Ch. 3 : Logic Coverage

Four Structures for Modeling Software

Graphs

Logic

Input Space

Syntax

Applied to

Source

FSMs

Specs

DNF

Source

Models

Specs

Use cases

Design

Integ

Input
Covering Logic Expressions  
(3.1)

• Logic expressions show up in many situations

• Covering logic expressions is required by the US Federal Aviation Administration for safety critical software

• Logical expressions can come from many sources
  – Decisions in programs
  – FSMs and statecharts
  – Requirements

• Tests are intended to choose some subset of the total number of truth assignments to the expressions

Logic Predicates and Clauses

• A predicate is an expression that evaluates to a boolean value

• Predicates can contain
  – boolean variables
  – non-boolean variables that contain >, <, ==, >=, <=, !=
  – boolean function calls

• Internal structure is created by logical operators
  – ¬ – the negation operator
  – ∧ – the and operator
  – ∨ – the or operator
  – → – the implication operator
  – θ – the exclusive or operator
  – ↔ – the equivalence operator

• A clause is a predicate with no logical operators
Examples

• \((a < b) \lor f(z) \land D \land (m \geq n \times o)\)

• **Four clauses:**
  – \((a < b)\) – relational expression
  – \(f(z)\) – boolean-valued function
  – \(D\) – boolean variable
  – \((m \geq n \times o)\) – relational expression

• **Most predicates have few clauses**
  – It would be nice to quantify that claim!

• **Sources of predicates**
  – Decisions in programs
  – Guards in finite state machines
  – Decisions in UML activity graphs
  – Requirements, both formal and informal
  – SQL queries

Translating from English

• “I am interested in SWE 637 and CS 652”
  \[\text{course} = \text{swe637} \lor \text{course} = \text{cs652}\]

• “If you leave before 6:30 AM, take Braddock to 495, if you leave after 7:00 AM, take Prosperity to 50, then 50 to 495”
  \[\text{time} < 6:30 \rightarrow \text{path} = \text{Braddock} \lor \text{time} > 7:00 \rightarrow \text{path} = \text{Prosperity}\]

  Hmm … this is incomplete!

  \[\text{time} < 6:30 \rightarrow \text{path} = \text{Braddock} \lor \text{time} \geq 6:30 \rightarrow \text{path} = \text{Prosperity}\]
Testing and Covering Predicates
(3.2)

• We use predicates in testing as follows:
  – Developing a model of the software as one or more predicates
  – Requiring tests to satisfy some combination of clauses

• Abbreviations:
  – \( P \) is the set of predicates
  – \( p \) is a single predicate in \( P \)
  – \( C \) is the set of clauses in \( P \)
  – \( C_p \) is the set of clauses in predicate \( p \)
  – \( c \) is a single clause in \( C \)

Predicate and Clause Coverage

• The first (and simplest) two criteria require that each predicate
  and each clause be evaluated to both true and false

  **Predicate Coverage (PC)**: For each \( p \) in \( P \), \( TR \) contains two
  requirements: \( p \) evaluates to true, and \( p \) evaluates to false.

• When predicates come from conditions on edges, this is
  equivalent to edge coverage

• PC does not evaluate all the clauses, so …

  **Clause Coverage (CC)**: For each \( c \) in \( C \), \( TR \) contains two
  requirements: \( c \) evaluates to true, and \( c \) evaluates to false.
Predicate Coverage Example

\[(a < b) \lor D) \land (m \geq n \times o)\]

**Predicate coverage**

<table>
<thead>
<tr>
<th>Predicate = true</th>
<th>Predicate = false</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = 5, b = 10, D = true, m = 1, n = 1, o = 1</td>
<td>a = 10, b = 5, D = false, m = 1, n = 1, o = 1</td>
</tr>
<tr>
<td>(5 &lt; 10) \lor true \land (1 \geq 1 \times 1)</td>
<td>(10 &lt; 5) \lor false \land (1 \geq 1 \times 1)</td>
</tr>
<tr>
<td>= true \lor true \land TRUE</td>
<td>= false \lor false \land TRUE</td>
</tr>
<tr>
<td>= true</td>
<td>= false</td>
</tr>
</tbody>
</table>

Clause Coverage Example

\[(a < b) \lor D) \land (m \geq n \times o)\]

