Solution to Exercises From 25 Feb 2004

1 An IEEE 802.5 token-ring system consists of 120 stations, one per repeater. Let \( P_l \) represent probability of link failure, \( P_r \) represent probability of repeater failure. Derive an expression for the probability that two nodes selected at random will not be able to communicate.

A station is unable to communicate on a token-ring network if (i) the node’s ring interface has failed, or, (ii) there is a break anywhere in the ring. The probability of (i) is \( P_l \), of (ii) is \( P_r \).

This is a single ring connecting 120 nodes. A failure in this network will prevent (at least) any two nodes chosen at random from communicating. What, then, is the probability of failure of the network? Consider

\[
P(\text{network does not fail}) = P(\text{no link failure}) \times P(\text{no repeater failure})
\]

\[
P(\text{network does fail}) = 1 - (1 - P_l)^{120} \times (1 - P_r)^{120}
\]

where \( n \) is the number of nodes, here 120.

Two nodes on this network will be unable to communicate if there has been a network failure. The final version of that probability is, then, \( 1 - (1 - P_l)^{120} \times (1 - P_r)^{120} \).

2 Another IEEE 802.5 token-ring system consists of three 40-repeater rings, interconnected by a transparent bridge that has an interface to each ring. Use \( P_l \) and \( P_r \) as before and let \( P_b \) represent probability of bridge failure. Derive an expression for the probability that any two stations selected at random will be unable to communicate.

For two nodes on the same ring we know the answer; see above. Here, we must additionally account for the fact that the two nodes chosen at random may be on different rings. Label the individual rings \( r_1 \), \( r_2 \), and \( r_3 \). We start by determining the probability that the two nodes we choose are on the same ring:

\[
P(\text{same}) = P(x \text{ on } r_1, y \text{ on } r_1) + P(x \text{ on } r_2, y \text{ on } r_2) + P(x \text{ on } r_3, y \text{ on } r_3)
\]

\[
= P(x \text{ on } r_1) \times P(y \text{ on } r_1) + P(x \text{ on } r_2) \times P(y \text{ on } r_2) + P(x \text{ on } r_3) \times P(y \text{ on } r_3)
\]

\[
= \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3}
\]

\[
= \frac{1}{3}
\]

Hence the probability that the two nodes we choose are on different rings is \( P(\text{different}) = \frac{2}{3} \).

The probability that two nodes on the same ring cannot communicate is as determined in question 1, above:

\[
P(\text{failsame}) = 1 - (1 - P_l)^{40} \times (1 - P_r)^{40}
\]

In the case where the nodes are on different rings, ask what could cause failure: failure of either ring or the bridge. The probability of the former we know, so we can say:

\[
P(\text{success}) = P(\text{no failure in } r_x) \times P(\text{no failure in } r_y) \times P(\text{no failure in bridge})
\]

\[
= \left( 1 - P_l ight)^j \times \left( 1 - P_r ight)^{j} \times \left[ 1 - \left( 1 - P_l ight)^k \times \left( 1 - P_r ight)^k \times P_b \right]
\]

where \( r_x \) denotes the ring on which node \( x \) appears, \( r_y \) the node on which node \( y \) appears, \( j \) is the size (number of nodes) of the ring node \( x \) is on, and \( k \) is the size of the ring node \( y \) is on; here, \( j = k = 40 \). The probability of failure will simply be \( P(\text{faildifferent}) = 1 - P(\text{success}) \).
We can combine all these now to produce the complete expression of the probability of failure:

\[ P(\text{two nodes cannot communicate}) = P_{\text{same}} \times P_{\text{failsame}} + P_{\text{different}} \times P_{\text{faildifferent}} \]

3. Evaluate 1 and 2 for \( P_l = 0.002, P_r = 0.005, P_b = 0.005 \).

For (1),

\[
P(\text{failure}) = 1 - (1 - P)_{\text{120}} \times (1 - P)_{\text{120}}
\]

\[
= 1 - (1 - 0.002)_{\text{120}} \times (1 - 0.005)_{\text{120}}
\]

\[
= 1 - (0.786 \times 0.548) = 0.569
\]

For (2),

\[
P(\text{failure}) = P_{\text{same}} \times P_{\text{failsame}} + P_{\text{different}} \times P_{\text{faildifferent}}
\]

\[
= \left[ \frac{1}{3} \times (1 - (1 - 0.002)^{40} \times (1 - 0.005)^{40}) \right] + \left[ \frac{2}{3} \times (1 - (1 - 0.002)^{40} \times (1 - 0.005)^{40} \times (1 - 0.005)) \right]
\]

\[
= \left[ \frac{1}{3} \times (1 - 0.923 \times 0.818) \right] + \left[ \frac{2}{3} \times (1 - 0.923 \times 0.818 \times 0.005) \right]
\]

\[
= (1/3 \times 0.245) + (2/3 \times 0.996)
\]

\[
= 0.082 + 0.664 = 0.746
\]

4. Describe in words the advantages and disadvantages of each system.

The single ring model has the advantage that it does not involve a bridge, but it does mean that any failure in the ring results in all 120 nodes being unable to communicate. Breaking the 120 nodes into three bridge-connected rings at least means that a failure in any one of the three rings need not impact communication among nodes in the other two, hence only 40 of the 120 nodes is impacted. What would the impact of a bridge failure be?