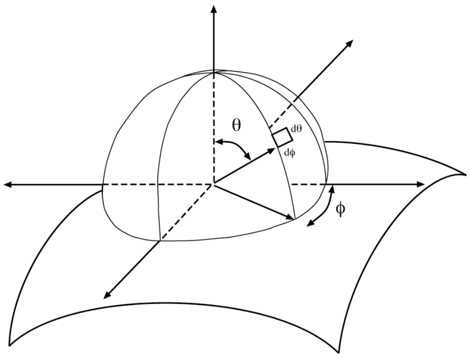
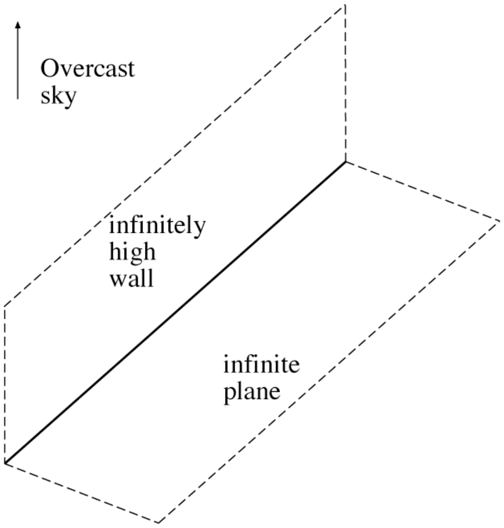
Radiometry

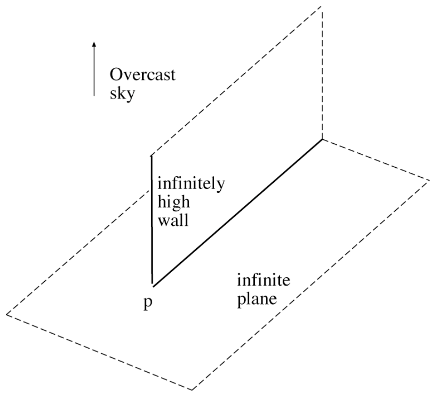
- Questions:
  - how “bright” will surfaces be?
  - what is “brightness”?
    - measuring light
    - interactions between light and surfaces
- Core idea - think about light arriving at a surface
- around any point is a hemisphere of directions
- Simplest problems can be dealt with by reasoning about this hemisphere

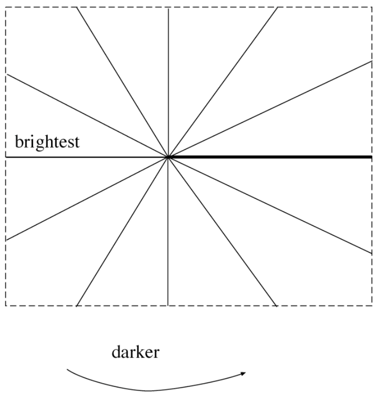


Lambert’s wall



More complex wall





Foreshortening

- **Principle:** two sources that look the same to a receiver must have the same effect on the receiver.
- **Principle:** two receivers that look the same to a source must receive the same amount of energy.

- “look the same” means produce the same input hemisphere (or output hemisphere)

- **Reason:** what else can a receiver know about a source but what appears on its input hemisphere? (ditto, swapping receiver and source)
- **Crucial consequence:** a big source (resp. receiver), viewed at a glancing angle, must produce (resp. experience) the same effect as a small source (resp. receiver) viewed frontally.

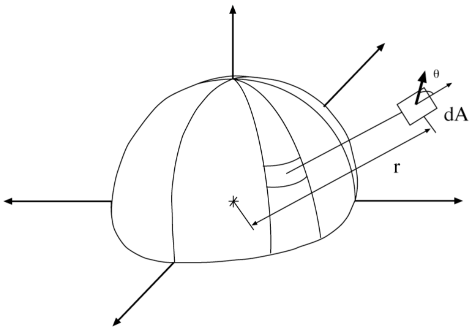
Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
- The solid angle subtended by a patch area dA is given by

$$d\omega = \frac{dA\cos\vartheta}{r^2}$$

- Another useful expression:

$$d\omega = \sin\vartheta\left(d\vartheta\right)\left(d\phi\right)$$



Measuring Light in Free Space

- **Desirable property:** in a vacuum, the relevant unit does not go down along a straight line.
- How do we get a unit with this property? Think about the power transferred from an infinitesimal source to an infinitesimal receiver.
- We have

  total power leaving s to r =

  total power arriving at r from s
- Also:

  Power arriving at r is proportional to:
  - solid angle subtended by s at r (because if s looked bigger from r, there'd be more)
  - foreshortened area of r (because a bigger r will collect more power

Radiance

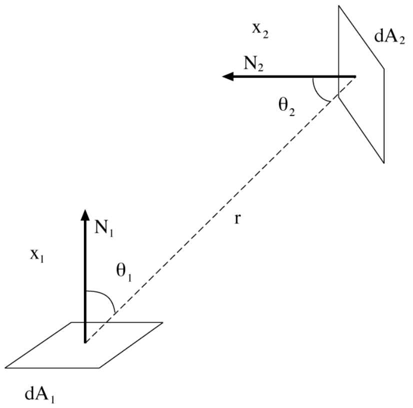
- All this suggests that the light transferred from source to receiver should be measured as:

  Radiant power per unit foreshortened area per unit solid angle
- This is radiance
- Units: watts per square meter per steradian ($wm^{-2}sr^{-1}$)
- Usually written as:

$$L(\underline{x},\vartheta,\varphi)$$

- Crucial property:
  In a vacuum, radiance leaving p in the direction of q is the same as radiance arriving at q from p
  - which was how we got to the unit

Radiance is constant along straight lines



- Power 1->2, leaving 1:

$$L(\underline{x}_1,\vartheta,\varphi)(dA_1\cos\vartheta_1)\left(\frac{dA_2\cos\vartheta_2}{r^2}\right)$$

- Power 1->2, arriving at 2:

$$L(\underline{x}_2,\vartheta,\varphi)(dA_2\cos\vartheta_2)\left(\frac{dA_1\cos\vartheta_1}{r^2}\right)$$

- But these must be the same, so that the two radiances are equal

Irradiance

- How much light is arriving at a surface?
- Sensible unit is *Irradiance*
- Incident power per unit area *not foreshortened*
- This is a function of incoming angle.
- A surface experiencing radiance L(x,θ,φ) coming in from dω experiences irradiance

$$L(\underline{x},\vartheta,\varphi)\cos\vartheta d\omega$$

- Crucial property:
  Total power arriving at the surface is given by adding irradiance over all incoming angles --- this is why it's a natural unit
- Total power is

$$\int_{\Omega} L(\underline{x},\vartheta,\varphi)\cos\vartheta\sin\vartheta d\vartheta d\varphi$$

Light at surfaces

- Many effects when light strikes a surface -- could be:
  - absorbed
  - transmitted
    - skin
  - reflected
    - mirror
  - scattered
    - milk
  - travel along the surface and leave at some other point
    - sweaty skin
- Assume that
  - surfaces don't fluoresce
    - e.g. scorpions, washing powder
  - surfaces don't emit light (i.e. are cool)
  - all the light leaving a point is due to that arriving at that point

The BRDF

- Assuming that
  - surfaces don't fluoresce
  - surfaces don't emit light (i.e. are cool)
  - all the light leaving a point is due to that arriving at that point
- Can model this situation with the Bidirectional Reflectance Distribution Function (BRDF)
- the ratio of the radiance in the outgoing direction to the incident irradiance

$$\rho_{bd}\left(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,\right)=\frac{L_o\left(\underline{x},\vartheta_o,\varphi_o\right)}{L_i\left(\underline{x},\vartheta_i,\varphi_i\right)\cos\vartheta_i d\omega}$$

BRDF

- Units: inverse steradians (sr^{-1})
- Symmetric in incoming and outgoing directions - this is the *Helmholtz reciprocity principle*
- Radiance leaving a surface in a particular direction:
  - add contributions from every incoming direction

$$\int_{\Omega}\rho_{bd}\left(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i,\right)L_i\left(\underline{x},\vartheta_i,\varphi_i\right)\cos\vartheta_i d\omega_i$$

Suppressing Angles - Radiosity

- In many situations, we do not really need angle coordinates
  - e.g. cotton cloth, where the reflected light is not dependent on angle
- Appropriate radiometric unit is radiosity
  - total power leaving a point on the surface, per unit area on the surface (Wm^{-2})
  - note that this is independent of the direction
- Radiosity from radiance?
  - sum radiance leaving surface over all exit directions, multiplying by a cosine because this is per unit area not per unit foreshortened area

$$B(\underline{x}) = \int_{\Omega} L_o(\underline{x},\vartheta,\varphi)\cos\vartheta d\omega$$

