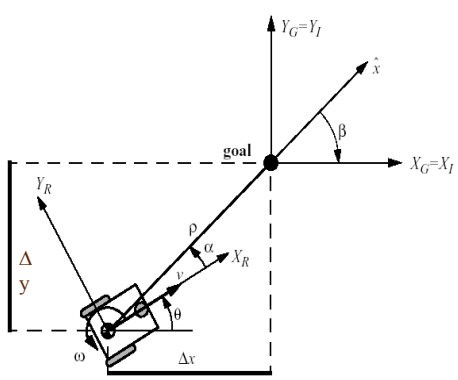
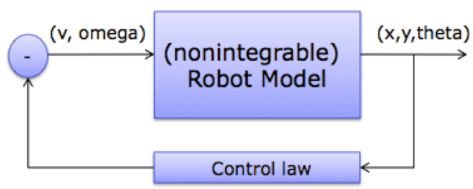
Robot Control Basics
CS 685

Previously Feedback control

- Kinematics of the differential drive robot in polar coordinates
- Designed state feedback controller (error feedback)
- At each instance of time compute a control law
- Given the current error between current and desired position





3.6.2

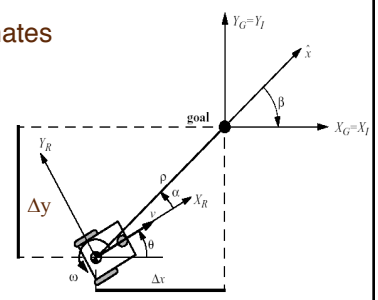
Kinematic Position Control: Coordinates Transformation

Coordinates transformation into polar coordinates with its origin at goal position:

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}$$

$$\alpha = -\theta + \operatorname{atan2}(\Delta y, \Delta x)$$

$$\beta = -\theta - \alpha$$



System description, in the new polar coordinates

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \frac{\sin\alpha}{\rho} & -1 \\ -\frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \qquad \begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ -\frac{\sin\alpha}{\rho} & 1 \\ \frac{\sin\alpha}{\rho} & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

For α from $I_1 = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$ for $I_2 = (-\pi, -\pi/2] \cup (\pi/2, \pi]$

3.6.2

Kinematic Position Control: The Control Law

- It can be shown, that with

  $$v = k_\rho \rho \qquad \omega = k_\alpha \alpha + k_\beta \beta$$

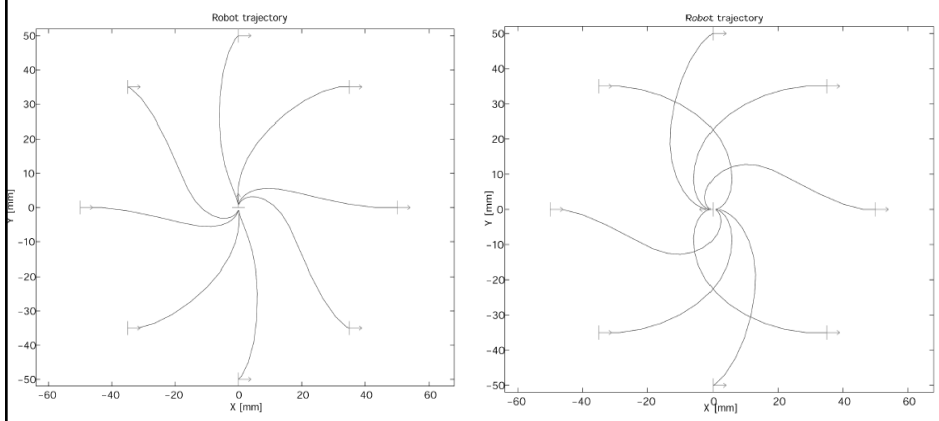
  the feedback controlled system

  $$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos\alpha \\ k_\rho \sin\alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin\alpha \end{bmatrix}$$

- will drive the robot to $(\rho, \alpha, \beta) = (0,0,0)$
- The control signal *v* has always constant sign,
  - the direction of movement is kept positive or negative during movement
  - parking maneuver is performed always in the most natural way and without ever inverting its motion.

3.6.2

Kinematic Position Control: Resulting Path



Potential Field Methods

- Idea robot is a particle, point with unit mass
- Environment is represented as a potential field (locally)
- Advantage – capability to generate on-line collision avoidance

Compute force acting on a robot – incremental path planning

q – the pose of the robot, here only position

$$F(q) = -\nabla U(q) \qquad q = [x, y]^T$$

Example: Robot can translate freely , we can control independently

Environment represented by a potential function

$$U(x, y)$$

Force is proportional to the gradient of the potential function

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = -\nabla U(x, y)$$

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Attractive Potential Field

- Linear function of distance

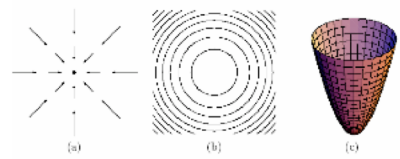
$$U_a(q) = \xi \|q - q_{goal}\| \qquad F_a(q) = -\nabla U_a(q) = -\xi \frac{(q - q_{goal})}{\|q - q_{goal}\|}$$

$$q = [x, y]^T \quad q_{goal} = [x_{goal}, y_{goal}]^T$$

- Quadratic function of distance

$$U_a(q) = \xi \frac{1}{2} \|q - q_{goal}\|^2 \quad F_a(q) = -\nabla U_a(q) = -\xi (q - q_{goal})$$

