An Additional Problem for Assignment 5

1. Suppose you are managing the construction of billboards on the Stephen Daedalus Memorial Highway, a heavily traveled stretch of road that runs west-east for M miles. The possible sites for billboards are given by numbers $x_l, x_2, \ldots, x_n$, each in the interval $[0, M]$ (specifying their position along the highway, measured in miles from its western end). If you place a billboard at location x_i, you receive a revenue of $r_i > 0$.

   Regulations imposed by the county's Highway Department require that no two of the billboards be within less than or equal to 5 miles of each other. You'd like to place billboards at a subset of the sites so as to maximize your total revenue, subject to this restriction.

   Example. Suppose $M = 20$, $n = 4$,

   $$\{x_1, x_2, x_3, x_4\} = \{6, 7, 12, 14\},$$

   and

   $$\{r_l, r_2, r_3, r_4\} = \{5, 6, 5, 1\}.$$

   Then the optimal solution would be to place billboards at x_l and x_3, for a total revenue of 10.

   Give an algorithm that takes an instance of this problem as input and returns the maximum total revenue that can be obtained from any valid subset of sites. The running time of the algorithm should be polynomial in n.