Pixelated Image Abstraction with Integrated User Constraints

Timothy Gerstner^a, Doug DeCarlo^a, Marc Alexa^c, Adam Finkelstein^b, Yotam Gingold^{a,d}, Andrew Nealen^a

^aRutgers University ^bPrinceton University ^cTU Berlin ^dColumbia University

Abstract

We present an automatic method that can be used to abstract high resolution images into very low resolution outputs with reduced color palettes in the style of pixel art. Our method simultaneously solves for a mapping of features and a reduced palette needed to construct the output image. The results are an approximation to the results generated by pixel artists. We compare our method against the results of two naive methods common to image manipulation programs, as well as the hand-crafted work of pixel artists. Through a formal user study and interviews with expert pixel artists we show that our results offer an improvement over the naive methods. By integrating a set of manual controls into our algorithm, we give users the ability to add constraints and incorporate their own choices into the iterative process.

21

22

24 25

26

27

28

Keywords: pixel art, image abstraction, non-photorealistic rendering, image segmentation, color quantization

1 1. Introduction

We see pixel art every day. Modern day handheld 2 devices such as the iPhone, Android devices and the 3 Nintendo DS regularly utilize pixel art to convey 4 information on compact screens. Companies like 5 Coca-Cola, Honda, Adobe, and Sony use pixel 6 art in their advertisements [1]. It is used to make icons for desktops and avatars for social 8 networks. While pixel art stems from the need 9 to optimize imagery for low resolution displays, 10 it has emerged as a contemporary art form in its 11 own right. For example, it has been featured by 12 Museum of Modern Art, and there are a number of 13 passionate online communities devoted to it. The 14 "Digital Orca" by Douglas Coupland is a popular 15 sight at the Vancouver Convention Center. France 16 recently was struck by a "Post-it War"¹, where 17 people use Post-It notes to create pixel art on their 18 windows, competing with their neighbors across 19 workplaces, small businesses, and homes. 20

Email address: timgerst@cs.rutgers.edu (Timothy Gerstner)

¹http://www.postitwar.com/

Preprint submitted to Computers and Graphics



Figure 1: Examples of pixel art. "Alice Blue" and "Kyle Red" by Alice Bartlett. Notice how faces are easily distinguishable even with this limited resolution and palette. The facial features are no longer proportionally accurate, similar to deformation in a caricature.

What makes pixel art both compelling and difficult is the limitations imposed on the medium. With a significantly limited palette and resolution to work with, the task of creating pixel art becomes one of carefully choosing the set of colors and placing each pixel such that the final image best depicts the original subject. This task is particularly difficult as pixel art is typically viewed at a distance

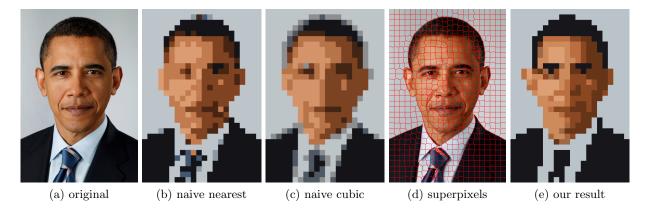


Figure 2: Pixel art images simultaneously use very few pixels and a tiny color palette. Attempts to represent image (a) using only 22×32 pixels and 8 colors using (b) nearest-neighbor or (c) cubic downsampling (both followed by median cut color quantization), result in detail loss and blurriness. We optimize over a set of superpixels (d) and an associated color palette to produce output (e) in the style of pixel art.

63

64

65

66

67

68

69

70

71

72

73

74

75

76

77

78

79

81

82

83

84

85

86

87

88

89

90

91

92

where the pixel grid is clearly visible, which has 29 been shown to contribute to the perception of the 30 image [2]. As seen in Figure 1, creating pixel art is 31 not a simple mapping process. Features such as the 32 eyes and mouth need to be abstracted and resized 33 in order to be represented in the final image. The 34 end product, which is no longer physically accurate, 35 still gives the impression of an identifiable person. 36

However, few, if any methods exist to automat-37 ically or semi-automatically create effective pixel 38 art. Existing downsampling methods, two of which 39 are shown in Figure 2, do not accurately capture 40 the original subject. Artists often turn to mak-41 ing pieces by hand, pixel-by-pixel, which can take 42 a significant amount of time and requires a certain 43 degree of skill not easily acquired by novices of the 44 art. Automated and semi-automated methods have 45 been proposed for other popular art forms, such as 46 line drawing [3, 4] and painting [5]. Methods such 47 as [6] and [7] not only abstract images, but do so 48 while retaining salient features. 49

We introduce an entirely automated process that 50 transforms high resolution images into low resolu-51 tion, small palette outputs in a pixel art style. At 52 the core of our algorithm is a multi-step iterative 53 process that simultaneously solves for a mapping 54 of features and a reduced palette to convert an in-55 put image into a pixelated output image. In the 56 first part of each iteration we use a modified version 57 of an image segmentation proposed by Achanta et 58 59 al. [8] to map regions of the input image to output pixels. In the second step, we utilize an adaptation 60 of mass-constrained deterministic annealing [9] to 61 find an optimal palette and its association to out-62

put pixels. These steps are interdependent, and the final solution is an optimization of both the spatial and palette sizes specified by the user. Throughout this process we utilize the perceptually uniform CIELAB color space [10]. The end result serves as an approximation to the process performed by pixel artists (Figure 2, right).

This paper presents an extended edition of Pixelated Image Abstraction [11]]. In addition to an expanded results section, we have added a set of user controls to bridge the gap between the manual process of an artist and the automated process of our algorithm. These controls allow the user to provide as much or as little input into the process as desired, to produce a result that leverages both the strengths of our automated algorithm and the knowledge and personal touch of the user.

Aside from assisting a class of artists in this medium, applications for this work include automatic and semi-automatic design of low-resolution imagery in handheld, desktop, and online contexts like Facebook and Flickr, wherever iconic representations of high-resolution imagery are used.

