Problem 1. (15 points) Consider a relational schema R(A,B,C) and its relation instance as follows:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>b2</td>
<td>c1</td>
</tr>
<tr>
<td>10</td>
<td>b2</td>
<td>c2</td>
</tr>
<tr>
<td>11</td>
<td>b4</td>
<td>c1</td>
</tr>
<tr>
<td>11</td>
<td>b3</td>
<td>c4</td>
</tr>
<tr>
<td>13</td>
<td>b1</td>
<td>c1</td>
</tr>
<tr>
<td>14</td>
<td>b3</td>
<td>c4</td>
</tr>
</tbody>
</table>

a. Given the above instance, which of the following dependencies may hold in the relation R(A,B,C)? If the dependency cannot hold, explain why by specifying the tuples that cause the violation (one counterexample is sufficient).

1. A→B
2. B→C
3. C→B
4. B→A
5. C→A
6. BC→A
7. AC→B
8. AC→C

b. Does the above relation have potential candidate keys? If it does, list all of them and explain why they are potential candidate keys. If it does not, why not?

Problem 2. (15 points) List all the functional dependencies (involving the attributes A, B, C) that are satisfied by the following relation instance:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Problem 3. (20 points) Consider the relational schema R(A,B,C,D) and the set of functional dependencies: F= { B→C, CD→A }.(Assume the set of integers is the domain of each attribute.) Do the following:

1. Find a non-empty instance of R (i.e., give a number of rows) that satisfies every FD in F.
2. Can you find an instance that satisfies every FD in F, but does not satisfy the FD B→D? If yes, give the instance. If not, explain why.
3. Can you find an instance that satisfies every FD in \( F \), but does not satisfy the FD \( BD \rightarrow A \)? If yes, give the instance. If not, explain why.

**Problem 4 (20 points)**

Consider the relation \( R \), which has attributes that hold schedules of courses and sections at a university:

\[
R = \{ \text{CourseNo}, \text{SecNo}, \text{OfferingDept}, \text{Credit-Hours}, \text{CourseLevel}, \text{InstructorSSN}, \text{Semester}, \text{Year}, \text{Days_Hours}, \text{RoomNo}, \text{NoOfStudents} \}.
\]

Suppose that the following functional dependencies hold on \( R \):

\[
\begin{align*}
\{ \text{CourseNo} \} & \rightarrow \{ \text{OfferingDept}, \text{Credit-Hours}, \text{CourseLevel} \} \\
\{ \text{CourseNo}, \text{SecNo}, \text{Semester}, \text{Year} \} & \rightarrow \{ \text{Days_Hours}, \text{RoomNo}, \text{NoOfStudents}, \text{InstructorSSN} \} \\
\{ \text{RoomNo}, \text{Days_Hours}, \text{Semester}, \text{Year} \} & \rightarrow \{ \text{InstructorSSN}, \text{CourseNo}, \text{SecNo} \}
\end{align*}
\]

a. Determine all the candidate keys for \( R \). Show your steps.

b. Is the relation \( R \) in BCNF? If not, show all the FDs that violate the BCNF, and explain why.

c. If \( R \) is not in BCNF, use the proper algorithm to find a lossless-join decomposition of \( R \) (under the given FDs) into BCNF relational schemas. Show your decomposition using a decomposition tree; for each step, indicates the functional dependency used. Does your decomposition preserve dependencies? Motivate your answer.

**Problem 5. (15 points)**

Consider the relational schema \( R \) \((A,B,C,D,E,G)\) and the set of functional dependencies \( F = \{ AB \rightarrow C, AC \rightarrow B, AD \rightarrow E, B \rightarrow D, BC \rightarrow A, E \rightarrow G \} \).

1. List all the candidate keys for \( R \).

2. Is relation \( R \) in BCNF? If not, show all the FDs that violate the BCNF condition and explain why.

**Problem 6. (15 points)** Consider the relational schema \( R \) \((A,B,C,D,E)\) and the set of functional dependencies \( F = \{ A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A \} \).

1. Consider the decomposition \((A,D,E)\) and \((B,C,E)\). Is it a lossless-join decomposition of \( R \) (under \( F \))? Explain your answer.

2. Consider the decomposition \((A,B,E)\) and \((B,C,D)\). Is it a lossless-join decomposition of \( R \) (under \( F \))? Explain your answer.

3. Using the proper algorithm, find a lossless-join decomposition of \( R \) (under \( F \)) into BCNF relation schemas.