Outline

- Knowledge-based agents
- Wumpus world
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - forward chaining
  - backward chaining
  - resolution
Knowledge bases

Knowledge base = set of sentences in a formal language

Declarative approach to building an agent (or other system):
TELL it what it needs to know
Then it can ASK itself what to do—answers should follow from the KB
Agents can be viewed at the knowledge level
i.e., what they know, regardless of how implemented
Or at the implementation level
i.e., data structures in KB and algorithms that manipulate them

A simple knowledge-based agent

function KB-AGENT(percept) returns an action

static: KB, a knowledge base
        t, a counter, initially 0, indicating time

TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
action ← ASK(KB, MAKE-ACTION-QUERY(t))
TELL(KB, MAKE-ACTION-SENTENCE(action, t))
t ← t + 1
return action

The agent must be able to:
Represent states, actions, etc.
Incorporate new percepts
Update internal representations of the world
Deduce hidden properties of the world
Deduce appropriate actions
Wumpus World PEAS description

Performance measure
gold +1000, death -1000
-1 per step, -10 for using the arrow

Environment
Squares adjacent to wumpus are smelly
Squares adjacent to pit are breezy
Glitter iff gold is in the same square
Shooting kills wumpus if you are facing it
Shooting uses up the only arrow
Grabbing picks up gold if in same square
Releasing drops the gold in same square

Sensors Breeze, Glitter, Smell
Actuators Left turn, Right turn,
Forward, Grab, Release, Shoot

Wumpus world characterization

Observable??
Wumpus world characterization

Observable?? No—only local perception

Deterministic??

- Observable?? No—only local perception
- Deterministic?? Yes—outcomes exactly specified
- Episodic??
Wumpus world characterization

Observable?? No—only local perception
Deterministic?? Yes—outcomes exactly specified
Episodic?? No—sequential at the level of actions
Static??

Wumpus world characterization

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Static?? Yes—Wumpus and Pits do not move
Discrete??
Wumpus world characterization

Observable?? No—only local perception
Deterministic?? Yes—outcomes exactly specified
Episodic?? No—sequential at the level of actions
Static?? Yes—Wumpus and Pits do not move
Discrete?? Yes
Single-agent?? Yes—Wumpus is essentially a natural feature
Exploring a wumpus world

Exploring a wumpus world
Exploring a wumpus world

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Other tight spots

Breeze in (1,2) and (2,1)
⇒ no safe actions

Assuming pits uniformly distributed,
(2,2) has pit w/ prob 0.86, vs. 0.31

Smell in (1,1)
⇒ cannot move

Can use a strategy of coercion:
  shoot straight ahead
  wumpus was there ⇒ dead ⇒ safe
  wumpus wasn’t there ⇒ safe

Logic in general

Logics are formal languages for representing information
such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the “meaning” of sentences;
  i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x \geq 2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number $y$

$x + 2 \geq y$ is true in a world where $x = 7$, $y = 1$

$x + 2 \geq y$ is false in a world where $x = 0$, $y = 6$
Entailment

Entailment means that one thing follows from another:

\[ KB \models \alpha \]

Knowledge base \( KB \) entails sentence \( \alpha \)
if and only if
\( \alpha \) is true in all worlds where \( KB \) is true
E.g., the KB containing “the Giants won” and “the Reds won”
entails “Either the Giants won or the Reds won”
E.g., \( x + y = 4 \) entails \( 4 = x + y \)
Entailment is a relationship between sentences (i.e., syntax)
that is based on semantics
Note: brains process syntax (of some sort)

Models

Logicians typically think in terms of models, which are formally
structured worlds with respect to which truth can be evaluated

We say \( m \) is a model of a sentence \( \alpha \) if \( \alpha \) is true in \( m \)

\( M(\alpha) \) is the set of all models of \( \alpha \)

Then \( KB \models \alpha \) if and only if \( M(KB) \subseteq M(\alpha) \)
E.g. \( KB = \) Giants won and Reds won
\( \alpha = \) Giants won
Entailment in the wumpus world

Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

Consider possible models for ?'s assuming only pits

3 Boolean choices  ⇒  8 possible models
$KB = \text{wumpus-world rules + observations}$

$\alpha_1 = \text{“[1,2] is safe”, } KB \models \alpha_1$, proved by model checking
$KB = \text{wumpus-world rules + observations}$

$\alpha_2 = \"[2,2] is safe\", KB \not\models \alpha_2$
Inference

$KB \vdash_i \alpha = \text{sentence } \alpha \text{ can be derived from } KB \text{ by procedure } i$

Consequences of $KB$ are a haystack; $\alpha$ is a needle.
Entailment = needle in haystack; inference = finding it

**Soundness:** $i$ is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

**Completeness:** $i$ is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
That is, the procedure will answer any question whose answer follows from what is known by the $KB$.

Propositional logic: Syntax

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols $P_1, P_2$ etc are sentences

If $S$ is a sentence, $\neg S$ is a sentence (negation)

If $S_1$ and $S_2$ are sentences, $S_1 \land S_2$ is a sentence (conjunction)

If $S_1$ and $S_2$ are sentences, $S_1 \lor S_2$ is a sentence (disjunction)

If $S_1$ and $S_2$ are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If $S_1$ and $S_2$ are sentences, $S_1 \iff S_2$ is a sentence (biconditional)
Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. \( P_{1,2} \quad P_{2,2} \quad P_{3,1} \)

\[ \begin{array}{c|c|c} \end{array} \]

true \quad true \quad false

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model \( m \):

\[ \begin{align*}
\neg S & \text{ is true iff } S \text{ is false} \\
S_1 \land S_2 & \text{ is true iff } S_1 \text{ is true and } S_2 \text{ is true} \\
S_1 \lor S_2 & \text{ is true iff } S_1 \text{ is true or } S_2 \text{ is true} \\
S_1 \Rightarrow S_2 & \text{ is true iff } S_1 \text{ is false or } S_2 \text{ is true} \\
i.e., & \text{ is false iff } S_1 \text{ is true and } S_2 \text{ is false} \\
S_1 \iff S_2 & \text{ is true iff } S_1 \text{ is true and } S_2 \text{ is true} \\
\end{align*} \]

Simple recursive process evaluates an arbitrary sentence, e.g.,

\[ \neg P_{1,2} \land (P_{2,2} \lor P_{3,1}) = \text{true} \land (\text{false} \lor \text{true}) = \text{true} \land \text{true} = \text{true} \]

Truth tables for connectives

<table>
<thead>
<tr>
<th>( P )</th>
<th>( Q )</th>
<th>( \neg P )</th>
<th>( P \land Q )</th>
<th>( P \lor Q )</th>
<th>( P \Rightarrow Q )</th>
<th>( P \iff Q )</th>
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Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

\[ \neg P_{1,1} \]
\[ \neg B_{1,1} \]
\[ B_{2,1} \]

“Pits cause breezes in adjacent squares”

\[ B_{1,1} \iff (P_{1,2} \lor P_{2,1}) \]
\[ B_{2,1} \iff (P_{1,1} \lor P_{2,2} \lor P_{3,1}) \]

“A square is breezy if and only if there is an adjacent pit”
Truth tables for inference

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<th>$B_{1,1}$</th>
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<th>$P_{1,2}$</th>
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Inference by enumeration

Depth-first enumeration of all models is sound and complete

function TT-ENTAILS?(KB, $\alpha$) returns true or false

symbols $\leftarrow$ a list of the proposition symbols in $KB$ and $\alpha$

return TT-CHECK-ALL($KB, \alpha, symbols, []$)

function TT-CHECK-ALL($KB, \alpha, symbols, model$) returns true or false

if EMPTY?(symbols) then
    if PL-TRUE?(KB, model) then return PL-TRUE?($\alpha, model$)
    else return true
else do
    $P \leftarrow$ FIRST(symbols); rest $\leftarrow$ REST(symbols)
    return TT-CHECK-ALL($KB, \alpha, rest, EXTEND(P, true, model)$ and
                         TT-CHECK-ALL($KB, \alpha, rest, EXTEND(P, false, model)$)