2. Related Work

One aspect of our problem is to reproduce an image as faithfully as possible while constrained to just a few output colors. Color quantization is a classic problem wherein a limited color palette is chosen based on an input image for indexed color displays. A variety of methods were developed in the 1980's and early 1990's prior to the

advent of inexpensive 24-bit displays, for exam- 146 94 ple [12, 13, 14, 15]. A similar problem is that of 95 selecting a small set of custom inks to be used in 96 147 printing an image [16]. These methods rely only 97 on the color histogram of the input image, and are 98 1/18 typically coupled to an independent dithering (or 99 halftoning) method for output in a relatively high 100 150 resolution image. In our problem where the spatial 101 151 resolution of the output is also highly constrained, 102 152 we optimize simultaneously the selection and place-103 ment of colors in the final image. 104 154

The problem of image segmentation has been 105 155 extensively studied. Proposed solutions include 106 156 graph-cut techniques, such as the method proposed 107 157 by Shi and Malik [17], and superpixel-based meth-108 158 ods QuickShift [18], Turbopixels [19], and SLIC [8]. 109 In particular, SLIC (Simple Linear Interative Clus-110

tering) produces regular sized and spaced regions ¹⁵⁹ 111 with low computational overhead given very few in-112

put parameters. These characteristics make SLIC 160 113 an appropriate starting point for parts of our 161 114 method. 115 162

Mass-constrained deterministic annealing 163 116 (MCDA) [9] is a method that uses a probabilistic 164 117 assignment while clustering. Similar to k-means, it 165 118 uses a fixed number of clusters, but unlike k-means 166 119 it is independent of initialization. Also, unlike 167 120 simulated annealing [20], it does not randomly 168 121 search the solution space and will converge to the 169 122 same result every time. We use an adapted version 170 123 of MCDA for color palette optimization. 171 124

Puzicha et al. [21] proposed a method that re-172 125 duces the palette of an image and applies half- 173 126 toning using a model of human visual perception. 174 127 While their method uses deterministic annealing 175 128 and the CIELAB space to find a solution that 176 129 optimizes both color reduction and dithering, our 177 130 method instead emphasizes palette reduction in 178 131 parallel with the reduction of the output resolution. 179 132 Kopf and Lischinski [22] proposed a method that 180 133

extracts vector art representations from pixel art. 181 134 This problem is almost the inverse of the one pre-182 135 sented in this paper. However, while their solution 183 136 focuses on interpolating unknown information, con-184 137 verting an image to pixel art requires compressing 185 138 known information. 139

Finally, we show that with minor modification 187 140 our algorithm can produce "posterized" images, 188 141 wherein large regions of constant color are sepa-142 189 rated by vectorized boundaries. To our knowledge, 190 143 little research has addressed this problem, though 191 144 it shares some aesthetic concerns with the *artistic* 192 145

thresholding approach of Xu and Kaplan [23].

3. Background

Our method for making pixel art builds upon two existing techniques, which we briefly describe in this section.

SLIC. Achanta et al. [8] proposed an iterative method to segment an image into regions termed "superpixels." The algorithm is analogous to kmeans clustering [24] in a five dimensional space (three color and two positional), discussed for example in Forsyth and Ponce [25]. Pixels in the input image p_i are assigned to superpixels p_s by minimizing

$$d(p_i, p_s) = d_c(p_i, p_s) + m\sqrt{\frac{N}{M}}d_p(p_i, p_s) \qquad (1)$$

where d_c is the color difference, d_p is the positional difference, M is the number of pixels in the input image, N is the number of superpixels, and m is some value in the range [0, 20] that controls the relative weight that color similarity and pixel adjacency have on the solution. The color and positional differences are measured using Euclidean distance (as are all distances in our paper, unless otherwise noted), and the colors are represented in LAB color space. Upon each iteration, superpixels are reassigned to the average color and position of the associated input pixels.

Mass Constrained Deterministic Annealing. MCDA [9] is a global optimization method for clustering that draws upon an analogy with the process of annealing a physical material. We use this method both for determining the colors in our palette, and for assigning one of these palette colors to each pixel—each cluster corresponds to a palette color.

MCDA is a fuzzy clustering algorithm that probabilistically assigns objects to clusters based on their distance from each cluster. It relies on a temperature value T, which can be viewed as proportional to the expected variance of the clusters. Initially, T is set to a high value T_0 , which makes each object equally likely to belong to any cluster. Each time the system locally converges T is lowered (and the variance of each cluster decreases). As this happens, objects begin to prefer favor particular clusters, and as T approaches zero each object becomes effectively assigned to a single cluster, at which point the final set of clusters is produced.

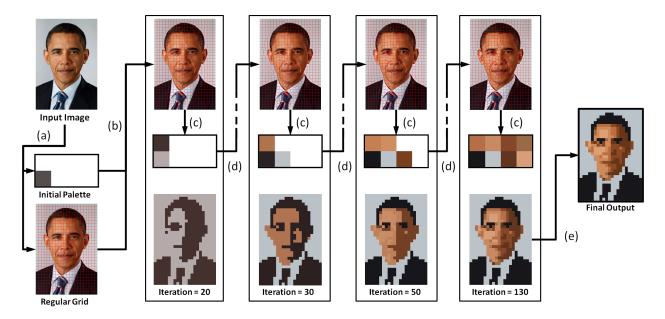


Figure 3: The pipeline of the algorithm. The superpixels (a) are initialized in a regular grid across the input image, and the palette is set to the average color of the M input pixels. The algorithm then begins iterating (b). Each iteration has two main steps: (c) the assignment of input pixels to superpixels, and (d) the assignment of superpixels to colors in the palette and updating the palette. This not only updates each color, but may also add new colors to the palette. After convergence, the palette is saturated (e) producing the final output.

In Section 4.3 we provide a formal definition of the
 conditional probability we use to assign superpixels
 to colors in the palette.

Since at high T having multiple clusters is re- $_{221}$ 196 dundant, MCDA begins with a single cluster, rep-197 resented internally by two sub-clusters. At the be-198 222 ginning of each iteration these sub-clusters are set 199 223 to slight permutations of their mean. At a high 200 224 T these clusters converge to the same value after 201 225 several iterations, but as the temperature is low-202 226 ered they begin to naturally separate. When this 203 227 occurs, the cluster is split into two separate clus-204 228 ters (each represented by their own sub-clusters). 205 229 This continues recursively until the (user specified) 206 230 maximum number of clusters is reached. 207

208 4. Method

Our automated algorithm is an iterative 233 209 procedure—an example execution is shown in Fig- 234 210 ure 3. The process begins with an input image of 235 211 width $w_{\rm in}$ and height $h_{\rm in}$ and produces an output $_{^{236}}$ image of width $w_{\rm out}$ and height $h_{\rm out}$ which con- $_{^{237}}$ 212 213 tains at most K different colors—the palette size. $_{238}$ 214 Given the target output dimensions and palette 239 215 size, each iteration of the algorithm segments the 240 216 pixels in the input into regions corresponding to 241 217

pixels in the output and solves for an optimal palette. Upon convergence, the palette is saturated to produce the final output. In this section, we describe our algorithm in terms of the following:

- **Input Pixels** The set of pixels in the input image, denoted as p_i where $i \in [1, M]$, and $M = w_{\text{in}} \times h_{\text{in}}$.
- **Ouput Pixels** The set of pixels in the output image, denoted as p_o where $o \in [1, N]$, and $N = w_{\text{out}} \times h_{\text{out}}$.
- **Superpixel** A region of the input image, denoted as p_s where $s \in [1, N]$. The superpixels are a partition of the input image.
- **Palette** A set of K colors c_k , $k \in [1, K]$ in LAB space.

