Software Model Checking: Theory and Practice

Lecture: Specification Checking - Temporal Logic
Objectives

- Understand why temporal logic can be a useful formalism for specifying properties of concurrent/reactive systems.
- Understand the intuition behind Computation Tree Logic (CTL) – the specification logic used e.g., in the well-known SMV model-checker.
- Be able to confidently apply Linear Temporal Logic (LTL) – the specification logic used in e.g., Bogor and SPIN – to specify simple properties of systems.
- Understand the formal semantics of LTL.
Outline

- CTL by example
- LTL by example
- LTL - formal definition
- Common properties to be stated for concurrent systems and how they can be specified using LTL
- Bogor’s support for LTL
To Do

- Show never claims being generated from LTL formula
- For you to do’s…
We want to reason about execution trees
- tree node = snapshot of the program's state

Reasoning consists of two layers
- defining predicates on the program states (control points, variable values)
- expressing temporal relationships between those predicates
Why Use Temporal Logic?

- Requirements of concurrent, distributed, and reactive systems are often phrased as constraints on *sequences of events or states* or constraints on *execution paths*.

- Temporal logic provides a formal, expressive, and compact notation for realizing such requirements.

- The temporal logics we consider are also strongly tied to various computational frameworks (e.g., automata theory) which provides a foundation for building verification tools.
Computational Tree Logic (CTL)

Syntax

\( \Phi : ::= P \)

\( | ! \Phi \ | \ \Phi \ \&\& \ \Phi \ | \ \Phi \ | \ | \ \Phi \ | \ \Phi \rightarrow \ \Phi \)  

\( | \ \text{AG} \ \Phi \ | \ \text{EG} \ \Phi \ | \ \text{AF} \ \Phi \ | \ \text{EF} \ \Phi \) 

\( | \ \text{AX} \ \Phi \ | \ \text{EX} \ \Phi \ | \ A[\ \Phi \ U \ \Phi] \ | \ E[\ \Phi \ U \ \Phi] \)
Computational Tree Logic (CTL)

Syntax

\( \Phi ::= P \)

\( \neg \Phi \mid \Phi \land \Phi \mid \Phi \lor \Phi \mid \Phi - > \Phi \)

\( \text{AG} \ \Phi \mid \text{EG} \ \Phi \mid \text{AF} \ \Phi \mid \text{EF} \ \Phi \)

\( \text{AX} \ \Phi \mid \text{EX} \ \Phi \mid \text{A}[\Phi \ U \ \Phi] \mid \text{E}[\Phi \ U \ \Phi] \)

Semantic Intuition

\( \text{AG} \ p \quad \text{…along All paths p holds Globally} \)

\( \text{EG} \ p \quad \text{…there Exists a path where p holds Globally} \)

\( \text{AF} \ p \quad \text{…along All paths p holds at some state in the Future} \)

\( \text{EF} \ p \quad \text{…there Exists a path where p holds at some state in the Future} \)
Computational Tree Logic (CTL)

Syntax

\[ \Phi ::= P \]

| !\Phi | \Phi \&\& \Phi | \Phi \| \Phi | \Phi \rightarrow \Phi |
| AG \Phi | EG \Phi | AF \Phi | EF \Phi |
| AX \Phi | EX \Phi | A[\Phi U \Phi] | E[\Phi U \Phi] |

Semantic Intuition

- **AX p**: ...along *All* paths, p holds in the *next* state
- **EX p**: ...there *Exists* a path where p holds in the *next* state
- **A[p U q]**: ...along *All* paths, p holds *Until* q holds
- **E[p U q]**: ...there *Exists* a path where p holds *Until* q holds
Computation Tree Logic