$O(2^n)$ for $n$ symbols; problem is co-NP-complete
Logical equivalence

Two sentences are logically equivalent iff true in same models:
\[ \alpha \equiv \beta \text{ if and only if } \alpha \models \beta \text{ and } \beta \models \alpha \]

\[
\begin{align*}
(\alpha \land \beta) & \equiv (\beta \land \alpha) \text{ commutativity of } \land \\
(\alpha \lor \beta) & \equiv (\beta \lor \alpha) \text{ commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv ((\alpha \land (\beta \land \gamma)) \text{ associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv ((\alpha \lor (\beta \lor \gamma)) \text{ associativity of } \lor \\
\neg(\neg\alpha) & \equiv \alpha \text{ double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv (\neg\beta \Rightarrow \neg\alpha) \text{ contraposition} \\
(\alpha \Rightarrow \beta) & \equiv (\neg\alpha \lor \beta) \text{ implication elimination} \\
(\alpha \iff \beta) & \equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) \text{ biconditional elimination} \\
\neg(\alpha \land \beta) & \equiv (\neg\alpha \lor \neg\beta) \text{ de Morgan} \\
\neg(\alpha \lor \beta) & \equiv (\neg\alpha \land \neg\beta) \text{ de Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) \text{ distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) \text{ distributivity of } \lor \text{ over } \land
\end{align*}
\]

Validity and satisfiability

A sentence is valid if it is true in all models,
e.g., True, \( A \lor \neg A \), \( A \Rightarrow A \), \( (A \land (A \Rightarrow B)) \Rightarrow B \)

Validity is connected to inference via the Deduction Theorem:
\[ KB \models \alpha \text{ if and only if } (KB \Rightarrow \alpha) \text{ is valid} \]

A sentence is satisfiable if it is true in some model
e.g., \( A \lor B \), \( C \)

A sentence is unsatisfiable if it is true in no models
e.g., \( A \land \neg A \)

Satisfiability is connected to inference via the following:
\[ KB \models \alpha \text{ if and only if } (KB \land \neg\alpha) \text{ is unsatisfiable} \]
i.e., prove \( \alpha \) by reductio ad absurdum
Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules
– Legitimate (sound) generation of new sentences from old
– Proof = a sequence of inference rule applications
  Can use inference rules as operators in a standard search alg.
– Typically require translation of sentences into a normal form

Model checking
truth table enumeration (always exponential in \( n \))
improved backtracking, e.g., Davis–Putnam–Logemann–Loveland
heuristic search in model space (sound but incomplete)
e.g., min-conflicts-like hill-climbing algorithms

Forward and backward chaining

Horn Form (restricted)
KB = conjunction of Horn clauses
Horn clause =
  ◊ proposition symbol; or
  ◊ (conjunction of symbols) ⇒ symbol
E.g., \( C \land (B \Rightarrow A) \land (C \land D \Rightarrow B) \)

Modus Ponens (for Horn Form): complete for Horn KBs

\[
\frac{\alpha_1, \ldots, \alpha_n, \quad \alpha_1 \land \cdots \land \alpha_n \Rightarrow \beta}{\beta}
\]

Can be used with forward chaining or backward chaining.
These algorithms are very natural and run in linear time
Forward chaining

Idea: fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until query is found

\[ P \Rightarrow Q \]
\[ L \land M \Rightarrow P \]
\[ B \land L \Rightarrow M \]
\[ A \land P \Rightarrow L \]
\[ A \land B \Rightarrow L \]
\[ A \]
\[ B \]