Our algorithm constructs a mapping for each superpixel that relates a region of input pixels with a single pixel in the output, as in Figure 4. The algorithm proceeds similarly to MCDA, with a superpixel refinement and palette association step performed upon each iteration, as summarized in Algorithm 1. Section 5.1 describes how the algorithm can be expanded to allow a user to indicate important regions in the input image.

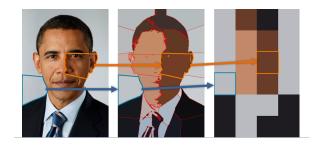


Figure 4: Pixels in the input image (left) are associated with superpixel regions (middle). Each superpixel region corresponds to a single pixel in the output image (right).

Algorithm 1

- \triangleright initialize superpixels, palette and temperature T (Section 4.1)
- \triangleright while $(T > T_f)$
 - ▶ **refine** superpixels with 1 step of modified SLIC (Section 4.2)
 - ▷ associate superpixels to colors in the palette (Section 4.3)
 - \triangleright refine colors in the palette (Section 4.3)
 - \triangleright if (palette converged)
 - \triangleright reduce temperature $T = \alpha T$
 - \triangleright expand palette (Section 4.3)
- \triangleright post-process (Section 4.4)

242 4.1. Initialization

276 The N superpixel centers are initialized in a 243 277 regular grid across the input image, and each input 244 278 pixel is assigned to the nearest superpixel (in (x, y)) 245 space, measured to the superpixel center). The 279 246 palette is initialized to a single color, which is $^{\scriptscriptstyle 280}$ 247 281 set to the mean value of the M input pixels. 248 282 All superpixels are assigned this mean color. See 249 Figure 3, step (a). 250

284 The temperature T is set to $1.1T_c$, where T_c 251 is the critical temperature of the set of M input $^{\mbox{\tiny 285}}$ 252 pixels, defined as twice the variance along the ²⁸⁶ 253 major principal component axis of the set in LAB $^{\ 287}$ 254 space [9]. The T_c of a set of objects assigned to ²⁸⁸ 255 a cluster is the temperature at which a cluster 289 256 will naturally split. Therefore, this policy ensures 290 257 that the initial temperature is easily above the 258 292 temperature at which more than one color in the 259 palette would exist. 293 260 294

261 4.2. Superpixel refinement

This stage of the algorithm assigns pixels in the 297 input image to superpixels, which correspond to 298

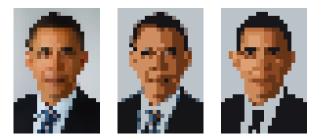


Figure 5: Our method uses palette colors when finding superpixels. Using the mean color of a superpixel works when the palette is unconstrained (left), but fails when using a constrained palette (middle). This is because the input pixels cluster into superpixels based on colors that do not exist in the final image, which creates a discrepancy. Using the palette colors to represent the superpixels (right) removes this discrepancy.

pixels in the output image—see steps (b) and (d) in Figure 3.

To accomplish this task, we use a single iteration of our modified version of SLIC. In the original SLIC algorithm, upon each iteration, every input pixel is assigned to the superpixel that minimizes $d(p_i, p_s)$, and the color of each superpixel is set to the mean color value of its associated input pixels, m_s . However, in our implementation, the color of each superpixel is set to the palette color that is associated with the superpixel (the construction of this mapping is explained in Section 4.3). This interdependency with the palette forces the superpixels to be optimized with respect to the colors in the palette rather than the colors in the input image. Figure 5 shows the results of using the mean color value instead of our optimized palette used in Figure 2.

However, this also means the color error will be generally higher. As a result, we've found that minimizing $d(p_i, p_s)$ using a value of m = 45 is more appropriate in this case (Achanta et al. [8] suggest m = 10). This increases the weight of the positional distance and results in a segmentation that contains superpixels with relatively uniform size.

Next, we perform two steps, one modifies each superpixel's (x, y) position for the next iteration, and one changes each superpixel's representative color. Each step is an additional modification to the original SLIC method and significantly improves the final result.

As seen in Figure 6 (left), SLIC results in superpixel regions which tend to be organized in 6connected neighborhoods (i.e. a hexagonal grid). This is caused by how the (x, y) position of each

295

296

264

265

266

267

268

269

270

271

272

273

274

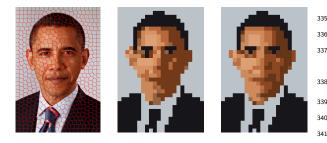


Figure 6: Without the Laplacian smoothing step, the superpixels (left) tend to have 6-connected neighborhoods. This causes small distortions in the output (center), which are particularly noticeable on the ear, eye and mouth, when compared to original output that uses the superpixels that included the smoothing step (right).

superpixel is defined as the average position of the 299 input pixels associated with it. This hexagonal grid 300 does not match the neighborhoods of the output 301 pixels, which are 8-connected (i.e. a rectangular 302 grid) and will give rise to undesirable distortions 303 of image features and structures in the output, as 304 seen in Figure 6(center). 305

We address this problem with Laplacian smooth-306 ing. Each superpixel center is moved a percentage 307 of the distance from its current position to the aver-308 age position of its 4-connected neighbors (using the 309 neighborhoods at the time of initialization). We 310 use 40%. As seen in Figure 2 (d), this improves the 311 correspondence between the superpixel and output 312 pixel neighborhoods. Specifically, it helps ensure 313 that superpixel regions that are adjacent in the in-314 put map are also adjacent pixels in the output. To 315 be clear, it is only in the next iteration when the 316 superpixels will be reassigned based on this new 317 center, due to the interleaved nature of our algo-366 318 rithm. 319

In our second additional step, the color repre-320 sentatives of the superpixels are smoothed. In the 321 original SLIC algorithm, the representative color for 322 each superpixel is the average color m_s of the input 323 pixels associated with it. However, simply using the 324 mean color can become problematic for continuous 325 regions in the image that contain a color gradient 326 (such as a smooth shadowed surface). While this 327 gradient appears natural in the input image, the 328 region will not appear continuous in the pixelated 329 output. 330

331 To remedy this, our algorithm adjusts the values of m_s using a bilateral filter. We construct an 332 image of size $w_{\text{out}} \times h_{\text{out}}$ where each superpixel 333 is assigned the same position as its corresponding 334

output pixel, with value m_s . The colors that results from bilaterally filtering this image, m_s' are used while iterating the palette.