Combination of two – far away use linear, closer by use parabolic well



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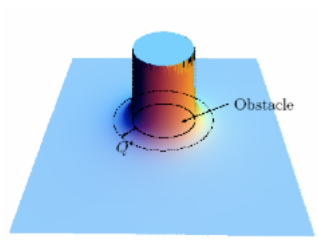
Repulsive Potential Field

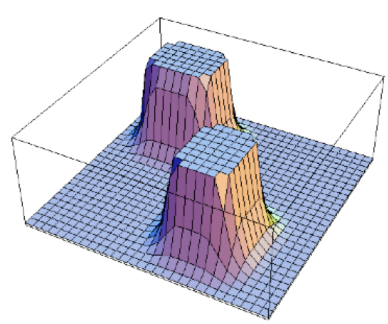
$$U_r(q) = \frac{1}{2} \nu \left(\frac{1}{\rho(q, q_{obst})} - \frac{1}{\rho_o} \right) \qquad \text{if} \quad \rho(q, q_{obst})\| \leq \rho_0$$

$$\text{else} \quad U_r(q) = 0$$

Minimal distance between the robot and the obstacle

$$\rho(q, q_{obst}) = \|q - q_{obst}\|$$





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Repulsive Potential Field

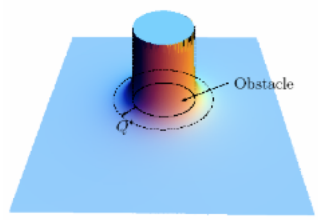
$$U_{rep} = \frac{1}{2}\nu\left(\frac{1}{\rho(q, q_{obst})} - \frac{1}{\rho_0}\right)^2 \quad \text{if} \quad \rho(q, q_{obst})\| \leq \rho_0$$

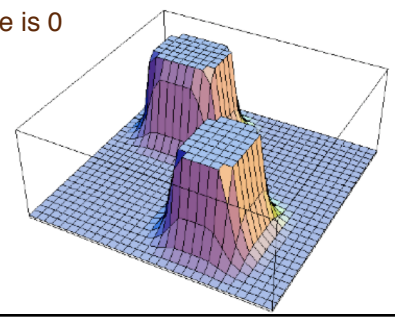
$$\text{else} \quad U_r(q) = 0$$

Minimal distance between the robot and the obstacle

$$F_{rep} = -\nabla U_{rep} = \nu\left(\frac{1}{\rho(q)} - \frac{1}{\rho_0}\right)\frac{1}{\rho(q)^2}\frac{q - q_{obs}}{\rho(q)}$$

Outside of sensitivity zone repulsive force is 0





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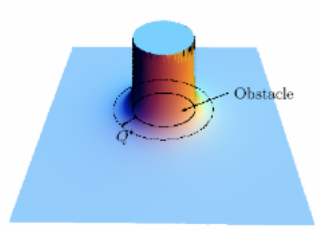
Repulsive Potential Field

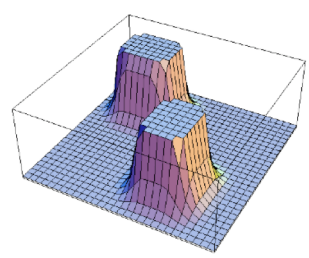
$$U_r(q) = \frac{1}{2}\nu\left(\frac{1}{\rho(q, q_{obst})} - \frac{1}{\rho_o}\right) \quad \text{if} \quad \rho(q, q_{obst})\| \leq \rho_0$$

$$\text{else} \quad U_r(q) = 0$$

Minimal distance between the robot and the obstacle

Previously – repulsive potential related to the square of the Inverse distance – here just proportional to inverse distance
Note: need to compute gradient to get the force





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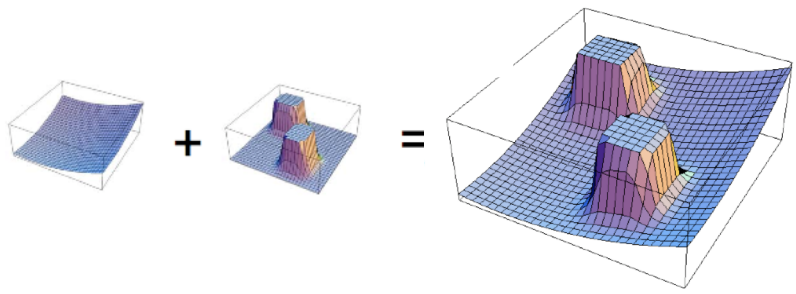
Potential Fields

Resulting force $F(q) = -\nabla(U_a(q) + U_r(q))$

Iterative gradient descent planning $q_{i+1} = q_i + \delta_i \frac{F(q)}{\|F(q)\|}$

