

GMU CS583 Algorithms Assignment 2

Name: G#:

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1. Solve the following recurrence relations and give an O -bound for each of them.

(a) $T(n) = T(n - 1) + n^c$, where $c \geq 1$ is a constant

(b) $T(n) = 2T(n/3) + 1$

(c) $T(n) = 5T(n/4) + n$

2. A **random graph** is defined as a graph of n nodes where each pair of nodes is connected by probability $p \in [0, 1]$. Let us denote $G(n, p)$ a random graph with n nodes and probability p to connect any pair of nodes. Calculate the expected number of edges in the random graph $G(n, p)$, using the approach of indicator variables.
3. Consider a very simple online auction system that works as follows. There are n *bidding agents*; agent i has a bid b_i , which is a positive natural number. We will assume that all bids b_i are distinct from one another.

The bidding agents appear in an order chosen uniformly at random, each proposes its bid b_i in turn, and at all times the system maintains a variable b^* equal to the highest bid seen so far. (Initially b^* is set to 0.)

What is the expected number of times that b^* is updated when this process is executed, as a function of the parameters in the problem?

Example. Suppose $b_1 = 20$, $b_2 = 25$, and $b_3 = 10$, and the bidders arrive in the order 1, 3, 2. Then b^* is updated for 1 and 2, but not for 3.