4.3. Palette refinement

342

343

345

346

347

348

349

350

351

352

353

354

355

356

357

358

359

360

361

362

363

364

367

Palette iteration is performed using MCDA [9]. Each iteration of the palette, as seen in step (c) in Figure 3, can be broken down into three basic steps: associating superpixels to colors in the palette, refining the palette, and expanding the palette. The associate and refine steps occur every iteration of our algorithm. When the palette has converged for the current temperature T, the expand step is performed.

It is important to note how we handle the subclusters mentioned in Section 3: we treat each subcluster as a separate color in the palette, and keep track of the pairs. The color of each c_k is the average color of its two sub-clusters. When the maximum size of the palette is reached (in terms of the number of distinct colors c_k), we eliminate the sub-clusters and represent each color in the palette as a single cluster.

Associate. The MCDA algorithm requires a probability model that states how likely a particular superpixel will be associated with each color in the palette. See Figure 7. The conditional probability $P(c_k|p_s)$ of a superpixel p_s being assigned color c_k depends on the color distance in LAB space and the current temperature, and is given by (after suitable normalization):

$$P(c_k|p_s) \propto P(c_k) e^{-\frac{||m_s' - c_k||}{T}}$$
(2)

 $P(c_k)$ is the probability that color c_k is assigned to any superpixel, given the existing assignment.

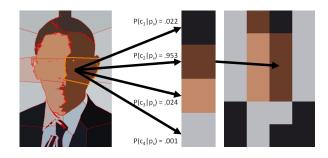


Figure 7: Each superpixel (left) is associated by some conditional probability $P(c_k|p_s)$ to each color in the palette (middle). The color with the highest probability is assigned to the superpixel and its associated output pixel in the final image (right).

³⁶⁸ Upon initialization, there is only one color, and ⁴¹³ ³⁶⁹ thus this value is initialized to 1. As more colors ⁴¹⁴ ³⁷⁰ are introduced into the palette, the value of this ⁴¹⁵ ³⁷¹ probability is computed by marginalizing over p_s : ⁴¹⁶

$$P(c_k) = \sum_{s=1}^{N} P(c_k|p_s) P(p_s)$$
(3)
(3)
(418)
(419)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(420)
(42

For the moment, $P(p_s)$ simply has a uniform distri-373 421 bution. This will be revisited in Section 5.1 when $_{422}$ 374 incorporating user-specified importance. The val- 423 375 ues of $P(c_k)$ are updated after the values of $P(c_k|p_s)$ 424 376 are computed using Equation 2. Each superpixel is 425 377 assigned to the color in the palette that maximizes 426 378 $P(c_k|p_s)$. Intermediate results of this assignment 379 427 can be seen in Figure 3 (bottom row). The expo-380 428 nential distribution in Equation 2 tends towards a 429 381 uniform distribution for large values of T, in which 382 430 case each superpixel will be evenly associated with 431 383 every palette color. As T decreases, superpixels fa- $_{432}$ 384 vor colors in the palette that are less distant. At 433 385 the final temperature, the generic situation after $_{434}$ 386 convergence has $P(c_k|p_s) = 1$ for a single color 435 387 in the palette and $P(c_k|p_s) = 0$ for the rest. In $_{436}$ 388 this case, deterministic annealing is equivalent to $_{437}$ 389 k-means clustering. 390 438

Refine. The next step is to refine the palette by reassigning each color c_k to a weighted average of all superpixel colors, using the probability of association with that color:

$$c_{k} = \frac{\sum_{s=1}^{N} m_{s}' P(c_{k}|p_{s}) P(p_{s})}{P(c_{k})}$$
(4)

395

This adapts the colors in the existing palette given the revised superpixels. Such changes in the palette can be seen in Figure 3, as the computation progresses.

Expand. Expansion only occurs during an 448 400 iteration if the palette has converged for the current 110 401 temperature T (convergence is measured by the 402 450 total change in the palette since last iteration being 451 403 less than some small value $\epsilon_{\rm palette}).~$ First, the $_{\rm ^{452}}$ 404 temperature is lowered by some factor α (we use 453 405 0.7). Next, the palette is expanded if the number 454 406 of colors is less than the number specified by the 455 407 user. For each c_k we check to see if the color 456 408 409 needs to be split into two separate colors in the 457 palette. As per MCDA, each color in the palette 458 410 is represented by two cluster points c_{k_1} and c_{k_2} . 459 411 We use $||c_{k_1} - c_{k_2}|| > \epsilon_{\text{cluster}}$ (where $\epsilon_{\text{cluster}}$ is 460 412

a sufficiently small number), to check for palette separation. If so, the two cluster points are added to the palette as separate colors, each with its own pair of cluster points. As seen in Figure 3, over the course of many iterations, the palette grows from a single color to a set of eight (which is the maximum number specified by the user in this example).

After resolving any splits, each color is represented by two sub-clusters with the same value (unless the maximum number of colors have been reached). In order for any color's sub-clusters to separate in the following iterations, c_{k_1} and c_{k_2} must be made distinctly different. To do so, we perturb the sub-clusters of each color by a small amount along the principal component axis of the cluster in LAB space. Rose [9] has shown this to be the direction a cluster will split. This perturbation allows the sub-clusters of each color to merge when $T > T_c$ and separate when $T < T_c$.

Algorithm 1 is defined so that the superpixel and palette refinement steps are iterated until convergence. The system converges when the temperature has reached the final temperature T_f and the palette converges. We use $T_f = 1$ to avoid truncation errors as the exponential component of the Equation 2 becomes small.

4.4. Palette Saturation

As a post-processing step, we provide the option to saturate the palette, which is a typical pixel artist technique, by simply multiplying the *a* and *b* channels of each color by a parameter $\beta > 1$. This value used in all our results is $\beta = 1.1$. Lastly, by converting to from LAB to RGB space, our algorithm outputs the final image.

5. User Controls

The algorithm described in Section 4 completely automates the selection of the color palette. This stands in marked contrast to the traditional, manual process of creating pixel art, where the artist carefully selects each color in the palette and its placement in the image. Therefore, we propose a set of user controls that leverage the results of our algorithm and bridges the gap between these two extremes. These controls allow the user to have in as much or as little control over the process as they want. This combines the power and speed of our automated method with the knowledge and creativity of the user.

