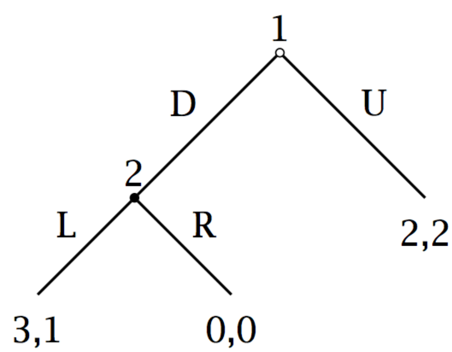
CSE 516: Homework 2

Due: March 5, 2015

Note: Please keep in mind the collaboration policy as specified in the course syllabus. If you discuss questions with others you **must** write their names on your submission, and if you use any outside resources you **must** reference them. Keep in mind that homework (in hardcopy) is due **at the beginning of lecture.** Grading will take into account how clearly you communicate your solutions, so please write your answers up carefully. There are seven questions on two pages.

1. (5 points) **Subgame perfection warmup:** Consider the following game in extensive form:



   What are the pure-strategy Nash equilibria of this game (when you consider it in normal form)? Are any of them not subgame perfect? Why?

2. (20 points) **A very unfair final:** 15 people show up for the final exam in CSE 777, "Gamification". The instructor orders them by their current score in the class, with the person with the highest score first and the person with the lowest score last (there are no ties). So now let's say Student 1 is the highest scoring, Student 2 the second highest, etc. In this order, they play a splitting game of the following form. 100 points are to be split between the students, with only integer splits allowed. Student i gets to propose a split among all the remaining students (students i through 15). Now all the remaining students (i through 15) vote on the split. If it gets at least 50% of the votes (therefore, ties are broken in favor of the proposal) the proposal is accepted and the proposed split constitutes the scores of the students on the final. If the proposal is rejected, the student who proposed it receives an F for the course (which is a strictly worse outcome for all students than receiving 0 on the final) and leaves the game, and student $i + 1$ gets to make the next proposal.

   (a) What is the (subgame perfect) outcome of this game?

   (b) How does this change if ties are now broken in favor of rejecting the proposal?

3. (15 points) **Scheduling game** Problem 2.3 in Chapter 2 of Parkes and Seuken (available on Piazza)

4. (15 points) **Network routing game** Problem 2.6 in Chapter 2 of Parkes and Seuken (available on Piazza)

5. (15 points) **Load balancing game** Problem 2.7 in Chapter 2 of Parkes and Seuken (available on Piazza)

6. (15 points) **On selling and rebuying** I'm auctioning off a book. Everyone in class receives a private signal s_i of the value of the book, and these signals are uniformly distributed in $[0, M]$. I'm going to run an auction where everyone gets to write down her or his bid b_i for the book on a piece of paper (along with his or her name), and put it in an envelope. I will sell the book to the highest bidder at her or his bid, b_i, but immediately buy it back from him or her at a price equal to the average of all the signals each individual received. Find a Bayes-Nash equilibrium for this game, where each bid b_i is a linear function of the received signal s_i.

7. (15 points) **Alternating ultimatums** Consider the following variant of the ultimatum game. Alice can make an offer to Bob to split a reward of 1 (the units are really large, say on the order of $1M, but it's convenient to think of it as one unit). She makes some offer $a \in [0, 1]$ and Bob decides whether to accept or not. If Bob accepts, they receive the utility from that split. However, if Bob rejects, then *he* gets to make the next offer – he offers Alice some $b \in [0, 1]$ and then she has to decide whether to accept or not. If she rejects, then she gets to make the next offer again, and so on forever. There is one more twist. Utilities are discounted, so that one unit some t rounds in the future is only worth γ^t ($\gamma \in (0, 1)$) of one unit now (so, for example, if Alice accepts an offer of $1/2$ three rounds from now, after her offer is rejected and Bob's next offer and her next offer are also both rejected, then she receives utility (from her current perspective) of $\gamma^3(1/2)$).

   Show that the following strategies are a subgame perfect equilibrium: At any time, Alice offers $a^* = 1/(1 + \gamma)$ and accepts any $b \leq b^*$, while Bob offers $b^* = 1/(1 + \gamma)$ and accepts any $a \leq a^*$.

   For 2 points and 10000 MASEcoins of extra credit, show that for *any* $x \in [0, 1]$, there is some *Nash* equilibrium of the game where the reward of 1 is divided as x going to Alice and $1 - x$ going to Bob.