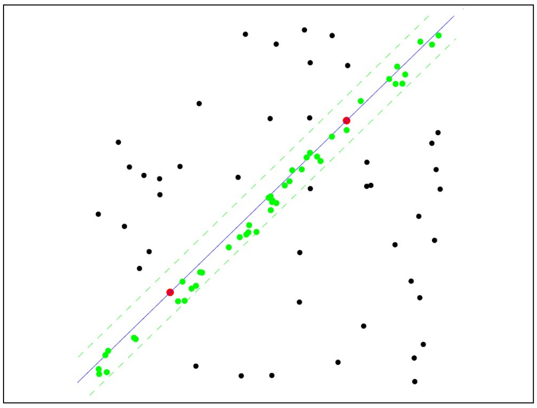
RANSAC for line fitting example

Uncontaminated sample



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

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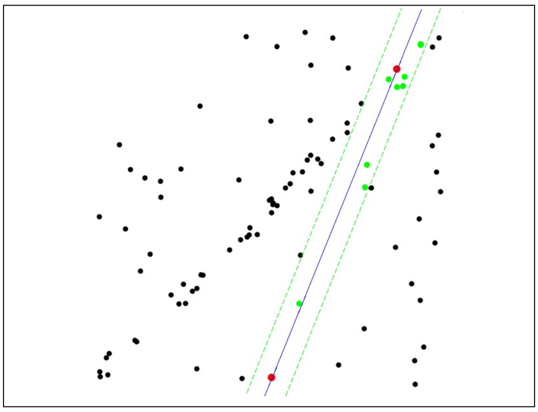
RANSAC for line fitting example



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

Source: R. Raguram

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RANSAC for line fitting

- Repeat ***N*** times:
- Draw ***s*** points uniformly at random
- Fit line to these ***s*** points
- Find inliers to this line among the remaining points (i.e., points whose distance from the line is less than ***t***)
- If there are ***d*** or more inliers, accept the line and refit using all inliers

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Choosing the parameters

- Initial number of points *s*
  - Typically minimum number needed to fit the model
- Distance threshold *t*
  - Choose *t* so probability for inlier is *p* (e.g. 0.95)
  - Zero-mean Gaussian noise with std. dev. σ: $t^2=3.84\sigma^2$
- Number of samples *N*
  - Choose *N* so that, with probability *p*, at least one random sample is free from outliers (e.g. *p*=0.99) (outlier ratio: *e*)

Source: M. Pollefeys

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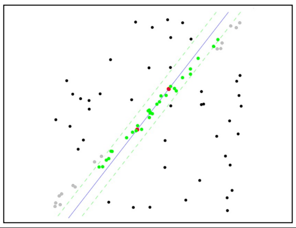
RANSAC pros and cons

- Pros
  - Simple and general
  - Applicable to many different problems
  - Often works well in practice
- Cons
  - Lots of parameters to tune
  - Doesn't work well for low inlier ratios (too many iterations, or can fail completely)
  - Can't always get a good initialization of the model based on the minimum number of samples



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Previously

- Camera model
- Stereo matching, triangulation

- Features
- Least Squares Fitting
- RANSAC

- Today
- Camera pose estimation in the world coordinate frame
- Relative Pose Estimation

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Calibration with a Rig

Use the fact that both 3-D and 2-D coordinates of feature points on a pre-fabricated object (e.g., a cube) are known.

Calibration with a Rig

- Given 3-D coordinates on known object

$\lambda \mathbf{x}' = [KR, KT]\mathbf{X}$ → $\lambda \mathbf{x}' = \Pi \mathbf{X}$

$$\lambda \begin{bmatrix} x^i \\ y^i \\ 1 \end{bmatrix} = \begin{bmatrix} \pi_1^T \\ \pi_2^T \\ \pi_3^T \end{bmatrix} \begin{bmatrix} X^i \\ Y^i \\ Z^i \\ 1 \end{bmatrix}$$

- Eliminate unknown scales

$$x^i(\pi_3^T \mathbf{X}) = \pi_1^T \mathbf{X},$$
$$y^i(\pi_3^T \mathbf{X}) = \pi_2^T \mathbf{X}$$

- Recover projection matrix $\Pi = [KR, KT] = [R', T']$

$$\min \|M\Pi^s\|^2 \quad \text{subject to} \quad \|\Pi^s\|^2 = 1$$

$$\Pi^s = [\pi_{11}, \pi_{21}, \pi_{31}, \pi_{12}, \pi_{22}, \pi_{32}, \pi_{13}, \pi_{23}, \pi_{33}, \pi_{14}, \pi_{24}, \pi_{34}]^T$$

- Factor the into $R \in SO(3)$ and using QR decomposition
- Solve for translation $T = K^{-1}T'$