417

439

440

441

442

443

444

445

446

The first user control, originally proposed in Pix-461 elated Image Abstraction [11], is an "importance 462 map" that acts as an additional input to our algo-463 rithm and lets the user emphasize areas of the im-464 age they believe to be important. The second and 465 third controls we propose, pixel and palette con-466 straints, are used after the automated algorithm 467 initially converges. Using these two controls, the 468 user can directly edit the palette colors and their 469 assignment in the output image, giving them full 470 control over the result. After each set of edits, the 471 user can choose to have our automated algorithm 472 continue to iterate using the current result as its 473 starting point with the user's edits as constraints 474 (see Section 5.4). To demonstrate the effectiveness 475 of these user controls, we developed a user interface 476 that was used to generate the results in Figure 16. 477

5.1. Importance Map 478

As stated in Section 4 our automated method 479 does not favor any image content. For instance, 480 nothing is in place that can distinguish between 481 foreground and background objects, or treat them 482 separately in the output. However, user input (or 483 the output of a computer vision system) can easily 484 be incorporated into our algorithm to prioritize the 485 foreground in the output. Thus, our system allows 486 additional input at the beginning of our method. 487 Users can supply a $w_{\rm in} \times h_{\rm in}$ grayscale image of 488 weights $W_i \in [0,1], i \in [1,M]$, used to indicate 489 the importance of each input pixel p_i . In our 490 interface, this is done by using a simple brush to 508 491 mark areas with the desired weight. We incorporate 509 492 this map when iterating the palette (Section 4.3) by 493 adjusting the prior $P(p_s)$. 494

Given the importance map, the value $P(p_s)$ for ⁵¹² 495 each superpixel is given by the average importance 513 496 of all input pixels contained in superpixel p_s (and 514 497 suitable normalization across all superpixels): 498

499
$$P(p_s) \propto \frac{1}{|p_s|} \sum_{\substack{i \in p_s \\ 518}} W_i$$
 (5) 517
518

 $P(p_s)$ thus determines how much each superpixel 520 500 affects the resulting palette, through Equations 3 521 501 and 4. This results in a palette that can better 522 502 represent colors in the regions of the input image 523 503 marked as important. 504

5.2. Pixel Constraints 505

In traditional pixel art, the artist needs to manu-527 506 ally choose the color of each pixel in the output. 528 507

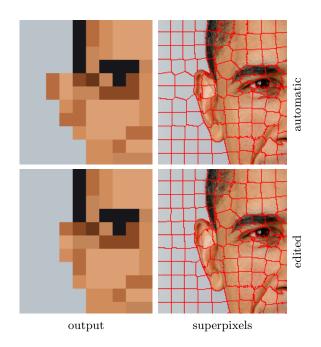


Figure 8: When the user provides constraints to the image, future iterations of the algorithm will update the superpixels in a way that seeks to decrease error under the new constraints. In this example, the original image (top left)), is modified by constraining several pixels of the ear to the background color (bottom left). As a result, the superpixels (top right) are redistributed to match the constraints (bottom right). The superpixels that used to be part of the ear now form segments of the background, and neighboring pixels in the output have changed to accommodate the new superpixel distribution.

In contrast, our automated algorithm makes the choice entirely for the user. By adding a simple constraint in to our program, we can allow the user to work in the area between these two extremes. For each pixel in the output, we allow the user to select a subset of colors in the palette. For each color not in this subset, we set the conditional probability of that color for this pixel, $P(c_k|p_s)$, to zero. This restricts the color assigned to the output pixel to the color with the highest conditional probability within the subset. Note this has the convenient property of being equivalent to the manual process when the subset is a single color, and to the automatic process when the subset is the entire palette.

As explained in Section 4.2, superpixels are represented using the color in the palette with the highest conditional probability, $P(c_k|p_s)$. Therefore, adding these constraints will affect the assignment of input pixels to superpixels in future iterations. As a result, when constraints are added by the user, neighboring superpixels will naturally

524

525

526

510

511

515

compensate as the algorithm attempts to decrease 578 529 error under these constraints, as seen in Figure 8. 579 530 In our interface, we implement this tool as a paint 580 531 brush, and allow the user to select one or more 581 532 colors from the palette to form the subset as they 582 533 paint onto the output image. Using the brush, they 534 583 are able to choose the precise color of specific pixels, 584 535 restrict the range of colors for others, and leave the 585 536 rest entirely to our algorithm. 586 537

5.3. Palette Constraints 538

589 Similarly, in traditional pixel art the artist needs 539 590 to manually choose each color of the palette. We 540 again provide a set of constraints to give the user 541 control over this process while using our algorithm. 542 593 After the palette has initially converged, the user 543 594 has the option to edit and fix colors in the palette. 544 595 This is done in one of two ways. The first is a trivial 545 method; the user directly modifies a specific color 546 in the palette. The second utilizes the information 596 547 already gathered by our algorithm. By choosing a 548 color in the palette c_k , and then a superpixel p_s 549 598 formed by our algorithm, we set c_k to the mean 550 color of that region, m_s , as found in Section 4.2. 551 While the first method allows the user to have direct 552 control, the second provides them with a way of 553 601 selecting a relatively uniform area of the original 554 602 image from which to sample a color, and without 555 603 having to specify specific values. 556

In addition to changing the color, the user has the 604 557 option to keep these colors fixed or free during any 605 558 future iterations of the algorithm. If they are fixed, 606 559 607 they will remain the same color for the rest of the 560 608 process. If they are not fixed, they will be free to 561 converge to a new value as our algorithm iterates, 562 600 starting with the initial color provided by the user's 563 edit. This gives the users another dimension of 564 611 palette control in addition to the ability to manually 565 612 choose the colors. 566 613

Note that when a color is changed in the palette, 567 areas of the original image may no longer be well 568 represented in the palette. Fortunately, during any 569 future iterations, our algorithm will naturally seek 570 to reduce this discrepancy by updating the unfixed 571 colors in the palette as it attempts to minimize error 572 and converge to a new local minimum. 573

5.4. Reiterating 574

After using any of the tools described in this sec-623 575 tion, the user has the option of rerunning our algo-576 rithm. However, rather than starting from scratch, 625 577

the algorithm begins with the results of the previous iteration, subject to the constraints specified by the user. When rerunning the algorithm, the temperature remains at the final temperature T_f it reached at convergence, and continues until the convergence condition described in Section 4.3 is met again. Note that while iterating, the algorithm maintains the user's constraints. Therefore the user can decide what the algorithm can and cannot update. Also note that since the algorithm is not starting from scratch, it is generally close to the next solution, and convergence occurs rapidly (usually less than a second). After the algorithm has converged, the user can continue making edits and rerunning the algorithm until satisfied. In this way the user becomes a part of the iterative loop, and both user and algorithm work to create a final solution.

6. Results

587

588

We tested our algorithm on a variety of input images at various output resolutions and color palette sizes (Figures 9–16). For each example, we compare *our method* to two naive approaches:

- *nearest method*: a bilateral filter followed by median cut color quantization, followed by nearest neighbor downsampling,
- *cubic method*: cubic downsampling followed by median cut color quantization. Unless otherwise stated, the results are generated using only our automated algorithm, and no user input was integrated into the result.