Radiosity

- Important relationship:
  - radiosity of a surface whose radiance is independent of angle (e.g. that cotton cloth)

$$\begin{aligned} B(\underline{x}) &= \int_{\Omega} L_o(\underline{x},\vartheta,\varphi)\cos\vartheta d\omega \\ &= L_o(\underline{x})\int_{\Omega}\cos\vartheta d\omega \\ &= L_o(\underline{x})\int_0^{\pi/2}\int_0^{2\pi}\cos\vartheta\sin\vartheta d\varphi d\vartheta \\ &= \pi L_o(\underline{x}) \end{aligned}$$

Suppressing the angles in the BRDF

- BRDF is a very general notion
  - some surfaces need it (underside of a CD; tiger eye; etc)
  - very hard to measure
    - ,illuminate from one direction, view from another, repeat
  - very unstable
    - minor surface damage can change the BRDF
    - e.g. ridges of oil left by contact with the skin can act as lenses
- for many surfaces, light leaving the surface is largely independent of exit angle
  - surface roughness is one source of this property

Directional hemispheric reflectance

- Directional hemispheric reflectance:
  - the fraction of the incident irradiance in a given direction that is reflected by the surface (whatever the direction of reflection)
  - unitless, range is 0-1
- Note that DHR varies with incoming direction
  - eg a ridged surface, where left facing ridges are absorbent and right facing ridges reflect.

$$\rho_{dh}(\vartheta_i,\varphi_i) = \frac{\int_{\Omega} L_o(\underline{x},\vartheta_o,\varphi_o)\cos\vartheta_o d\omega_o}{L_i(\underline{x},\vartheta_i,\varphi_i)\cos\vartheta_i d\omega_i}$$

$$= \int_{\Omega} \rho_{bd}(\underline{x},\vartheta_o,\varphi_o,\vartheta_i,\varphi_i)\cos\vartheta_o d\omega_o$$

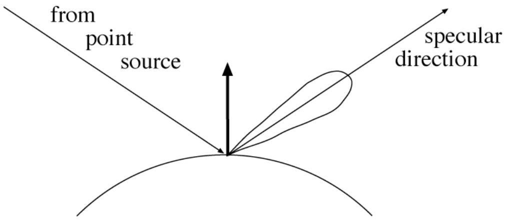
Lambertian surfaces and albedo

- For some surfaces, the DHR is independent of illumination direction too
  - cotton cloth, carpets, matte paper, matte paints, etc.
- For such surfaces, radiance leaving the surface is independent of angle
- Called **Lambertian surfaces** (same Lambert) or **ideal diffuse surfaces**
- Use radiosity as a unit to describe light leaving the surface
- DHR is often called **diffuse reflectance**, or **albedo**
- for a Lambertian surface, BRDF is independent of angle, too.
- Useful fact:

$$\rho_{brdf} = \frac{\rho_d}{\pi}$$

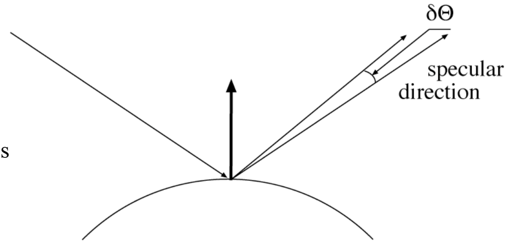
Specular surfaces

- Another important class of surfaces is specular, or mirror-like.
  - radiation arriving along a direction leaves along the specular direction
  - reflect about normal
  - some fraction is absorbed, some reflected
  - on real surfaces, energy usually goes into a lobe of directions
  - can write a BRDF, but requires the use of funny functions



Phong’s model

- There are very few cases where the exact shape of the specular lobe matters.
- Typically:
  - very, very small --- mirror
  - small -- blurry mirror
  - bigger -- see only light sources as “specularities”
  - very big -- faint specularities
- Phong’s model
  - reflected energy falls off with



$$\cos^{n}\left(\delta\vartheta\right)$$

Lambertian + specular

- Widespread model
  - all surfaces are Lambertian plus specular component
- Advantages
  - easy to manipulate
  - very often quite close true
- Disadvantages
  - some surfaces are not
    - e.g. underside of CD’s, feathers of many birds, blue spots on many marine crustaceans and fish, most rough surfaces, oil films (skin!), wet surfaces
  - Generally, very little advantage in modelling behaviour of light at a surface in more detail -- it is quite difficult to understand behaviour of L+S surfaces