**Clause coverage**

<table>
<thead>
<tr>
<th>(a &lt; b) = true</th>
<th>(a &lt; b) = false</th>
<th>D = true</th>
<th>D = false</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = 5, b = 10</td>
<td>a = 10, b = 5</td>
<td>D = true</td>
<td>D = false</td>
</tr>
<tr>
<td>m \geq n \times o = true</td>
<td>m \geq n \times o = false</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m = 1, n = 1, o = 1</td>
<td>m = 1, n = 2, o = 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Two tests**

1) a = 5, b = 10, D = true, m = 1, n = 1, o = 1
2) a = 10, b = 5, D = false, m = 1, n = 2, o = 2
Problems with PC and CC

- PC does not fully exercise all the clauses, especially in the presence of short circuit evaluation

- CC does not always ensure PC
  - That is, we can satisfy CC without causing the predicate to be both true and false
  - This is definitely not what we want!

- The simplest solution is to test all combinations …

Combinatorial Coverage

- CoC requires every possible combination
- Sometimes called Multiple Condition Coverage

**Combinatorial Coverage (CoC):** For each p in P, TR has test requirements for the clauses in Cp to evaluate to each possible combination of truth values.

<table>
<thead>
<tr>
<th>a &lt; b D</th>
<th>m &gt;= n*o</th>
<th>((a &lt; b) ∨ D) ∧ (m &gt;= n*o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Combinatorial Coverage

- This is simple, neat, clean, and comprehensive ...
- But quite expensive!
- $2^N$ tests, where $N$ is the number of clauses
  - Impractical for predicates with more than 3 or 4 clauses
- The literature has lots of suggestions – some confusing
- The general idea is simple:
  
  Test each clause independently from the other clauses

- Getting the details right is hard
- What exactly does “independently” mean?
- The book presents this idea as “making clauses active” ...

Active Clauses

- Clause coverage has a weakness: The values do not always make a difference
- Consider the first test for clause coverage, which caused each clause to be true:
  - $(5 < 10) \lor \text{true} \land (1 \geq 1*1)$
- Only the first clause counts!
- To really test the results of a clause, the clause should be the determining factor in the value of the predicate

**Determination:** A clause $C_i$ in predicate $p$, called the major clause, determines $p$ if and only if the values of the remaining minor clauses $C_j$ are such that changing $C_i$ changes the value of $p$

- This is considered to make the clause active
Determining Predicates

\[ P = A \lor B \]
- if \( B = true \), \( p \) is always true.
- so if \( B = false \), \( A \) determines \( p \).
- if \( A = false \), \( B \) determines \( p \).

\[ P = A \land B \]
- if \( B = false \), \( p \) is always false.
- so if \( B = true \), \( A \) determines \( p \).
- if \( A = true \), \( B \) determines \( p \).

- **Goal**: Find tests for each clause when the clause determines the value of the predicate
- This is formalized in several criteria that have subtle, but very important, differences

Active Clause Coverage

**Active Clause Coverage (ACC)**: For each \( p \) in \( P \) and each major clause \( c_i \) in \( C_p \), choose minor clauses \( c_j \), \( j \neq i \), so that \( c_i \) determines \( p \). TR has two requirements for each \( c_i : c_i \) evaluates to true and \( c_i \) evaluates to false.

\[ p = a \lor b \]
- 1) \( a = true, b = false \)
- 2) \( a = false, b = false \)
- 3) \( a = false, b = true \)
- 4) \( a = false, b = false \)

- This is a form of MCDC, which is required by the FAA for safety critical software
- **Ambiguity**: Do the minor clauses have to have the same values when the major clause is true and false?
Resolving the Ambiguity

\[
p = a \lor (b \land c)
\]

Major clause : a
a = true, b = false, c = true
a = false, b = false, c = false

Is this allowed?

- This question caused confusion among testers for years
- Considering this carefully leads to three separate criteria:
  - Minor clauses do not need to be the same
  - Minor clauses do need to be the same
  - Minor clauses force the predicate to become both true and false

General Active Clause Coverage

General Active Clause Coverage (GACC) : For each \( p \) in \( P \) and each major clause \( c_i \) in \( C_p \), choose minor clauses \( c_j, j \neq i \), so that \( c_i \) determines \( p \). TR has two requirements for each \( c_i \): \( c_i \) evaluates to true and \( c_i \) evaluates to false. The values chosen for the minor clauses \( c_j \) do not need to be the same when \( c_i \) is true as when \( c_i \) is false, that is, \( c_j(c_i = true) = c_j(c_i = false) \) for all \( c_j \) OR \( c_j(c_i = true) \neq c_j(c_i = false) \) for all \( c_j \).