All of our results use the parameter settings from Section 4. Each result was produced in generally less than a minute on an Intel 2.67Ghz i7 processor with 4GB memory. Each naive result is saturated using the same method described in Section 4.4. Please note it is best to view the results up-close or zoomed-in, with each pixel being distinctly visible.

In Figure 9, we show the effects of varying the number of colors in the output palette. Our automatic method introduces fewer isolated colors than the nearest method, while looking less washed out than the cubic method. As the palette size shrinks, our method is better able to preserve salient colors, such as the green in the turban. Our method's palette assignment also improves the visibility of the eyes and does not color any of the face pink.

614

615

616

617

618

610

620

621

622



Figure 9: Varying the palette size (output images are $64{\times}58).$

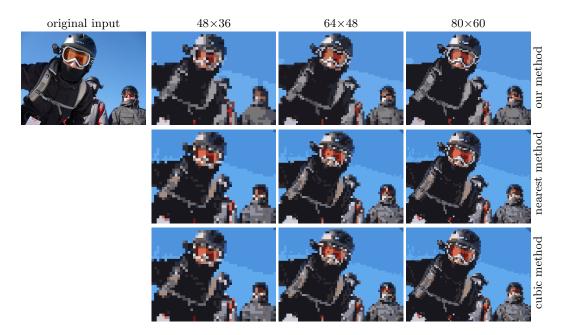


Figure 10: Varying the output resolution (palette has 16 colors).

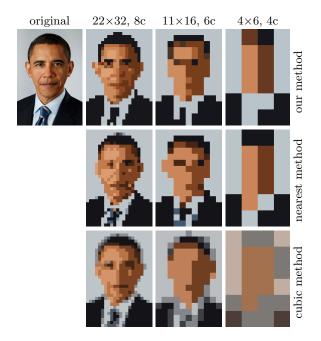


Figure 11: Examples of very low resolution and small palette sizes.

Similar results are seen in Figure 10 when we 626 vary the output resolution. Again we see that the 627 cubic method produces washed-out images and the 628 654 nearest method has a speckled appearance. At all 629 resolutions, our method preserves features such as 655 630 the goggles more faithfully, and consistently chooses 656 631 more accurate skin tones for the faces, whereas both $^{\ \ 657}$ 632 naive methods choose gray. 633

Using our automated algorithm, the image of ⁶⁵⁹ 634 660 Barack Obama is recognizable even at extremely 635 small output resolutions and palette sizes (Fig-661 636 ure 11). At 22×32 and 11×16 , our method more ⁶⁶² 637 clearly depicts features such as the eyes while col-663 638 oring regions such as the hair and tie more consis-639 tently. At 11×16 , the nearest method produces a ₆₆₅ 640 result that appears to distort facial features, while 666 641 the cubic method produces a result that "loses" the $_{\rm _{667}}$ 642 eyes. At 6×4 , results are very abstract, but our ₆₆₈ 643 method's output could still be identified as a per- 669 644 son or as having originated from the input. 645 670

In Figure 12, we compare our automated output 671 to manual results created by expert pixel artists. 672 647 While our results exhibit the same advantages seen 673 648 in the previous figures over the naive methods, they 674 649 650 do not match the results made by artists. Expert 675 artists are able to heavily leverage their human 676 651 understanding of the scene to emphasize and de- 677 652 emphasize features and make use of techniques such 678 653

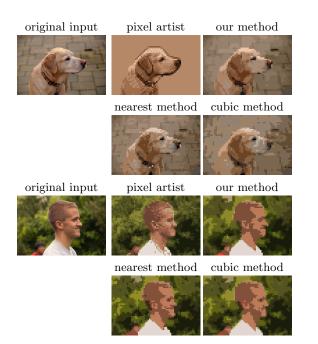


Figure 12: Comparing to the work of expert pixel artists $(64 \times 43).$ The results generate from our method and the naive methods use 16 colors in the first example, 12 in the second. The pixel artists use 8 colors in the first example. 11 in the second.

as dithering and edge highlighting. While there are many existing methods to automatically dither an image, at these resolutions the decision on when to apply dithering is nontrivial, and uniform dithering can introduce undesired textures to surfaces (such as skin).

Figure 13 contains additional results computed using various input images. Overall, our automated approach is able to produce less noisy, sharper images with a better selection of colors than the naive techniques we compared against.

To verify our analysis, we conducted a formal user study with 100 subjects using Amazon Mechanical Turk. Subjects were shown the original image and the results of our automated method and the two naive methods. The results were scaled to approximately 256 pixels along their longest dimension using nearest neighbor upsampling, so that users could clearly see the pixel grid. We asked subjects the question, "Which of the following best represents the image above?" Subjects responded by choosing a result image. The stimulus sets and answer choices were randomized to remove bias. The study consisted of the example images and parameters shown in our paper, excluding the results gen-

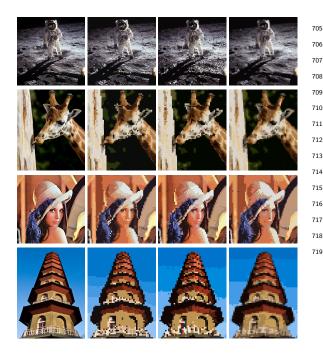


Figure 13: Additional results at various resolution and palette sizes. Columns (left to right): input image, output of our algorithm, output of the nearest method, output of the cubic method.

erated using user input, and each stimulus was duplicated four times (sixty total).

We accounted for users answering randomly by 681 eliminating the results of any subject who gave 682 inconsistent responses (choosing the same answer 683 for less than three of the four duplicates) on more 684 than a third of the stimuli. This reduced the 685 number of valid responses to forty. The final results 686 show that users choose our results 41.49% of the 687 time, the nearest method 34.52% of the time, and 688 the cubic method 23.99% of the time. Using a one-689 way analysis of variance (ANOVA) on the results, 690 we found a p value of 2.12×10^{-6} , which leads 691 us to reject the null hypothesis that subjects all 692 chose randomly. Using Tukey's range test we found 693 that our automated method is significantly different 694 from the nearest method with a 91% confidence 695 interval, and from the cubic method with a 99%696 confidence interval. While we acknowledge that the 697 question asked is difficult one given that it is an 698 aesthetic judgment, we believe the results of this 699 study still show subjects prefer the results of our 700 method over the results of either naive method. 701

We also received feedback from three expert pixel
artists on our automated method; each concluded
that the automated results are, in general, an im-

provement over the naive approaches. Ted Martens, creator of the Pixel Fireplace, said that our algorithm "chooses better colors for the palette, groups them well, and finds shapes better." Adam Saltsman, creator of Canabalt and Flixel, characterized our results as "more uniform, more reasonable palette, better forms, more readable." Craig Adams, art director of Superbrothers: Sword & Sworcery EP, observed that "essential features seem to survive a bit better [and] shapes seem to come through a bit more coherently. I think the snowboarder's goggles are the clearest example of an essential shape—the white rim of the goggle—being coherently preserved in your process, while it decays in the 'naive' process."



