Homework 1  
Due date: February 6

*Be as concise as possible.*

1. Set up the website where you will be posting your coding solutions and email the link to the instructor.

2. (5) Consider rigid body transformations in the plane. Draw a right triangle defined by three points $A = (2, 1), B = (4, 1), C = (4, 6)$.
   
   - Consider a rotation matrix
     
     $$
     T_1 = \begin{bmatrix}
     \cos \theta & -\sin \theta \\
     \sin \theta & \cos \theta 
     \end{bmatrix}
     $$
   
   a. What is the determinant of the matrix?
   
   - Consider transformation matrix
     
     $$
     T_2 = \begin{bmatrix}
     \sin \theta & \cos \theta \\
     \cos \theta & -\sin \theta 
     \end{bmatrix}
     $$
   
   a. Is the matrix orthonormal? What is the determinant of the matrix?
   
   c. Is $T_2$ rigid body transformation? What is the difference between $T_1$ and $T_2$, how are the results different?

3. (5) Point $P_A = [p_1, p_2, p_3]^T$ expressed in a stationary frame A is rotated about axis $Z_A$ by $\theta$ degrees and then rotated around axis $X_A$ by $\phi$ degrees. Give a rotation matrix that accomplishes these two rotations. Both of the rotations are around stationary frame.

4. Let $R \in SO(3)$ be a rotation matrix generated by rotating about a unit vector $\omega$ by $\theta$ radians that satisfies $R = exp(\hat{\omega} \theta)$.

   Consider following rotation matrix:

   $$
   R = \begin{bmatrix}
   0.1729 & -0.1468 & 0.9739 \\
   0.9739 & 0.1729 & -0.1468 \\
   -0.1468 & 0.9739 & 0.1729
   \end{bmatrix}
   $$

   Use the formulas given in class to compute the rotation axis and the associated angle. b) Use Matlab function `eig` to compute the eigenvalues and eigenvectors of the above rotation matrix $R$. What is the eigenvector associated with unit eigenvalue? Can you explain it’s physical meaning?
5. (10) Write a Matlab program to simulate the motion a differential drive robot.

- The function should take as an input vector $\xi_0$ specifying the initial pose $[x_0, y_0, \theta_0]$ and velocities $v, \omega$ and time $t$ denotes number of time steps and $\delta t$ the length of the time step. You should return resulting path as three vectors each $1 \times n$ long where $n$ is the number of time steps. The output will correspond to pose indexed by time.

$$[x, y, \theta] = \text{diffDrive}([x_0, y_0, \theta_0], v, \omega, t, \delta t)$$

- For the following example assume that at time $t = 0$ the configuration (pose) of the robot is $\xi_0 = [x, y, \theta] = [100, 50, 45^\circ]$. Robot starts moving with some angular and linear velocity $\omega = 2^\circ/s$ and $v = 1 \text{ m/s}$. How is the path affected by the choice of $\delta t$? Hand in the plot of the code and the plot of the path in x-y plane.