$q = [x, y]^T$



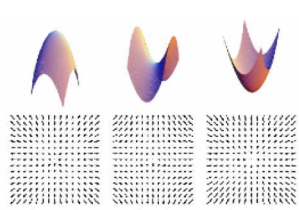
Potential Fields

- Simple way to get to the bottom, follow the gradient

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = -\nabla U(x, y) \qquad \dot{q} = -\nabla U(q)$$

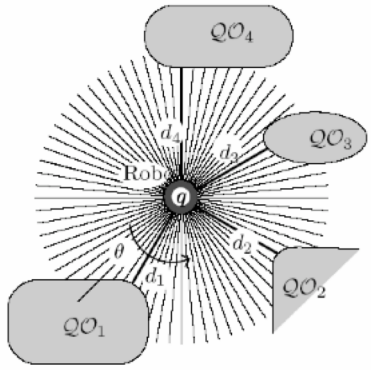
$$q_{i+1} = q_i + \delta_i \frac{F(q)}{\|F(q)\|}$$

- i.e. Gradient descent strategy $\nabla U(q) = 0$
- A critical, stationary point is such that
- Equation is stationary at the critical point



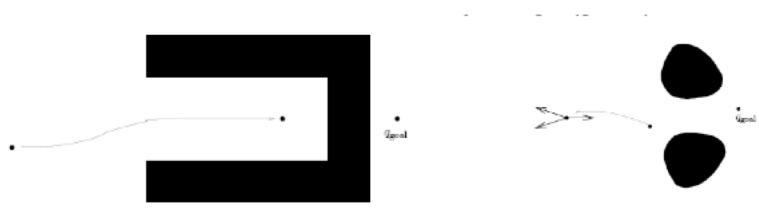
Computing Distances

- In practice the distances are computed using sensors
- Obstacles are not circular
- Consider a range sensor which returns the closest distance
  to the obs



Potential Functions

- How do we know we have single global minimum ?



- If global minimum is not guaranteed, need to do something else then gradient descent
- Design functions in such a way that global minimum can be guaranteed

Potential function

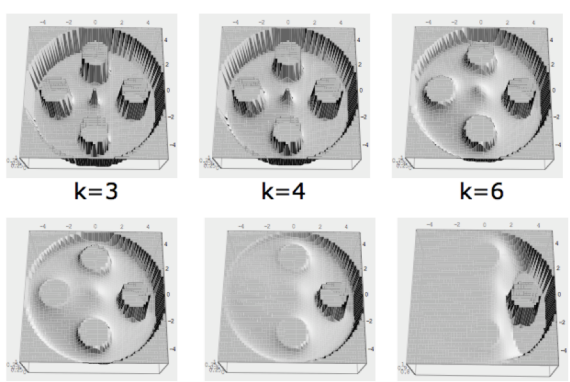
- Heuristics for escaping the local minima
- Can be used in local and global context
- Numerical techniques, Random walk methods

- Navigation functions (Rimon & Kodistchek, 92)
- Navigations in sphere worlds and worlds diffeomorphic to them

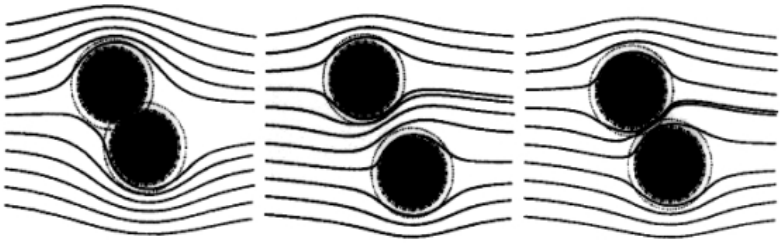
Navigation functions

$$\phi(q) = -\frac{d^2(q, q_{goal})}{[d(q, q_{goal})^{2k} + \beta(q)]^{1/k}} \quad \beta(q)$$ Obstacle term

- For sufficiently large k – this is a navigation function [Rimon-Koditschek, 92]

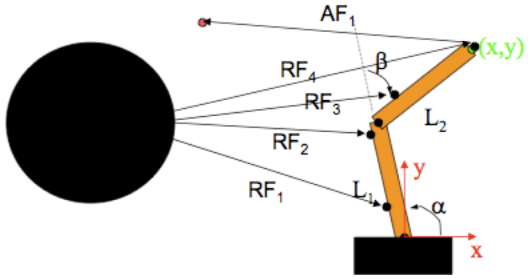


Potential Field Path Planning: Using Harmonic Potentials

- Hydrodynamics analogy
  - robot is moving similar to a fluid particle following
- 
- Note:
  - Complicated, only simulation shown

Potential fields for Rigid Bodies

- So far robot was considered a point- gradient of the potential function – force acting on a point
- How to generalize to manipulators of objects ?
- Idea – forces acting on objects – forces acting on multiple points of the object (black board)
- http://www.cs.cmu.edu/~motionplanning/
- For robots, pick enough control points to pin down the robot – define forces in workspace – map them to configuration space



Potential Fields Issues

- Local minima
- Heuristics for escaping the local minima
- Can be used in local and global context
- Numerical techniques
- Random walk methods
- To be continued