Figure 14: The original pixel art image (© Nintendo Co., Ltd.) is converted to a vectorized version using Kopf and Lischinski's method [22]. The vectorized version is then converted back to a pixelated version using our automated method and the two naive methods.

In Section 2, we mentioned that the method of 720 Kopf and Lischinski [22] is essentially the inverse 721 process of our method; it takes a pixel art piece and 722 converts it to a smooth, vectorized output. To see 723 how well our method actually serves as the inverse 724 process, we took the vectorized output of their 725 method as the input of our automated algorithm, 726 setting the size of output image and palette to the 727 same as their input. The results, compared to 728 those of the naive methods, are shown in Figure 14. 729 Visually our method appears to outperform either 730 naive method, and obtains a result that is similar 731 to their original input. To quantify the effectiveness 732 of our method, we took the sum of the Euclidean 733 distance in LAB space of every pixel between our 734 output and their input. We did the same for 735 the naive methods. We found the total error for 736 our method, the nearest method and the cubic 737 method to be 1.63×10^3 , 3.05×10^3 and 9.84×10^3 , 738 respectively. In other words, our method has 47% 739 less error than the nearest method, and 83.5% less 740 error than the cubic method. 741

775 In Figure 15, we present results from our method 742 using an the importance map as an additional in-743 put to the algorithm. The results are closer to those 777 744 778 created by expert pixel artist. Figure 15(left) allo-745 cates more colors in the palette to the face of the 779 746 person, similar to the manual result in Figure 12.⁷⁸⁰ 747 Figure 15(right) also shows an improvement over 748 the non-weighted results in Figure 9. For both ex-749 783 amples, the importance map emphasizes the face 750 and de-emphasizes the background: consequently. 751 more colors are allocated to the face in each exam-752 ple at the expense of the background. 753

In Figure 16, we demonstrate the advantage of ⁷⁸⁷ 754 788 allowing the user to also place pixel and palette 755 constraints during our iterative process. In Fig-789 756 790 ure 16(top), the user provides minimal, but effective 757 changes, such as improving the jawline, and remov-758 ing a skin color in favor of a blue in the palette for 759 the tie. They also introduce a simple striped pat-760 794 tern into the tie, which still represents the original 761 image, but no longer has a direct correspondence. 795 762 796 and would not be achievable by our algorithm alone. 763 These changes took less than a minute to make. 764

The improved result in Figure 16(middle row) is 765 achieved by interleaving multiple steps of user con-766 straints and iterations of our algorithm. The user 767 768 is also able to incorporate the advanced techniques 799 observed in Figure 12 such as dithering and edge 800 769 highlighting, which are not natively built into our 801 770 algorithm. 771

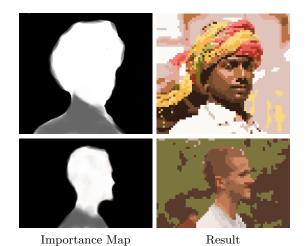


Figure 15: Results using an importance map. $(top)64 \times 58$, 16 colors (bottom) 64×43 , 12 colors

The image in Figure 16(bottom) is a failure case for our automated algorithm, due to the lighting and high variation in the background. However, even with this initially poor output, by interleaving the iterative process with user constraints (such as restricting the background to a single color) the results are significantly improved.

Finally, while not the direct goal of our work, we briefly mention a secondary application of our method, image *posterization*. This artistic technique uses just a few colors (originally motivated by the use of custom inks in printing) and typically seeks a vectorized output. Adobe Illustrator provides a feature called LiveTrace that can posterize the image in Figure 2(a), yielding Figure 17(a)with only 6 colors. To our knowledge, little research has addressed this problem, though it shares some aesthetic concerns with the artistic thresholding approach of Xu and Kaplan [23]. A simple modification to our optimization that omits the smoothing step (Figure 6-left) and then colors the original image via associated superpixels gives us Figure 17(b), which makes a more effective starting point for vectorization. The resulting Figure 17(c) offers improved spatial and color fidelity, based on a good faith effort to produce a similar style in Illustrator.

7. Conclusion, Limitations and Future work

We present a multi-step iterative process that simultaneously solves for a mapping of features and a reduced palette to convert an input image to a pixelated output image. Our method demonstrates sev-

772

773

774

776

782

784

785

791

792

793

797

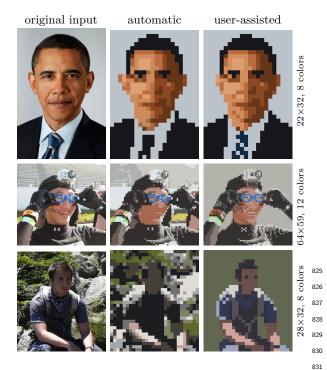


Figure 16: The results of the automatic method compared to the results obtained by integrating user input into the iterative process with our interface. Note that the user ⁸³³ can choose to make only a few key edits (top), or they 834 can leverage their understanding of the image to drastically 835 improve images that are otherwise difficult for the automated algorithm (bottom).

eral advantages over the naive methods. Our results 839 803 have a more vibrant palette, retain more features ⁸⁴⁰ 804 of the original image, and produce a cleaner output ⁸⁴¹ 805 with fewer artifacts. While the naive methods pro-842 806 duce unidentifiable results at very low resolutions 843 807 and palette sizes, our approach is still able to cre-844 808 ate iconic images that conjure the original image. 809 Thus our method makes a significant step towards 846 810 the quality of images produced by pixel artists. 847 811

848 Nevertheless, our method has several limitations 812 849 which we view as potential avenues for future 813 850 research. While pixel artists view the results of our 814 851 automated algorithm as an improvement, they also 815 express the desire to have a greater control over the 816 final product. 817

To address these concerns, we implemented sev-818 eral controls that allow the user to give as much or $_{854}$ 819 little feedback into the automated process as they 820 821 desire. By incorporating an importance map we 855 give the user the ability to guide the palette selec-822 tion, and by giving the user the ability to provide 857 823 pixel and palette constraints and interleave them 858 824

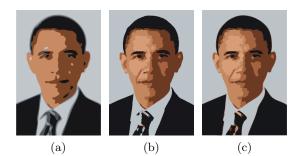


Figure 17: (a) vectorized photo posterized with Illustrator (6 colors). (b) Optimization without Laplacian smoothing, coloring associated input pixels (6 colors). (c) Vectorizing b in Illustrator yields similar style with better spatial and color fidelity than a.

with our algorithm, we remove the gap between the manual and automated methods of producing pixel art.