- This is complicated!
- It is possible to satisfy GACC without satisfying predicate coverage
- We really want to cause predicates to be both true and false!
Restricted Active Clause Coverage

Restricted Active Clause Coverage (RACC) : For each \( p \) in \( P \) and each major clause \( ci \) in \( Cp \), choose minor clauses \( cj, j \neq i \), so that \( ci \) determines \( p \). TR has two requirements for each \( ci \):

- \( ci \) evaluates to true and \( ci \) evaluates to false. The values chosen for the minor clauses \( cj \) must be the same when \( ci \) is true as when \( ci \) is false, that is, it is required that \( cj(ci = true) = cj(ci = false) \) for all \( cj \).

- This has been a common interpretation by aviation developers
- RACC often leads to infeasible test requirements
- There is no logical reason for such a restriction

Correlated Active Clause Coverage

Correlated Active Clause Coverage (CACC) : For each \( p \) in \( P \) and each major clause \( ci \) in \( Cp \), choose minor clauses \( cj, j \neq i \), so that \( ci \) determines \( p \). TR has two requirements for each \( ci \):

- \( ci \) evaluates to true and \( ci \) evaluates to false. The values chosen for the minor clauses \( cj \) must cause \( p \) to be true for one value of the major clause \( ci \) and false for the other, that is, it is required that \( p(ci = true) \neq p(ci = false) \).

- A more recent interpretation
- Implicitly allows minor clauses to have different values
- Explicitly satisfies (subsumes) predicate coverage
CACC and RACC

\[
\begin{array}{ccc}
\text{a} & \text{b} & \text{c} & \text{a} \land (\text{b} \lor \text{c}) \\
1 & T & T & T \\
2 & T & F & T \\
3 & T & T & T \\
5 & F & T & T \\
6 & F & T & F \\
7 & F & T & F \\
\end{array}
\]

CACC can be satisfied by choosing any of rows 1, 2, 3 AND any of rows 5, 6, 7 – a total of nine pairs

\[
\begin{array}{ccc}
\text{a} & \text{b} & \text{c} & \text{a} \land (\text{b} \lor \text{c}) \\
1 & T & T & T \\
5 & F & T & T \\
2 & T & F & T \\
6 & F & T & F \\
3 & T & F & T \\
7 & F & F & T \\
\end{array}
\]

RACC can only be satisfied by one of the three pairs above

Inactive Clause Coverage

- The active clause coverage criteria ensure that “major” clauses do affect the predicates
- Inactive clause coverage takes the opposite approach – major clauses do not affect the predicates

Inactive Clause Coverage (ICC): For each \( p \) in \( P \) and each major clause \( ci \) in \( Cp \), choose minor clauses \( cj, j \neq i \), so that \( ci \) does not determine \( p \). TR has four requirements for each \( ci \):
  1. \( ci \) evaluates to true with \( p \) true,
  2. \( ci \) evaluates to false with \( p \) true,
  3. \( ci \) evaluates to true with \( p \) false, and
  4. \( ci \) evaluates to false with \( p \) false.
General and Restricted ICC

• Unlike ACC, the notion of correlation is not relevant
  – ci does not determine p, so cannot correlate with p

• Predicate coverage is always guaranteed

**General Inactive Clause Coverage (GICC):** For each \( p \) in \( P \) and each major clause \( ci \) in \( Cp \), choose minor clauses \( cj, j \neq i \), so that \( ci \) does not determine \( p \). The values chosen for the minor clauses \( cj \) do not need to be the same when \( ci \) is true as when \( ci \) is false, that is, \( cj(ci = true) = cj(ci = false) \) for all \( cj \) OR \( cj(ci = true) \not= cj(ci = false) \) for all \( cj \).

**Restricted Inactive Clause Coverage (RICC):** For each \( p \) in \( P \) and each major clause \( ci \) in \( Cp \), choose minor clauses \( cj, j \neq i \), so that \( ci \) does not determine \( p \). The values chosen for the minor clauses \( cj \) must be the same when \( ci \) is true as when \( ci \) is false, that is, it is required that \( cj(ci = true) = cj(ci = false) \) for all \( cj \).