AG p

AG p

AG p

AG p

AG p

AG p

AG p

AG p

AG p

AG p

AG p
Computation Tree Logic

EG p

Specification Checking: Temporal Logic
Computation Tree Logic

AF $p$

Specification Checking: Temporal Logic
Computation Tree Logic

EF p
Computation Tree Logic

AX p

Specification Checking: Temporal Logic
Computation Tree Logic

EX p

Specification Checking : Temporal Logic
Computation Tree Logic

$A[p \ U q]$
Computation Tree Logic

$E[p \cup q]$
Example CTL Specifications

For any state, a request (e.g., for some resource) will eventually be acknowledged

$$\text{AG(requested} \rightarrow \text{AF acknowledged)}$$
Example CTL Specifications

From any state, it is possible to get to a restart state

\[ \text{AG(EF restart)} \]
Example CTL Specifications

An upwards travelling elevator at the second floor does not change its direction when it has passengers waiting to go to the fifth floor.

$$\text{AG}((\text{floor}=2 \land \text{direction}=\text{up} \land \text{button5pressed}) \rightarrow \text{A}[\text{direction}=\text{up} \lor \text{floor}=5])$$
Semantics for CTL (excerpts)

- For $p \in AP$:
  \[ s \models p \iff p \in L(s) \quad s \models \neg p \iff p \notin L(s) \]
- $s \models f \land g \iff s \models f \text{ and } s \models g$
- $s \models f \lor g \iff s \models f \text{ or } s \models g$
- $s \models EXf \iff \exists \pi = s_0s_1... \text{ from } s: \ s_1 \models f$
- $s \models E(f U g) \iff \exists \pi = s_0s_1... \text{ from } s$
  \[ \exists j \geq 0 \ [ \ s_j \models g \text{ and } \forall i : 0 \leq i < j \ [s_i \models f ] ] \]
- $s \models EGf \iff \exists \pi = s_0s_1... \text{ from } s \ \forall i \geq 0: \ s_i \models f$

Source: Orna Grumberg
CTL Notes

- Invented by E. Clarke and E. A. Emerson (early 1980’s)
- Specification language for Symbolic Model Verifier (SMV) model-checker
- SMV is a *symbolic* model-checker instead of an *explicit-state* model-checker
- Symbolic model-checking uses Binary Decision Diagrams (BDDs) to represent boolean functions (both transition system and specification)
Linear Time Logic

Restrict path quantification to "ALL" (no "EXISTS")
Linear Time Logic

Restrict path quantification to "ALL" (no "EXISTS")

Reason in terms of branching traces instead of branching trees
**Linear Time Logic (LTL)**

**Syntax**

\[ \Phi ::= P \mid \neg \Phi \mid \Phi \land \Phi \mid \Phi \lor \Phi \mid \Phi \rightarrow \Phi \mid [\ ] \Phi \mid <> \Phi \mid \Phi U \Phi \mid X \Phi \]

...primitive propositions
...propositional connectives
...temporal operators

**Semantic Intuition**

\[ [\ ] \Phi \quad \text{...always} \ \Phi \]

\[ <> \Phi \quad \text{...eventually} \ \Phi \]

\[ \Phi U \Gamma \quad \text{...} \ \Phi \text{ until } \Gamma \]
Linear Time Logic

Along all paths, it must be the case that globally (i.e., in each state we come to) eventually $p$ will hold.

Expresses a form of fairness

- $p$ must occur infinitely often along the path
- To check $\Phi$ under the assumption of fair traces, check $[]<>p \rightarrow \Phi$
Linear Time Logic

“Along all paths, eventually it is the case that $p$ holds at each state” (i.e., “eventually permanently $p$”)

“Any path contains only finitely many $\neg p$ states”
Linear Time Logic

\[ p \ W q = []p \parallel (p \ U q) \]

- “p unless q”, or “p waiting for q”, or “p weak-until q”
Semantics of LTL is given with respect to a (usually infinite) path or trace
- $\pi = s_1 s_2 s_3 \ldots$

We write $\pi_i$ for the suffix starting at $s_i$, e.g.,
- $\pi_3 = s_3 s_4 s_5 \ldots$

A system satisfies an LTL formula $f$ if each path through the system satisfies $f$. 
Semantics of LTL