The results of combining these user constraints into our iterative algorithm are encouraging. For future work, we wish to expand on our proposed method and user controls to increase the interaction between the automated algorithm and the user. Our goal is to create a complete system that incorporates the speed and power of an automated method to assist artists in their entire process, without restricting the control of the artists over the final result.

As such, the next step is to explore how the user's feedback can help inform more advanced pixel art techniques in our algorithm, such as those that would produce edge highlighting and dithering. We'd also like to look into ways of automatically performing palette transfers, which would allow potential applications of this work to include, for example, reproduction of an image in repeating tiles like Lego, or design for architectural facades composed of particular building materials like known shades of brick. Currently, our algorithm is limited to working with colors that are similar to the original image due to the nature of how we minimize error, and such an application is not possible without the user applying a large number of constraints.

Acknowledgments

We thank the anonymous reviewers for their helpful feedback. We also wish to thank pixel artists Craig Adams, Ted Martens, and Adam Saltsman for their advice and comments. This research is

852

853

832

836

837

supported in part by the Sloan Foundation, the 918 859 NSF (CAREER Award CCF-06-43268 and grants 919 860 920 IIS-09-16129, IIS-10-48948, IIS-11-17257, CMMI-861 921 11-29917, IIS-09-16845, DGE-05-49115), and gen-862 922 erous gifts from Adobe, Autodesk, Intel, mental 863 923 images, Microsoft, NVIDIA, Side Effects Software, 924 864 925 and the Walt Disney Company. The following copy-865 926 righted images are used with permission: Figure 1 866 927 by Alice Bartlett, Figure 9 by Louis Vest, Figure 13 928 867 (giraffe) by Paul Adams, and Figure 13 (pagoda) 929 868 930 by William Warby. The pixel art in Figure 12 is 869 931 copyright Adam Saltsman (top) and Ted Martens 932 870 (bottom). 933 871 934 935

References 872

877

878

879

880

881

882

883

884 885

886

887

888

889

891

892

893

894

895

896

897 898

899

900

- [1] Vermehr K, Sauerteig S, Smital S. eboy. 873 http://hello.eboy.com; 2012. 874
- Marr D, Hildreth E. Theory of edge detection. Inter-875 national Journal of Computer Vision 1980;. 876
 - 942 [3] DeCarlo D, Finkelstein A, Rusinkiewicz S, Santella A. 943 Suggestive contours for conveying shape. ACM Trans 944 Graph 2003:22(3):848-55. 945
 - Judd T, Durand F, Adelson EH. Apparent ridges for [4]line drawing. ACM Trans Graph 2007;26(3):19-
 - Gooch B, Coombe G, Shirley P. [5]Artistic vision: painterly rendering using computer vision techniques. In: Non-Photorealistic Animation and Rendering (NPAR). ISBN 1-58113-494-0; 2002, p. 83-90. doi:http://doi.acm.org/10.1145/508530.508545. URL http://doi.acm.org/10.1145/508530.508545.
- [6]DeCarlo D, Santella A. Stylization and abstraction of photographs. ACM Trans Graph 2002;21:769-76. doi:http://doi.acm.org/10.1145/566654.566650. URL 890 http://doi.acm.org/10.1145/566654.566650.
 - Winnemöller H, Olsen SC, Gooch B. Real-time video [7] abstraction. ACM Trans Graph 2006;25:1221-6.
 - Achanta R, Shaji A, Smith K, Lucchi A, Fua P. [8] Süsstrunk S. SLIC Superpixels. Tech. Rep.; IVRG CVLAB; 2010.
 - [9] Rose K. Deterministic annealing for clustering, compression, classification, regression, and related optimization problems. Proceedings of the IEEE 1998;86(11):2210-39. doi:10.1109/5.726788.
- [10] Sharma G, Trussell HJ. Digital color imaging. IEEE 901 Transactions on Image Processing 1997;6:901-32. 902
- Gerstner T, DeCarlo D, Alexa M, Finkelstein A, [11] 903 Gingold Y, Nealen A. Pixelated image abstraction. 904 In: Proceedings of the International Symposium on 905 906 Non-Photorealistic Animation and Rendering (NPAR). 907 2012..
- [12]Gervautz M, Purgathofer W. Graphics gems. chap. A 908 simple method for color quantization: octree quantiza-909 tion. ISBN 0-12-286169-5; 1990, p. 287-93. 910
- Heckbert P. Color image quantization for frame buffer [13]911 912 display. SIGGRAPH Comput Graph 1982;16:297-307.
- [14]Orchard M, Bouman C. Color quantization of images. 913 IEEE Trans on Signal Processing 1991;39:2677–90. 914
- Wu X. Color quantization by dynamic programming 915 [15]and principal analysis. ACM Trans Graph 1992;11:348-916 72.917

- [16] Stollnitz EJ, Ostromoukhov V, Salesin DH. Reproducing color images using custom inks. In: Proceedings of SIGGRAPH. ISBN 0-89791-999-8; 1998, p. 267-74.
- Shi J, Malik J. Normalized cuts and image segmenta-[17]tion. IEEE Transactions on Pattern Analysis and Machine Intelligence 1997;22:888-905.
- Vedaldi A, Soatto S. Quick shift and kernel methods [18]for mode seeking. In: In European Conference on Computer Vision, volume IV. 2008, p. 705-18.
- [19]Levinshtein A, Stere A, Kutulakos KN, Fleet DJ, Dickinson SJ, Siddiqi K. Turbopixels: Fast superpixels using geometric flows. 2009.
- [20]Kirkpatrick S, Gelatt CD, Vecchi MP. Optimization by simulated annealing. Science 1983;220:671-80.
- Puzicha J, Held M, Ketterer J, Buhmann JM, Fellner [21]DW. On spatial quantization of color images. IEEE Transactions on Image Processing 2000;9:666-82
- [22]Kopf J, Lischinski D. Depixelizing pixel art. ACM Trans Graph 2011:30(4):99-
- Xu J, Kaplan CS, Mi X. Computer-generated paper-[23]cutting. In: Proceedings of Pacific Graphics. 2007, p. 343 - 50.
- [24]MacQueen JB. Some methods for classification and analysis of multivariate observations. In: Proceedings of 5th Berkeley Symposium on Mathematical Statistics and Probability. 1967, p. 281–97.
- Forsyth DA, Ponce J. Computer Vision: A Modern [25]Approach. Prentice Hall; 2002.

936

937

938

939

940