More details

- Direct calibration by recovering and decomposing the projection matrix

$$\lambda \begin{bmatrix} x^i \\ y^i \\ 1 \end{bmatrix} = \begin{bmatrix} \pi_1^T \\ \pi_2^T \\ \pi_3^T \end{bmatrix} \begin{bmatrix} X^i \\ Y^i \\ Z^i \\ 1 \end{bmatrix} \rightarrow Z \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} \\ \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x_i = \frac{\pi_{11}X_i + \pi_{12}Y_i + \pi_{13}Z_i + \pi_{14}}{\pi_{31}X_i + \pi_{32}Y_i + \pi_{33}Z_i + \pi_{34}} \quad y_i = \frac{\pi_{21}X_i + \pi_{22}Y_i + \pi_{23}Z_i + \pi_{24}}{\pi_{31}X_i + \pi_{32}Y_i + \pi_{33}Z_i + \pi_{34}}$$

$$x_i(\pi_{31}X_i + \pi_{32}Y_i + \pi_{33}Z_i + \pi_{34}) = \pi_{11}X_i + \pi_{12}Y_i + \pi_{13}Z_i + \pi_{14}$$
$$y_i(\pi_{31}X_i + \pi_{32}Y_i + \pi_{33}Z_i + \pi_{34}) = \pi_{21}X_i + \pi_{22}Y_i + \pi_{23}Z_i + \pi_{24}$$

$$x^i(\pi_3^T \mathbf{X}) = \pi_1^T \mathbf{X}, \quad y^i(\pi_3^T \mathbf{X}) = \pi_2^T \mathbf{X}$$

2 constraints per point

$$[X_i, Y_i, Z_i, 1, 0, 0, 0, 0, -x_iX_i, -x_iY_i, -x_iZ_i, -x_i]\Pi_s = 0$$
$$[0, 0, 0, 0, X_i, Y_i, Z_i, 1, -y_iX_i, -y_iY_i, -y_iZ_i, -y_i]\Pi_s = 0$$

$$\Pi_s = [\pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{21}, \pi_{22}, \pi_{23}, \pi_{24}, \pi_{31}, \pi_{32}, \pi_{33}, \pi_{34}]^T$$

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More details

- Recover projection matrix $\Pi = [KR, KT] = [R', T']$

  $\min \|M\Pi^s\|^2$ subject to $\|\Pi^s\|^2 = 1$

  $\Pi^s = [\pi_{11}, \pi_{21}, \pi_{31}, \pi_{12}, \pi_{22}, \pi_{32}, \pi_{13}, \pi_{23}, \pi_{33}, \pi_{14}, \pi_{24}, \pi_{34}]^T$
- Collect the constraints from all N points into matrix M (2N x 12)
- Solution eigenvector associated with the smallest eigenvalue $M^T M$
- Unstack the solution and decompose into rotation and translation
- Factor the R' into $R \in SO(3)$ and K using QR decomposition
- Solve for translation $T = K^{-1}T'$

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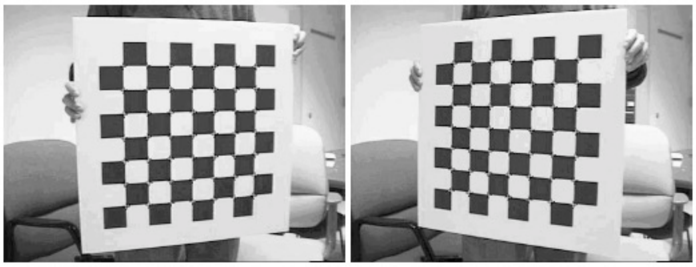
Calibration with a planar pattern



$$H \doteq K[r_1, r_2, T] \quad \in \mathbb{R}^{3\times 3} \qquad \lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = K[r_1, r_2, T] \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix},$$

To eliminate unknown depth, multiply both sides by $\widehat{x'}$

$$\widehat{x'} H [X, Y, 1]^T = 0.$$

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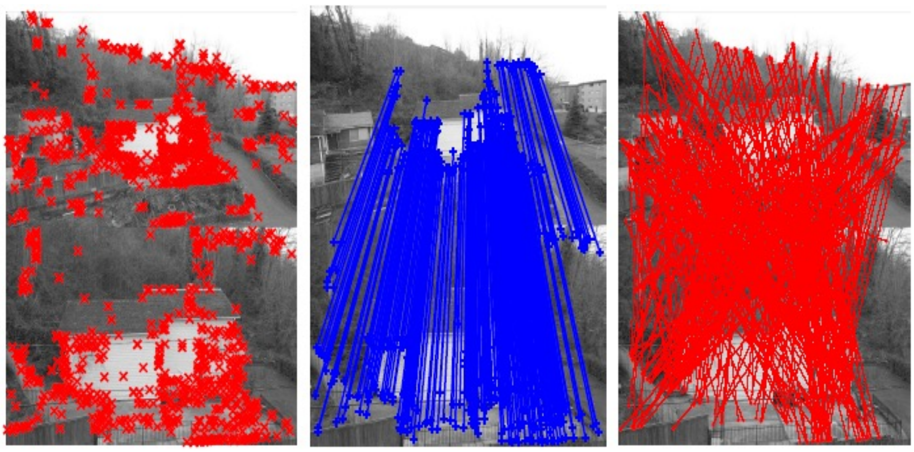
Robust technique



(a) correspondences. (b) identified inliers. (c) identified outliers.

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