- For $p \in \text{AP}$:
  - $\pi \models p \iff p \in L(s_1)$ \hspace{1cm} $\pi \models \neg p \iff p \notin L(s_1)$
  - $\pi \models f \land g \iff \pi \models f$ and $\pi \models g$
  - $\pi \models f \lor g \iff \pi \models f$ or $\pi \models g$
  - $\pi \models Xf \iff \pi_2 \models f$
  - $\pi \models <>f \iff \exists i \geq 1. \pi_i \models f$
  - $\pi \models []f \iff \forall i \geq 1. \pi_i \models f$
  - $\pi \models (f U g) \iff \exists i \geq 1. \pi_i \models g$
    and $\forall j : 1 \leq j < i-1. \pi_j \models f$
LTL Notes

- Invented by Prior (1960’s), and first use to reason about concurrent systems by A. Pnueli, Z. Manna, etc.
- LTL model-checkers are usually explicit-state checkers due to connection between LTL and automata theory
- Most popular LTL-based checker is SPIN (G. Holzman)
Comparing LTL and CTL

- CTL is not strictly more expression than LTL (and vice versa)
-CTL* invented by Emerson and Halpern in 1986 to unify CTL and LTL
- We believe that almost all properties that one wants to express about software lie in intersection of LTL and CTL
As for regular properties, Bogor defines an extension for LTL properties

- Property extension is the same

**LTL extension**

- Implemented by
  
  …bogor.module.property.ltl.LinearTemporalLogicModule

- Supports
  
  - Atomic propositions and literals (e.g., true/false)
  - Propositional connectives (e.g., and, or)
  - Temporal operators (e.g., always, eventually)
LTL extension

extension LTL for edu.ksu.cis.projects.bogor.module.property.ltl.LinearTemporalLogicModule
{
    typedef Formula;

    expdef LTL.Formula prop(string);
    expdef LTL.Formula literal(boolean);
    expdef LTL.Formula always(LTL.Formula);
    expdef LTL.Formula eventually(LTL.Formula);
    expdef LTL.Formula negation(LTL.Formula);
    expdef LTL.Formula until(LTL.Formula, LTL.Formula);
    expdef LTL.Formula release(LTL.Formula, LTL.Formula);
    expdef LTL.Formula equivalence(LTL.Formula, LTL.Formula);
    expdef LTL.Formula implication(LTL.Formula, LTL.Formula);
    expdef LTL.Formula conjunction(LTL.Formula, LTL.Formula);
    expdef LTL.Formula disjunction(LTL.Formula, LTL.Formula);

    expdef boolean temporalProperty(Property.ObservableDictionary, LTL.Formula);
}

Specification Checking : Temporal Logic
An Example

Mutual exclusion in ReadersWriters

fun mutualExclusion() returns boolean
  = LTL.temporalProperty(
      Property.createObservableDictionary(
        Property.createObservableKey(
          "someReading", activeReaders>0),
        Property.createObservableKey(
          "someWriting", activeWriters>0)
      ),
      LTL.always(
        LTL.implication(
          LTL.prop("someReading"),
          LTL.negation(LTL.prop("someWriting")))
      )
  );
Bogor Configuration

Use the defaults except for these settings

edu.ksu.cis.projects.bogor.module.IStateFactory=
edu.ksu.cis.projects.bogor.module.property.fsa.FSAStateFactory

edu.ksu.cis.projects.bogor.ast.transform.ISystemTransformer=
edu.ksu.cis.projects.bogor.module.property.ltl.LtlSystemTransformer

edu.ksu.cis.projects.bogor.module.ISearcher=
edu.ksu.cis.projects.bogor.module.property.buechi.NestedFSASearcher

edu.ksu.cis.projects.bogor.module.IStateManager.stateAugmenter=
edu.ksu.cis.projects.bogor.module.property.fsa.FSAStateAugmenter

LtlFunId=mutualExclusion