

Assignment 3

1. We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$ which is a real number in the range $0 \leq r(u, v) \leq 1$ that represents the reliability of a communication channel from vertex u to vertex v . We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent.

Give an efficient algorithm to find the most reliable path between two given vertices.

2. You are given a set of cities, along with the pattern of highways between them, in the form of an undirected graph $G = (V, E)$. Each stretch of highway $e \in E$ connects two of the cities, and you know its length in miles, l_e . You want to get from city s to city t . There's one problem: your car can only hold enough gas to cover L miles. There are gas stations in each city, but not between cities. Therefore, you can only take a route if every one of its edges has length $l_e \leq L$.

(a) Given the limitation on your car's fuel tank capacity, show how to determine in linear time whether there is a feasible route from s to t .

(b) You are now planning to buy a new car, and you want to know the minimum fuel tank capacity that is needed to travel from s to t . Give an efficient algorithm to determine this.

3. Alice wants to throw a party and is deciding whom to call. She has n people to choose from, and she has made up a list of which pairs of these people know each other. She wants to pick as many people as possible, subject to two constraints: at the party, each person should have at least five other people whom they know and five other people whom they don't know.

Give an efficient algorithm that takes as input the list of n people and the list of pairs who know each other and outputs the best choice of party invitees.