Forward chaining algorithm

function PL-FC-ENTAILS?(KB, q) returns true or false

local variables: count, a table, indexed by clause, initially the number of premises
inferred, a table, indexed by symbol, each entry initially false
agenda, a list of symbols, initially the symbols known to be true

while agenda is not empty do
    \[ p \leftarrow \text{POP(agenda)} \]
    unless inferred[p] do
        inferred[p] \leftarrow true
        for each Horn clause \( c \) in whose premise \( p \) appears do
            decrement count[c]
            if count[c] = 0 then do
                if HEAD[c] = q then return true
                PUSH(HEAD[c], agenda)
        return false
Forward chaining example

Q
P
M
L
B
A

Forward chaining example

Q
P
M
L
B
A
Forward chaining example
Forward chaining example
Forward chaining example
Proof of completeness

FC derives every atomic sentence that is entailed by $KB$

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model $m$, assigning true/false to symbols
3. Every clause in the original $KB$ is true in $m$
   
   **Proof:** Suppose a clause $a_1 \land \ldots \land a_k \Rightarrow b$ is false in $m$
   
   Then $a_1 \land \ldots \land a_k$ is true in $m$ and $b$ is false in $m$
   
   Therefore the algorithm has not reached a fixed point!
4. Hence $m$ is a model of $KB$
5. If $KB \models q$, $q$ is true in every model of $KB$, including $m$

---

Backward chaining

Idea: work backwards from the query $q$:

- to prove $q$ by BC,
  - check if $q$ is known already, or
  - prove by BC all premises of some rule concluding $q$

Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal

1) has already been proved true, or
2) has already failed
Backward chaining example

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Backward chaining example

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Backward chaining example

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Backward chaining example

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Backward chaining example
Backward chaining example

Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing, e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving, e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be much less than linear in size of KB
Resolution

Conjunctive Normal Form (CNF—universal)

\[ \text{conjunction of disjunctions of literals} \]

E.g., \((A \lor \neg B) \land (B \lor \neg C \lor \neg D)\)

Resolution inference rule (for CNF): complete for propositional logic

\[
\ell_1 \lor \cdots \lor \ell_k, \quad m_1 \lor \cdots \lor m_n
\]

\[
\ell_1 \lor \cdots \lor \ell_{i-1} \lor \ell_{i+1} \lor \cdots \lor \ell_k \lor m_1 \lor \cdots \lor m_{j-1} \lor m_{j+1} \lor \cdots \lor m_n
\]

where \(\ell_i\) and \(m_j\) are complementary literals. E.g.,

\[
P_{1,3} \lor P_{2,2}, \quad \neg P_{2,2} \quad \frac{}{P_{1,3}}
\]

Resolution is sound and complete for propositional logic

---

Conversion to CNF

\(B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})\)

1. Eliminate \(\Leftrightarrow\), replacing \(\alpha \Leftrightarrow \beta\) with \((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)\).

\((B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})\)

2. Eliminate \(\Rightarrow\), replacing \(\alpha \Rightarrow \beta\) with \(\neg \alpha \lor \beta\).

\((\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg (P_{1,2} \lor P_{2,1}) \lor B_{1,1})\)

3. Move \(\neg\) inwards using de Morgan’s rules and double-negation:

\((\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})\)

4. Apply distributivity law (\(\lor\) over \(\land\)) and flatten:

\((\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})\)
**Resolution algorithm**

Proof by contradiction, i.e., show $KB \land \neg \alpha$ unsatisfiable

```plaintext
function PL-RESOLUTION(KB, \alpha) returns true or false

clauses ← the set of clauses in the CNF representation of $KB \land \neg \alpha$
new ← {} 

loop do
    for each $C_i, C_j$ in clauses do
        resolvents ← PL-RESOLVE($C_i, C_j$)
        if resolvents contains the empty clause then return true
        new ← new \cup resolvents
        if new \subseteq clauses then return false
    clauses ← clauses \cup new
```

**Resolution example**

$KB = (B_{1,1} \leftrightarrow (P_{1,2} \lor P_{2,1})) \land \neg B_{1,1} \land \alpha = \neg P_{1,2}$
Logical agents apply inference to a knowledge base to derive new information and make decisions.

Basic concepts of logic:
- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundness: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses.
Resolution is complete for propositional logic.
Propositional logic lacks expressive power.