Logic Coverage Criteria Subsumption
Making Clauses Determine a Predicate

• Finding values for minor clauses $c_j$ is easy for simple predicates
• But how to find values for more complicated predicates ?
• Definitional approach:
  – $p_{c=true}$ is predicate $p$ with every occurrence of $c$ replaced by $true$
  – $p_{c=false}$ is predicate $p$ with every occurrence of $c$ replaced by $false$
• To find values for the minor clauses, connect $p_{c=true}$ and $p_{c=false}$ with exclusive OR

$$p_c = p_{c=true} \oplus p_{c=false}$$
• After solving, $p_c$ describes exactly the values needed for $c$ to determine $p$

Examples

<table>
<thead>
<tr>
<th>$p = a \lor b$</th>
<th>$p = a \land b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_a = p_{a=true} \oplus p_{a=false}$</td>
<td>$p_a = p_{a=true} \oplus p_{a=false}$</td>
</tr>
<tr>
<td>= (true $\lor$ b) XOR (false $\lor$ b)</td>
<td>= (true $\land$ b) $\oplus$ (false $\land$ b)</td>
</tr>
<tr>
<td>= true XOR b</td>
<td>= b $\oplus$ false</td>
</tr>
<tr>
<td>= $\neg$ b</td>
<td>= b</td>
</tr>
<tr>
<td>$p = a \lor (b \land c)$</td>
<td>$p = a \land b$</td>
</tr>
<tr>
<td>$p_a = p_{a=true} \oplus p_{a=false}$</td>
<td>$p_a = p_{a=true} \oplus p_{a=false}$</td>
</tr>
<tr>
<td>= (true $\lor$ (b $\land$ c)) $\oplus$ (false $\lor$ (b $\land$ c))</td>
<td>= (true $\land$ b) $\oplus$ (false $\land$ b)</td>
</tr>
<tr>
<td>= true $\oplus$ (b $\land$ c)</td>
<td>= b $\oplus$ false</td>
</tr>
<tr>
<td>= $\neg$ (b $\land$ c)</td>
<td>= b</td>
</tr>
<tr>
<td>= $\neg$ b $\lor$ $\neg$ c</td>
<td>= b</td>
</tr>
</tbody>
</table>

• “$NOT b \lor NOT c$” means either $b$ or $c$ can be false
• RACC requires the same choice for both values of $a$, CACC does not
Repeated Variables

- The definitions in this chapter yield the same tests no matter how the predicate is expressed

- \((a \lor b) \land (c \lor b) == (a \land c) \lor b\)

- \((a \land b) \lor (b \land c) \lor (a \land c)\)
  - Only has 8 possible tests, not 64

- Use the simplest form of the predicate, and ignore contradictory truth table assignments

A More Subtle Example

\[ p = (a \land b) \lor (a \land \neg b) \]

\[
\begin{align*}
p_a &= p_{a=true} \oplus p_{a=false} \\
&= ((true \land b) \lor (true \land \neg b)) \oplus ((false \land b) \lor (false \land \neg b)) \\
&= (b \lor \neg b) \oplus false \\
&= true \oplus false \\
&= true
\end{align*}
\]

\[ p_b = p_{b=true} \oplus p_{b=false} \]

\[
\begin{align*}
p_b &= (a \land true) \lor (a \land \neg true)) \oplus ((a \land false) \lor (a \land \neg false)) \\
&= (a \lor false) \oplus (false \lor a) \\
&= a \oplus a \\
&= false
\end{align*}
\]

- \(a\) always determines the value of this predicate
- \(b\) never determines the value – \(b\) is irrelevant!
Infeasible Test Requirements

- Consider the predicate:

\[(a > b \land b > c) \lor c > a\]

- \((a > b) = true, (b > c) = true, (c > a) = true\) is infeasible

- As with graph-based criteria, infeasible test requirements have to be recognized and ignored

- Recognizing infeasible test requirements is hard, and in general, undecidable

- Software testing is inexact – engineering, not science

Logic Coverage Summary

- Predicates are often very simple
  - PC may be enough
  - COC is practical

- Control software often has many complicated predicates, with lots of clauses

- Question … why don’t complexity metrics count the number of clauses in predicates?